 Name: solution

December 12, 2022

**INSTRUCTIONS**

Begin each problem in the space provided.

Write on the front side of the paper only. **Work appearing on the back side of the paper will not be graded.** Extra paper is available in the exam room.

If your solution does not follow a logical thought process, it will be assumed to be in error.

**You must turn in your crib sheet with your exam.**

In which section are you enrolled?

◯ Hess - 10:30-11:20 am

◯ Hess - 12:30-1:20 pm

◯ Akin - 3:30-4:20 pm

**PROBLEM No. 1** (25 points)

Problem 1 consists of 10 questions. Each question is worth 2.5 points.

(a) A spur gear set has a gear ratio of $m_G = 4$ and a diametral pitch of $P = 10$ teeth per inch. The center-to-center distance between the gear and the pinion is 5 inches. What are the pitch diameters of the pinion ($d_P$) and the gear ($d_G$)?

$d_P =$ 2 in

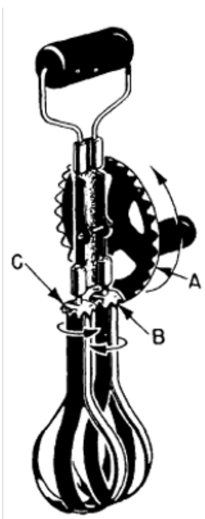
$d_G =$ 8 in

$r_p + r_G = 5\,in$

$4 = \frac{d_G}{d_p} = \frac{r_G}{r_p} \rightarrow r_G = 4r_p$

$r_p + 4r_p = 5in \rightarrow r_p = 1in$

(b) In the old-fashioned hand-cranked egg beater shown, the large gear ($A$) rotates in the vertical plane and drives a smaller gear ($B$). Gear $B$ rotates in the horizontal plan and drives gear $C$, where gears $B$ and $C$ are the same size.




What type of gear is gear $B$?

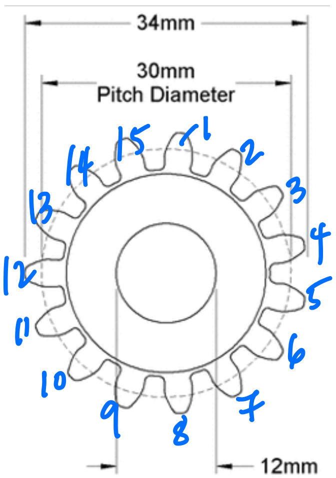
- ○ miter gear
- ○ screw gear
- ● bevel gear
- ○ worm gear

gears on $\perp$ axes

(c) Select all of the correct statements below.

- ☒ Gear interference can be corrected by undercutting the gear teeth
- ☐ Undercutting strengthens gear teeth
- ☐ Gears should be designed so that exactly one pair of teeth are in contact at all times
- ☒ The radial component of the contact force in gears serves no useful purpose

(d) Determine the module for the gear shown below. The gear is drawn to scale.




$$m = \frac{30 \text{ mm}}{15 \text{ teeth}} = 2 \text{ mm}$$

(e) A four-stage compound spur gear train for an overall ratio of approximately 892:1. What is the approximate gear ratio in each stage?

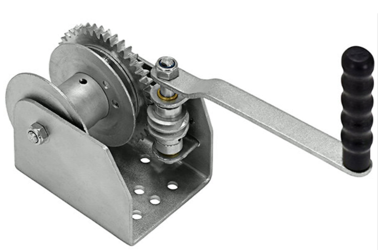
- ○ 3.5
- ○ 4.5
- ● 5.5
- ○ 6.5

$\sqrt[4]{892} = 5.465 \rightarrow$ choose 5.5

(f) Compared to coarse-thread fasteners, which of the following statements are true?

- ☐ Fine thread fasteners are more resistant to threads stripping
- ☒ Fine thread fasteners have better adjustment accuracy
- ☐ Fine thread fasteners have faster assembly speed
- ☒ Fine thread fasteners are better applications where vibrations may be present

(g) Winches are devices used to pull heavy loads. They usually have a rope or cable that is wound around a drum, which is turned by a handle or crank.



What are two reasons that worm gears are well-suited to be used in a winch?

