1 Introduction to Machine Design

A **machine** is an arrangement of parts having the purpose of doing work.

Parts may also be referred to as machine elements or machine components.

Machine components can fail in a variety of ways.

— Become obsolete

— Wear out

— Break (or permanently deform)

Design is a decision making process.

— It is <u>not</u> inventing a new device.

— It is defining the shape, size, and material of a part such that the part will not fail under the conditions expected in service.

  - The general concept, application, and shape of a part is often known at the beginning of the design process.
  - Failure is predicted when the factor of safety is 1.

In ME 354, the machine design decision making process will include the following considerations.

1. Geometry and kinematics
2. Loads
3. Stress
4. Strength
5. Static failure
6. Fatigue failure
7. Factor(s) of safety

By the end of ME 354, you will be able to:

— Apply fundamental concepts from statics, dynamics, and mechanics of materials to the design of machine components and/or systems.

— Apply static and fatigue failure theories to the design of machine components and/or systems.

— Select standard machine components and materials.

— Apply problem-solving and communication skills through design projects

The **factor of safety** (n) is always unitless and compares a loss-of-function parameter to the maximum allowed, or predicted, value of that parameter.

$$n = \frac{loss - of - function\,parameter}{maximum\,allowed,\,or\,predicted,\,parameter\,value}$$

The parameter can be load, stress, deflection, etc.

A part can have multiple factors of safety. Each factor of safety characterizes a potential failure mode. The most likely failure mode is the one with the smallest factor of safety.

The choice of a factor of safety may be dictated by a design code or by a standard. More often, engineering judgement is required to choose a factor of safety (see Table 1).

Given the uncertainties involved, a factor of safety should not be specified to more than 1 decimal place.

Table 1: General recommendations for choosing a factor of safety.

Application	**Factor of Safety**
For use with highly reliable materials where loading and environmental conditions are not severe and where weight is an important consideration.	1.2 - 1.5
For use with reliable materials where loading and environmental conditions are not severe.	
For use with ordinary materials where loading and environmental conditions are not severe	
For use with less tried and for brittle materials where loading and environmental conditions are not severe	
For use with materials where properties are not reliable and where loading and environmental conditions are not severe, or where reliable materials are used under difficult and environmental conditions	5+

TRUE · FALSE · IT DEPENDS

Rank the following in order from lowest factor of safety to highest factory of safety.

Estimate the factor of safety for each application.

_______ A screwdriver

_______ Cast-iron wheels

_______ Structural steel

_______ Aircraft components

TRUE · FALSE · IT DEPENDS

You are designing a flywheel for an energy storage application.

The factor of safety for the flywheel is most likely:

- ◯ 1.2
- ◯ 2
- ◯ 5
- ◯ 10

2 Static Failure Analysis

2.1 Combined Stress and Identification of Critical Elements

A **critical element** is a specific location within a machine component where failure is most likely to occur.

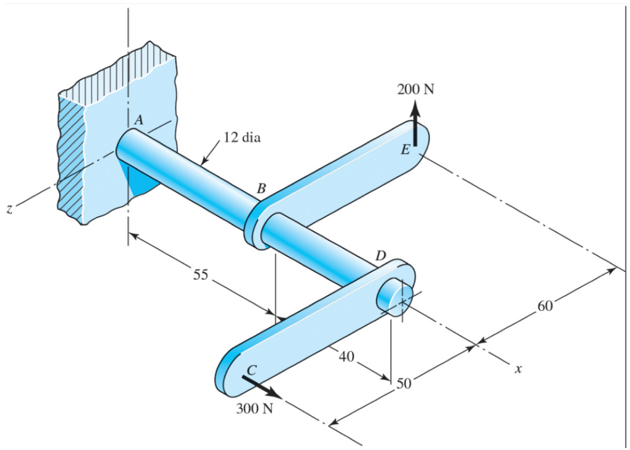
The recipe to identify the critical element(s) of a machine component is:

1. Draw the free body diagram of the machine component.
2. Solve for all reactions.
3. Determine, and sketch, the internal loads for each segment of the machine component.
   – Axial
   – Torsion
   – Bending
   – Transverse shear
4. Identify the location(s) in the machine component where the internal load(s) is/are extreme. The location(s) identified are the machine component's **critical cross-section**(s).
5. For each internal load, determine the stress distribution at the critical cross-section.
6. Identify critical element(s) on each critical cross-section.
7. Represent the state of stress for each critical element on a stress element.