1. worm gears are self locking
2. high gear ratios are possible

(h) Bolt threads strip due to cyclic loading.

- ○ True
- ● False

(i) Torque control is the most accurate way to control preload in a threaded fastener.

- ○ True
- ● False

(j) What is the most interesting thing you have learned this semester? It does not need to be something learned in ME 354.

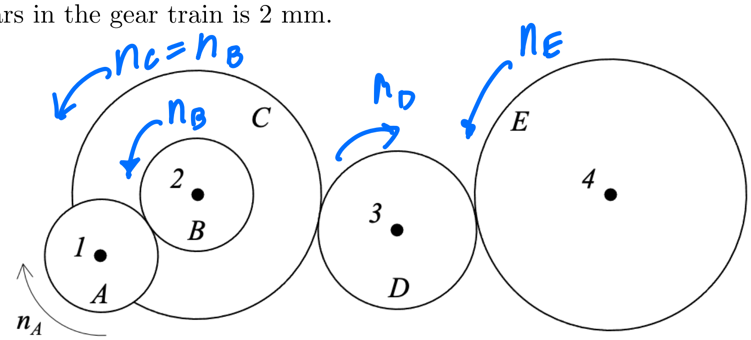
it depends.

**PROBLEM No. 2** (25 points)

The compound gear train shown below includes 5 gears located on 4 shafts. Gears $B$ and $C$ are on the common shaft 2.

Gear $A$ rotates clockwise (CW) at 1500 rpm and has a power input of 5 kW.

The module of all gears in the gear train is 2 mm.




Determine the following.

a) Complete the following table. Justify your answers and/or include free body diagrams where applicable.

b) The train value, $e$.

c) The power delivered by gear $E$.

d) Identify any gears that function as idlers in the gear train. Justify your answer.

| Gear | # of Teeth | Diameter (mm) | Direction | Speed (rpm) | Torque (N-m) | Transmitted load (N) |
|---|---|---|---|---|---|---|
| $A$ | 15 | 30 | CW | 1500 | 31.8 | 2122 |
| $B$ | 15 | 30 | CCW | 1500 | 31.8 | 2122 |
| $C$ | 50 | 100 | CCW | 1500 | 31.8 | 636.6 |
| $D$ | 25 | 50 | CW | 3000 | 15.9 | 636.6 |
| $E$ | 60 | 120 | CCW | 1250 | 38.2 | 636.6 |

a) $d = \#\text{ of teeth} \cdot \text{module}$

$d_A = 15 \cdot 2\text{mm} = 30 \text{ mm}$

$$w_B = \frac{N_A}{N_B} w_A = \frac{15}{15} \cdot 1500 = 1500 \text{ rpm}$$

**PROBLEM No. 2** (continued)

$\omega_C = \omega_B$ because B and C on common shaft

$$\omega_D = \frac{N_C}{N_D}\omega_C = \frac{50}{25}\cdot 1500 = 3000 \text{ rpm}$$

$$\omega_E = \frac{N_D}{N_E}\omega_D = \frac{25}{60}\cdot 3000 = 1250 \text{ rpm}$$

$$\text{torque}_A = \frac{\text{power}}{\omega} = \frac{5000 \text{ Nm/s}}{1500\frac{\text{rev}}{\text{min}}\cdot\frac{2\pi}{\text{rev}}\frac{\text{min}}{60\text{s}}} = 31.8 \text{ N-m}$$

$$W^t_A = \frac{T_A}{r_A} = \frac{31.8 \text{ N-m}}{0.015 \text{ m}} = 2122 \text{ N}$$

$$W^t_B = W^t_A$$

$$T_B = W^t_B r_B = 2122\text{N}\cdot 0.015\text{m} = 31.8 \text{ N-m}$$

B and C are on common shaft $\rightarrow T_B = T_C$

$$W^t_C = \frac{T_C}{r_C} = \frac{31.8 \text{ N-m}}{0.05 \text{ m}} = 636.6 \text{ N} = W^t_D$$

conserved in gear set.

$$T_D = W^t_D r_D = 636.6\text{N}\cdot 0.025 \text{ m} = 15.9 \text{ N-m}$$

$$T_E = W^t_E r_E = 636.6\text{N}\cdot 0.06\text{m} = 38.2 \text{ N-m}$$

b) $$\frac{\omega_A}{\omega_E} = \frac{1500}{1250} = 1.2$$

-or- $$\frac{T_E}{T_A} = \frac{38.2 \text{ N-m}}{31.8 \text{ N-m}} = 1.2$$

—or—

$$e = \pm \frac{\#\text{ of driven gears}}{\#\text{ of driving gears}} = - \frac{N_B \cancel{N_D} N_E}{N_A N_C \cancel{N_D}} = \frac{15 \cdot 60}{15 \cdot 50} = 1.2$$

3 meshes →
e is negative → gear E rotates opposite direction of gear A

all three methods work to find the train ratio.

c) $Power_E = T_E \omega_E = 38.2\,N \cdot m \cdot 1250 \frac{rev}{min} \cdot \frac{2\pi}{rev} \cdot \frac{min}{60s} = 5\,kW$

→ power is conserved thru the gear train

d) gear D functions as the idler because it cancels from calculation of e (see part b)

**PROBLEM No. 3** (20 points)

An external gearset consists of two spur gears. The pinion has 15 teeth and drives a 45 tooth gear.

The gears have a 20° pressure angle and a 1-inch face width. The diametral pitch is 6 teeth/inch.

The gears are grade 1 steel, through-hardened at 200 Brinell made to No. 6 quality standards.

A pinion life of $10^8$ cycles is desired, with a 90% reliability.

The pinion rotates at 3000 rpm and the input power is 10 hp.

Determine the following.

a) Complete the table on the next page for the pinion using the parameters given in the problem statement.

— Show your work and include units where applicable.

— Reference information is provided on subsequent pages.

— If needed, use the lower boundary of the shaded region when finding $Y_N$ and $Z_N$.

b) Calculate the gear bending stress ($\sigma$) and the bending factor of safety ($S_F$) using the AGMA equations.

c) Calculate the gear contact stress ($\sigma_c$) and the wear factor of safety ($S_H$) using the AGMA equations.

d) How do $S_F$ and $S_H$ found in parts (b) and (c) change if the number of gear teeth is increased to 60?

| | | | |
|---|---|---|---|
| $d_P$ | 2.5 in | $J$ | 0.25 from Fig 14-6 |
| $N_P$ | 15 teeth | $S_t$ | 28,260 psi |
| $P_d$ | 6 teeth/inch | $Y_N$ | 0.928 |
| $V$ | 1963.5 ft/min | $K_T$ | 1 assume T<250° |
| $W^t$ | 168 lbf | $K_R$ | 0.85 |
| $K_o$ | 1 | $C_p$ | 2300 $\sqrt{psi}$ |
| $K_v$ | 1.6 from Fig 14-9 | $I$ | 0.12 |
| $K_s$ | 1 | $S_c$ | 93,500 psi |
| $F$ | 1 inch | $Z_N$ | 0.879 |
| $K_m$ | 1.2225 | $C_H$ | 1 |
| $K_B$ | 1 | | |

$$d_p = \frac{N_p}{P_d} = \frac{15 \text{ teeth}}{6 \text{ teeth/in}} = 2.5 \text{ in}$$

$$V = \frac{\pi d n}{12} = \frac{\pi \cdot 2.5 \text{in} \cdot 3000 \text{ rpm}}{12} = 1963.5 \text{ ft/min}$$

$$W^t = \frac{33000 H}{V} = \frac{33000 \cdot 10}{1963.5} = 168 \text{ lbf}$$

b) $$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$= 168 \text{ lbf} \cdot 1 \cdot 1.6 \cdot 1 \cdot \frac{6 \text{ teeth/in}}{1 \text{ in}} \cdot \frac{1.2225 \cdot 1}{0.25} = 7886 \text{ psi}$$

$$S_F = \frac{S_t' Y_N / K_T K_R}{\sigma} = \frac{28260 \cdot 0.928 / 1 \cdot 0.85}{7886} = 3.9$$

c) $$\sigma_c = C_p \left( W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I} \right)^{1/2}$$

$$= 2300\sqrt{psi} \left( 168 \text{ lbf} \cdot 1 \cdot 1.6 \cdot 1 \cdot \frac{1.2225}{2.5 \text{in} \cdot 1 \text{in}} \cdot \frac{1}{0.12} \right)^{1/2}$$

$$= 76121 \text{ psi}$$

$$S_H = \frac{S_c Z_N C_H / K_T K_R}{\sigma_c} = \frac{93500 \cdot 0.879 / 1 \cdot 0.85}{76121} = 1.3$$

d) the variables impacted by the # of gear teeth (really, by the change in gear ratio) are J and I.