A worksheet to identify the critical element(s) of a machine component is available.

TRUE · FALSE · IT DEPENDS

The cantilevered bracket shown is acted on by a 200 N force at location E and a 300 N force at location C.



The internal loads acting in segment AB are (select all that apply):

◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

The internal loads acting in segment BD are (select all that apply):

◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

The internal loads acting in segment BE are (select all that apply):

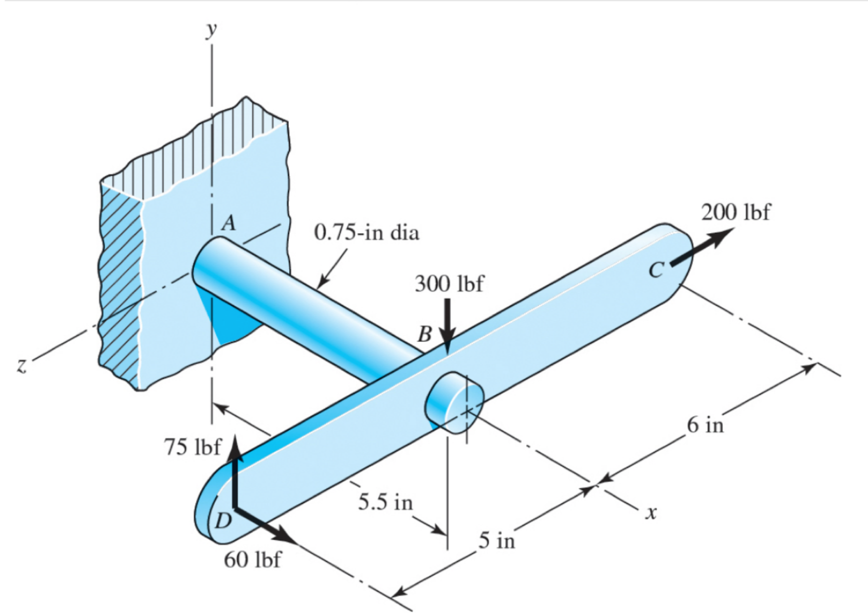
◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

The internal loads acting in segment CD are (select all that apply):

◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

TRUE · FALSE · IT DEPENDS

The cantilevered bracket shown is acted on by loads at locations B, C, and D.



The internal loads acting in segment AB are (select all that apply):

◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

The internal loads acting in segment BC are (select all that apply):

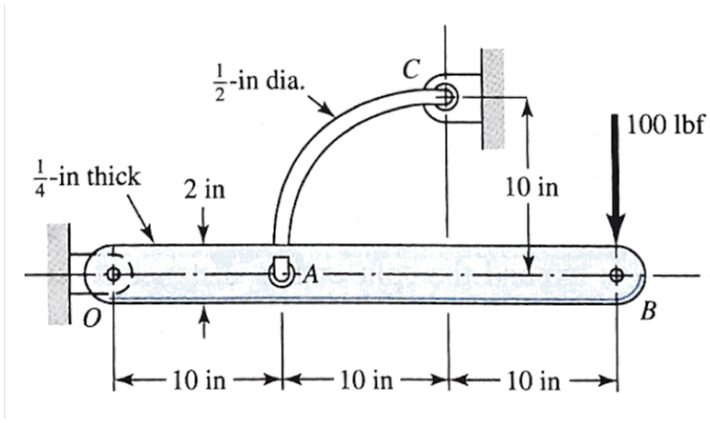
◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

The internal loads acting in segment BD are (select all that apply):

◯ Axial
◯ Torsion
◯ Bending
◯ Transverse shear

TRUE · FALSE · IT DEPENDS

The beam shown is pinned to the wall at O, is supported by member AC, and is subjected to a load at B.