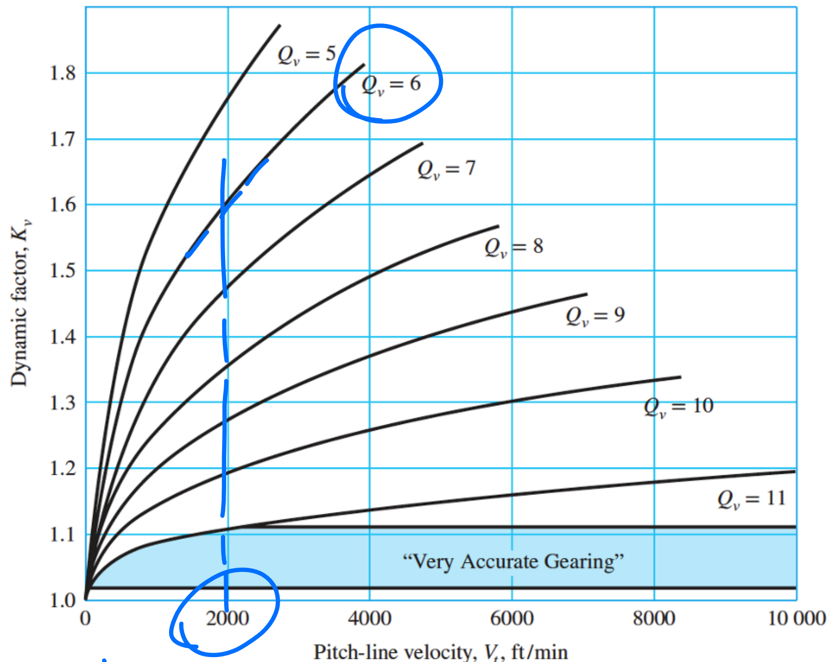
For J, because $N_P = 15$, J is independent of # of gear teeth. → $S_F$ is unchanged

For $I = \dfrac{\cos 20 \sin 20}{2 \cdot 1} \cdot \dfrac{\cancel{3}\ 4}{\cancel{3}\ 4 + 1} = 0.1285$

I being slightly larger would slightly decrease $\sigma_c$ and therefore slightly increase $S_H$.

**Figure 14–9**

Dynamic factor $K_v$. The equations to these curves are given by Equation (14–27) and the end points by Equation (14–29). *(ANSI/AGMA 2001-D04, Annex A)*




$V \approx 2000$ ft/min

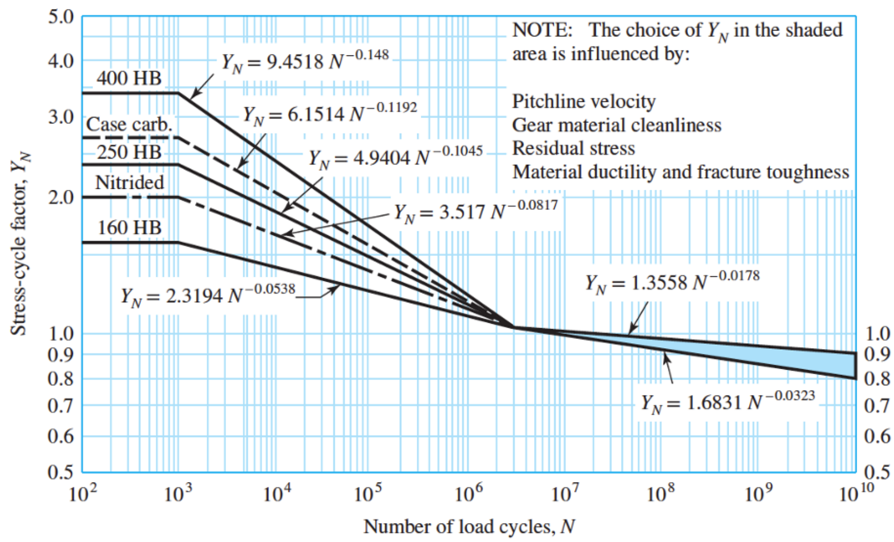



**Figure 14–14**

Repeatedly applied bending strength stress-cycle factor $Y_N$. *(ANSI/AGMA 2001-D04.)*

$$Y_N = 1.6831\,(10^8)^{-0.0323}$$
$$= 0.928$$

**Table 14–10 Reliability Factors $K_R$ ($Y_Z$)**

| Reliability | $K_R$ ($Y_Z$) |
|---|---|
| 0.9999 | 1.50 |
| 0.999 | 1.25 |
| 0.99 | 1.00 |
| 0.90 | 0.85 |
| 0.50 | 0.70 |