The critical cross-section is located:

- ◯ Just to the right of point O.
- ◯ Just to the left of point A.
- ◯ Just to the right of point A.
- ◯ Just to the left of point B.
- ◯ Cannot be determined with the information given.

2.2 Stress elements

The state of stress at a specific location within a machine component is visualized using a **stress element**.

— Normal stresses (σ_x, σ_y, σ_z)

- Tensile normal stresses point away from the stress element.
- Compressive normal stresses point toward the stress element.

— Shear stresses (τ_{xy}, τ_{yz}, τ_{xz})

- Shear stress τ_{mn} is located on the $m-$face of the stress element and acts in the $n-$direction. The $m-$face is perpendicular to the $m-$direction.
- For an equilibrium stress state, $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, and $\tau_{xz} = \tau_{zx}$.

— A state of **plane stress** occurs when the stresses on one surface of the 3D stress element are zero.

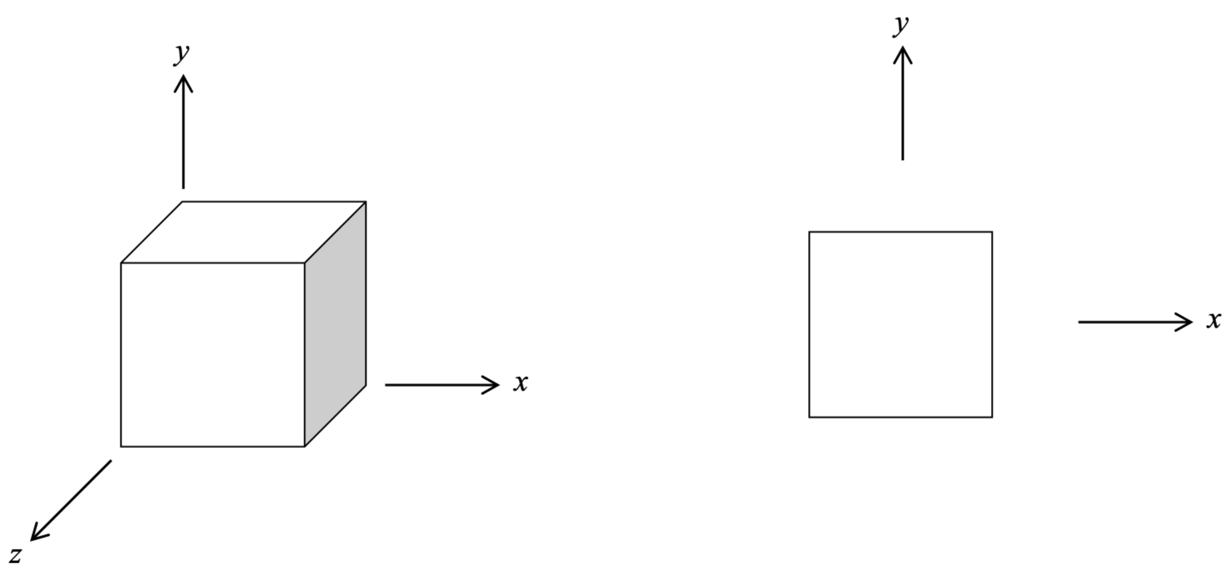


Figure 1: Stress elements are used to represent the state of stress at a specific location.

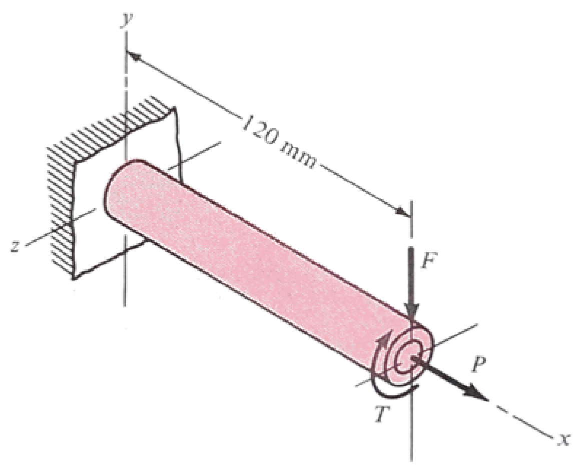
2.3 Example Problems: Combined Stress and Critical Elements

Example 1 A cantilevered tube is made of an aluminum alloy with $S_y = 276$ MPa.

The applied loads are $F = 1.75$ kN, $P = 9.0$ kN, and $T = 72$ Nm.

The tube's outer diameter is 50 mm and the inner diameter is 45 mm.

The cross-sectional area is 3.73 cm^2, $I = 10.55$ cm^4, and $J = 21.1$ cm^4.



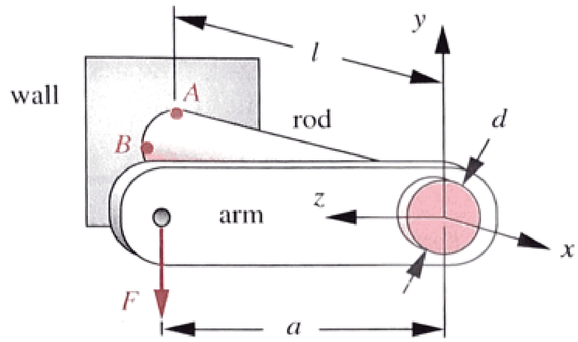
Determine the following.

(a) The critical element(s) of the tube.

(b) Show the state of stress on a stress element for each critical element.

Example 2 The bracket is made of 2024-T4 aluminum with a yield strength of 47 ksi.

The rod length (l) is 6 inches and the arm length (a) is 8 inches. The rod diameter (d) is 1.5 inches. The applied load (F) is 1000 lbf.



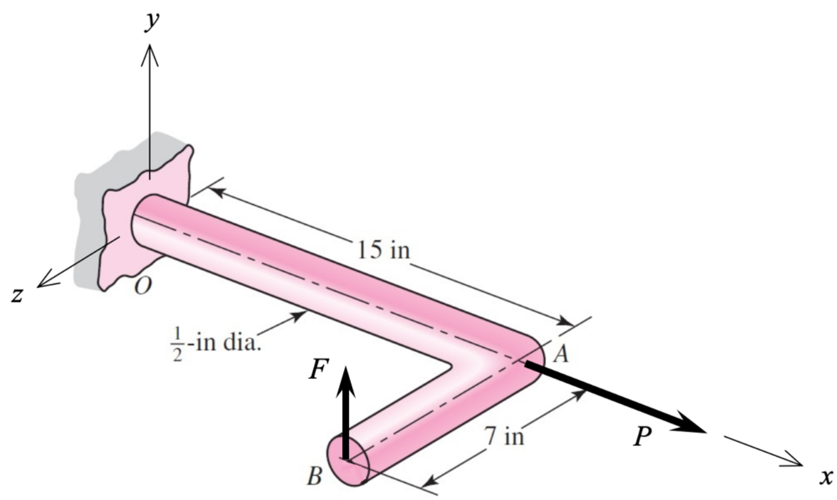
Determine the following.

(a) The critical element(s) of the bracket.

(b) Show the state of stress on a stress element for each critical element.

Example 3 The cantilevered bar is subjected to two constant loads. Force $P = 20$ lbf acts at point A and is in the $+x-$direction. Force F acts at point B and is in the $+y-$direction.

The bar is a solid rod with a constant 1/2-in diameter.