*Source: ANSI/AGMA 2001-D04.*

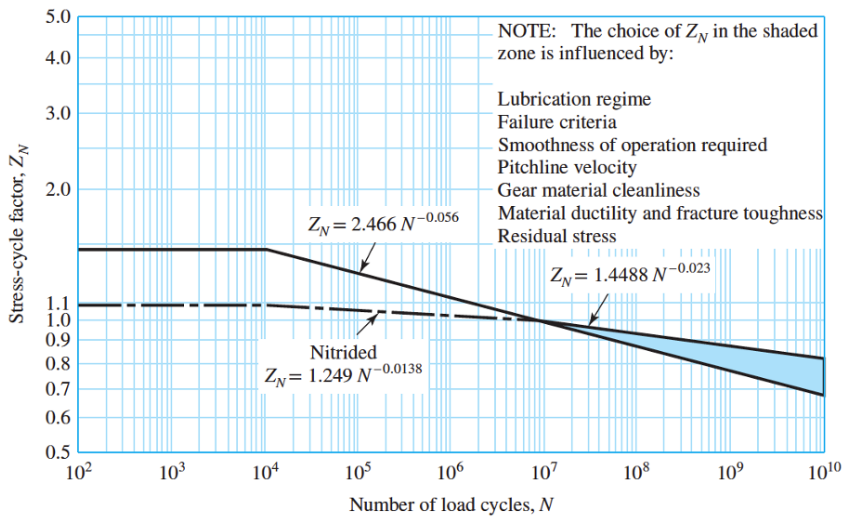



**Figure 14–15**

Pitting resistance stress-cycle factor $Z_N$. *(ANSI/AGMA 2001-D04.)*

$Z_N = 2.466(10^8)^{-0.056}$

$= 0.879$

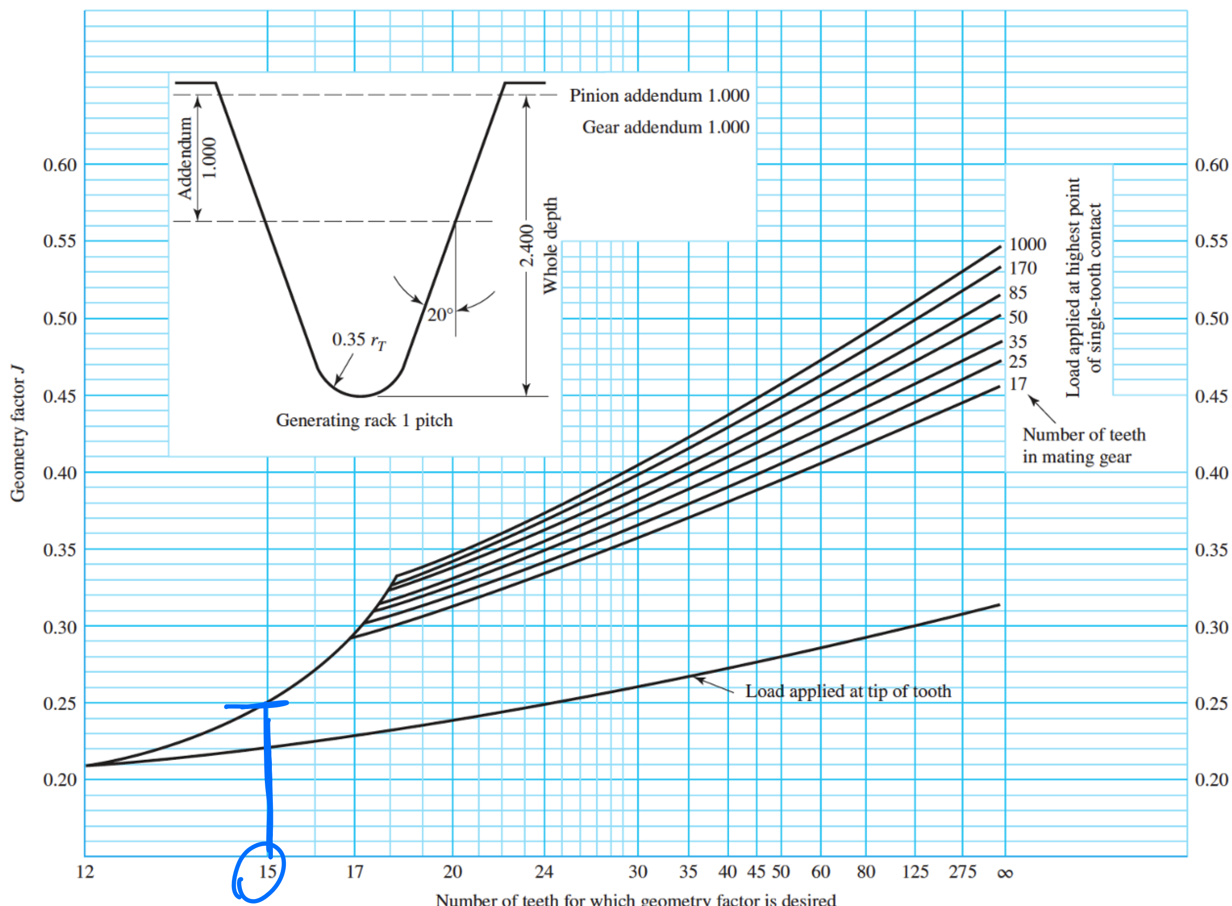



**Figure 14–6**

$N_p = 15$

Equation 14-23 for an external spur gear set where $m_N = 1$, $m_G$ is the gear ratio, and $\phi_t$ the pressure angle.

$$I = \frac{\cos\phi_t \sin\phi_t}{2\,m_N}\frac{m_G}{m_G+1} = \frac{\cos 20 \sin 20}{2\cdot 1}\cdot\frac{3}{3+1}$$

$= 0.12$

**PROBLEM No. 4** (30 points)

As shown in the figure below, two steel plates are compressed with one bolt and nut.

Two identical washers are also used under the head of the bolt and nut.

The M16×2 grade 8.8 steel bolt has $A_t = 157$ mm$^2$ and strengths $S_{ut} = 830$ MPa, $S_y = 660$ MPa, $S_p = 600$ MPa, and $S_e = 129$ MPa.

The steel washers are 2 mm thick.

The nut height is 14.8 mm.

The plates are steel. The top plate is 20 mm thick and the bottom plate is 40 mm thick.

The bolt will be re-used. $\rightarrow$ $F_i = 0.75 F_p = 0.75\, S_p A_t$

The external load applied to the bolted joint fluctuates between $P_{min} = 20$ kN and $P_{max} = 80$ kN.