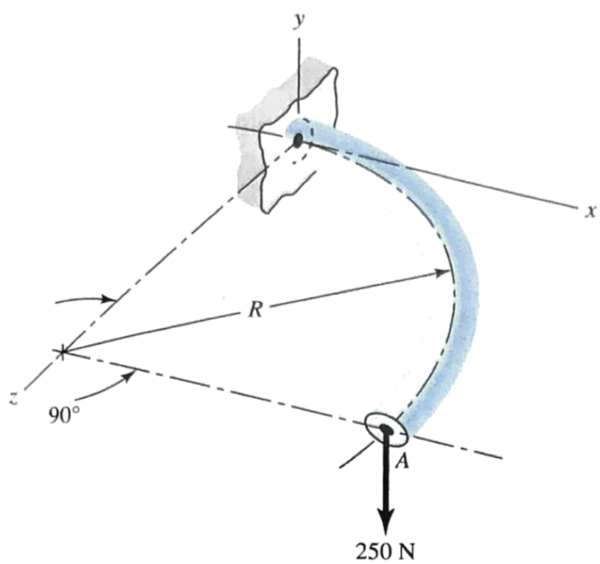
Determine the following.

(a) The critical element(s) of the bar.

(b) Show the state of stress on a stress element for each critical element.

Example 4 The curved, cantilever bar shown is acted upon by a 250 N force in the -y-direction at point A.

Radius $R = 80$ mm and the bar diameter is 6 mm.

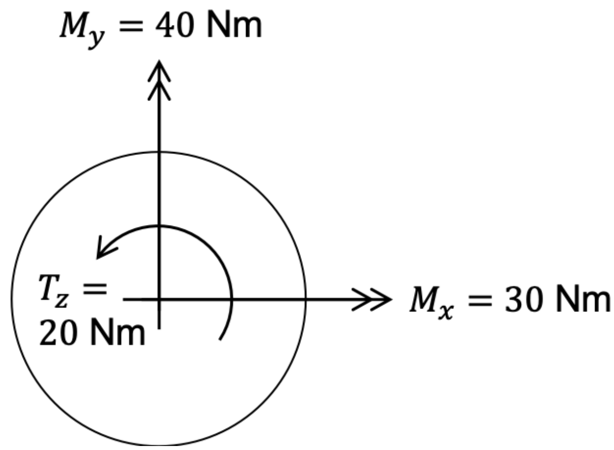


Determine the following.

(a) The critical element(s) of the bar.

(b) Show the state of stress on a stress element for each critical element.

Example 5 A particular cross-section of a part is loaded as shown in the figure below.



Determine the following.

(a) The critical element(s) on the cross-section.

(b) Show the state of stress on a stress element for each critical element.

2.4 How Machine Components Break

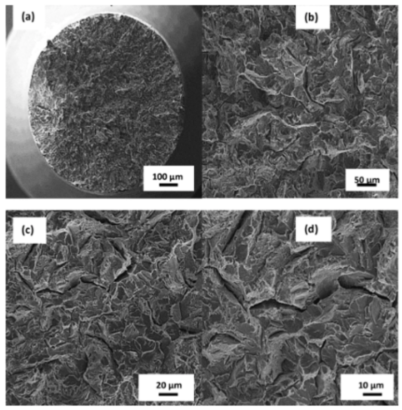
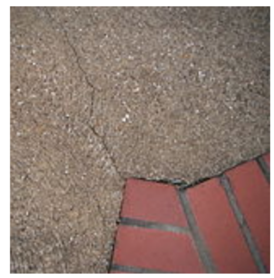
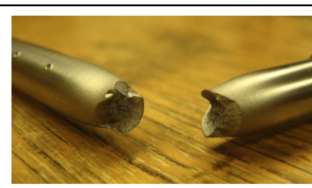
	Static		Fatigue
	Brittle	Ductile	
1. Crack Formation	At/near stress concentration and/or due to voids in the material		
			
2. Crack Propagation	Unstable; crack will propagate without additional load	Stable due to local plastic deformation; resists propagation unless load is increased	Incremental with each load/unload cycle
3. Catastrophic Failure	Sudden	Exhibits yielding first	Sudden
			

Figure 2: The stages of machine component failure include crack formation, crack propagation, and catastrophic failure.

2.5 Static Failure Theories

A **static load** cannot change in any manner.

— Unchanging in magnitude.

— Unchanging in direction.

— Unchanging in location of application

A machine component **fails** when it separates into two, or more, pieces or when it is permanently distorted.

Several **theories** exist for predicting failure due to static loading (see Figure 3).

— Each theory is a different comparison of strength and stress.

— Failure is predicted when the factor of safety is 1.

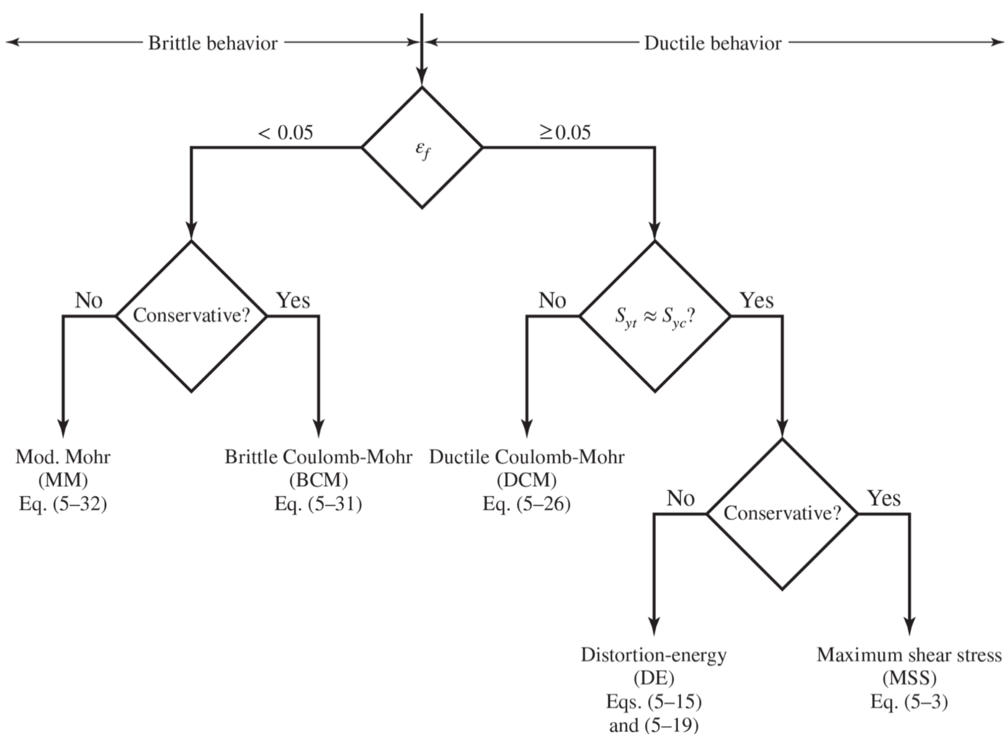


Figure 3: Static failure theories.

2.6 Strength vs. Stress

Strength and stress both have units of force per area (e.g., MPa, psi).

Strength is a material property.

- Strength depends on material composition, heat treatment, and processing.
- Tensile test:
  - Test specimen is loaded in tension.
  - Load and deflection are observed.
  - Results plotted as a stress-strain diagram.
  - Used to determine yield strength (S_y) and ultimate tensile strength (S_{ut}).
- Compression test:
  - Similar to tensile test, but more difficult to conduct.
    - Specimen may buckle during testing.
    - Stresses are difficult to distribute uniformly.
    - Ductile materials bulge after yielding.
  - Used to determine ultimate compressive strength (S_{uc}).
    - For most ductile materials, compressive strengths are the same as tensile strengths.
- Torsion test:
  - Test specimen is loaded in torsion.
  - Load and deflection are observed.
  - Results plotted as a torque-twist diagram.
  - Used to determine torsional yield strength (S_{sy}) and modulus of rupture (S_{su}).