Determine the following:

a) A suitable length for the bolt (round up the length to the nearest 2.5 mm)

b) The bolt stiffness, $k_b$

c) The overall stiffness of the members, $k_m$

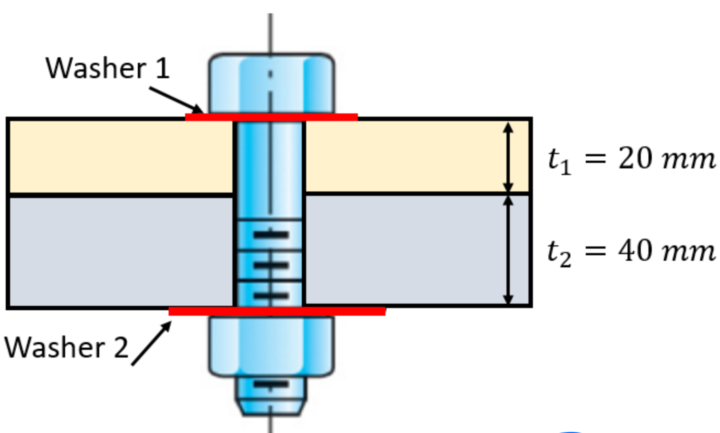
d) The joint stiffness constant, $C$

e) For the bolt determine the following:

  i The factor of safety guarding against the static loading, $n_p$

  ii The factor of safety guarding against the joint separating, $n_o$

  iii The factor of safety guarding against fatigue, $n_f$




a) $L > l + H = \overbrace{2 \cdot 2mm + 20\ mm + 40mm}^{l = 64\ mm} + 14.8\ mm$

$= 78.8\ mm \rightarrow$ round up to $80\ mm$

**PROBLEM No. 4** (continued)

b) follow process in Table 8-7.

$l = 64\ mm$

$L_T = 2d + 6\ mm = 2 \cdot 16\ mm + 6\ mm = 38\ mm$

$l_d = L - L_T = 80 - 38 = 42\ mm$

$l_t = l - l_d = 64 - 42 = 22\ mm$

$A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4} (16\ mm)^2 = 201.1\ mm^2$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{201.1 \cdot 157 \cdot 207}{201.1 \cdot 22 + 157 \cdot 42} = 593.1\ GPa \cdot mm$$

$E = 207\ GPa$ from eqn. sheet for steel.

$A_t = 157\ mm^2$ (given)

$$k_b = 593.1\ GPa \cdot mm \cdot \frac{1000\ MN/m^2}{GPa} \cdot \frac{m}{1000\ mm} = 593.1\ MN/m$$

c) $k_b = A E d \exp(B d/l)$

$$= 0.78715 \cdot 207 \cdot 10^3 \frac{MN}{m^2} \cdot 0.016\ m \cdot \exp\left(0.62873 \cdot 16/64\right)$$

$$= 3050\ MN/m$$

d) $C = \dfrac{k_b}{k_b + k_m} = \dfrac{593.1}{593.1 + 3050} = 0.162$

e) $n_p = \dfrac{S_p A_t}{CP + F_i}$   use $P_{max}$ for $P$ in static factors of safety

$F_i = 0.75\, S_p A_t = 0.75 \cdot 600 \cdot 10^6 \dfrac{N}{m^2} \cdot 157\, mm^2 \cdot \left(\dfrac{m}{1000\, mm}\right)^2$

$= 70650\ N$

$n_p = \dfrac{600 \cdot 10^6 \frac{N}{m^2} \cdot 157\, mm^2 \cdot \left(\frac{m}{1000\, mm}\right)^2}{0.162 \cdot 80000\ N + 70650\ N} = 1.1$

$n_o = \dfrac{F_i}{P(1-C)} = \dfrac{70650\ N}{80000\ N\,(1-0.162)} = 1$ (really 1.05)

$n_f = \dfrac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)}$

$\sigma_a = \dfrac{C(P_{max} - P_{min})}{2A_t} = \dfrac{0.162(80000 - 20000)N}{2 \cdot 157\, mm^2 \cdot \left(\frac{m}{1000\, mm}\right)^2}$

$= 30.96\ MPa$

$\sigma_m = \dfrac{C(P_{max} + P_{min})}{2A_t} + \sigma_i$

$= \dfrac{0.162(80000 + 20000)N}{2 \cdot 157\, mm^2 \cdot \left(\frac{m}{1000\, mm}\right)^2} + 0.75 \cdot \overset{S_p}{600}\ MPa = 501.6\ MPa$

$$n_f = \frac{129\,MPa\,(830\,MPa - 450\,MPa)}{830 \cdot 30.96 + 129\,(501.6 - 450)} = 1.5$$