Stress depends on load, geometry, and temperature.

- Residual stresses occur when a region is constrained by adjacent regions from expanding, contracting, or releasing elastic strains.

TRUE · FALSE · IT DEPENDS

A material's modulus of elasticity (E) depends on which of the following?

◯ Heat treatment (e.g., annealed, quenched and tempered)

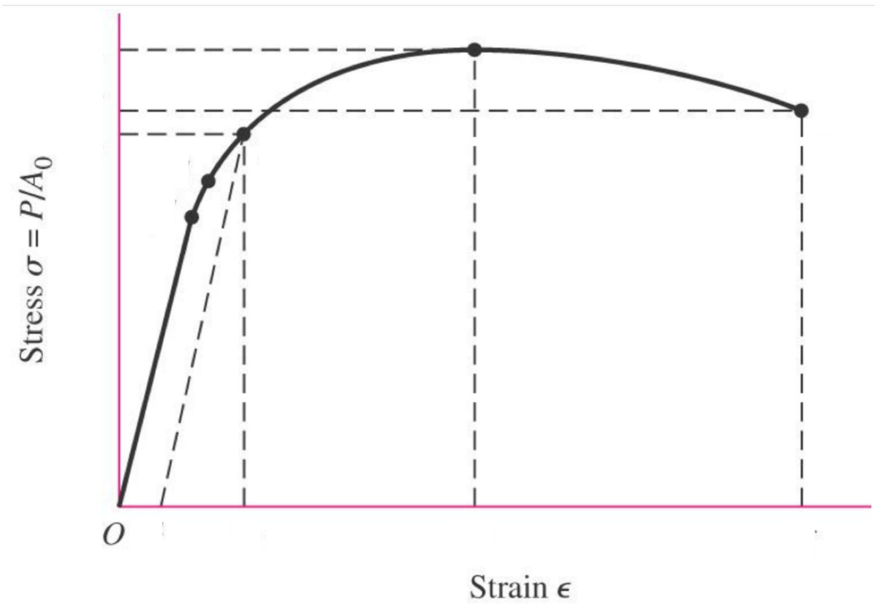
◯ Processing (e.g., cold-drawn, hot-rolled)

◯ Alloying

◯ All of the above

◯ None of the above

TRUE · FALSE · IT DEPENDS



The stress-strain curve for a tensile test is shown above. Load P is applied to a tensile test specimen. The test specimen's initial cross-sectional area is A_0.

Label the stress-strain curve with the following.

— **Proportional limit** (pl): The point until which Hooke's Law can be applied.

— **Elastic limit** (el): The point at which additional stress causes permanent deformation.

— **Yield strength** (S_y): The stress at which the material will retain a 0.2% permanent elongation after the load is removed.

— **Ultimate tensile strength** (S_{ut}): The maximum stress the material can withstand.

— **Fracture limit** (S_f): The point at which the material will fracture.

— If load is removed in the **elastic region**, the material will return to its original shape.

— If load is removed in the **uniform plastic region**, the material will retain permanent deformation and strain hardening.

  - During strain hardening, the yield point is moved to the right on the stress-strain curve. Moving the yield point to the right increases the material's yield strength.

— The specimen's cross-sectional area decreases in the **nonuniform plastic necking region**.

2.7 Coulomb-Mohr theory for ductile materials

The **Ductile Coulomb-Mohr (DCM)** theory applies to ductile materials with different compressive and tensile strengths.

— For some magnesium alloys, S_{ut} is about twice S_{uc}.

— For gray cast irons, S_{uc} is three or four times S_{ut}.

The Mohr theory was developed as a graphical method.

— Based on three simple tests: tension, compression, and shear.

— The failure curve is tangent to the three Mohr's circles.

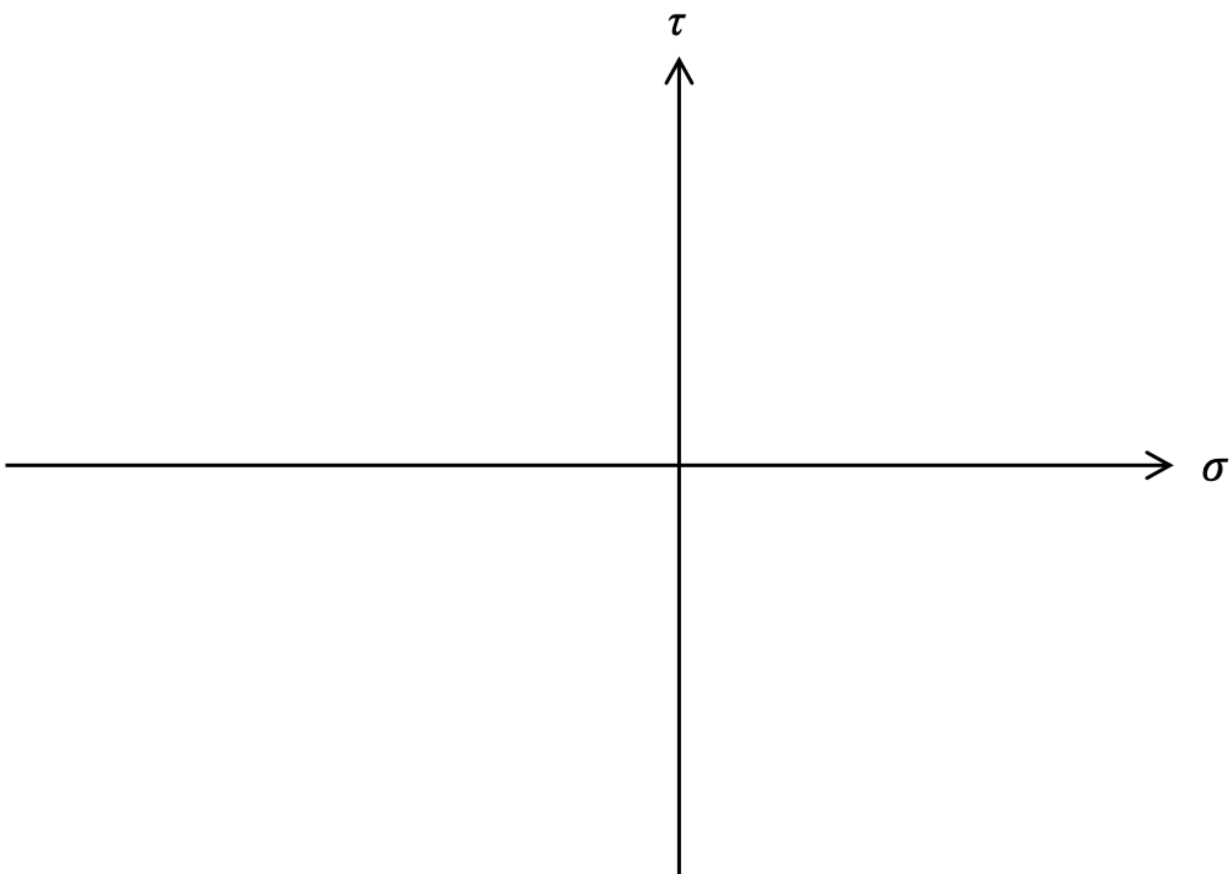


Figure 4: Mohr's theory is based on three simple tests.

The Coulomb-Mohr theory is based on two simple tests: tension and compression.

— The failure curve is a straight line.

— It is easier to calculate the shear yield strength than measure it: $S_{sy} = 0.577\, S_y$

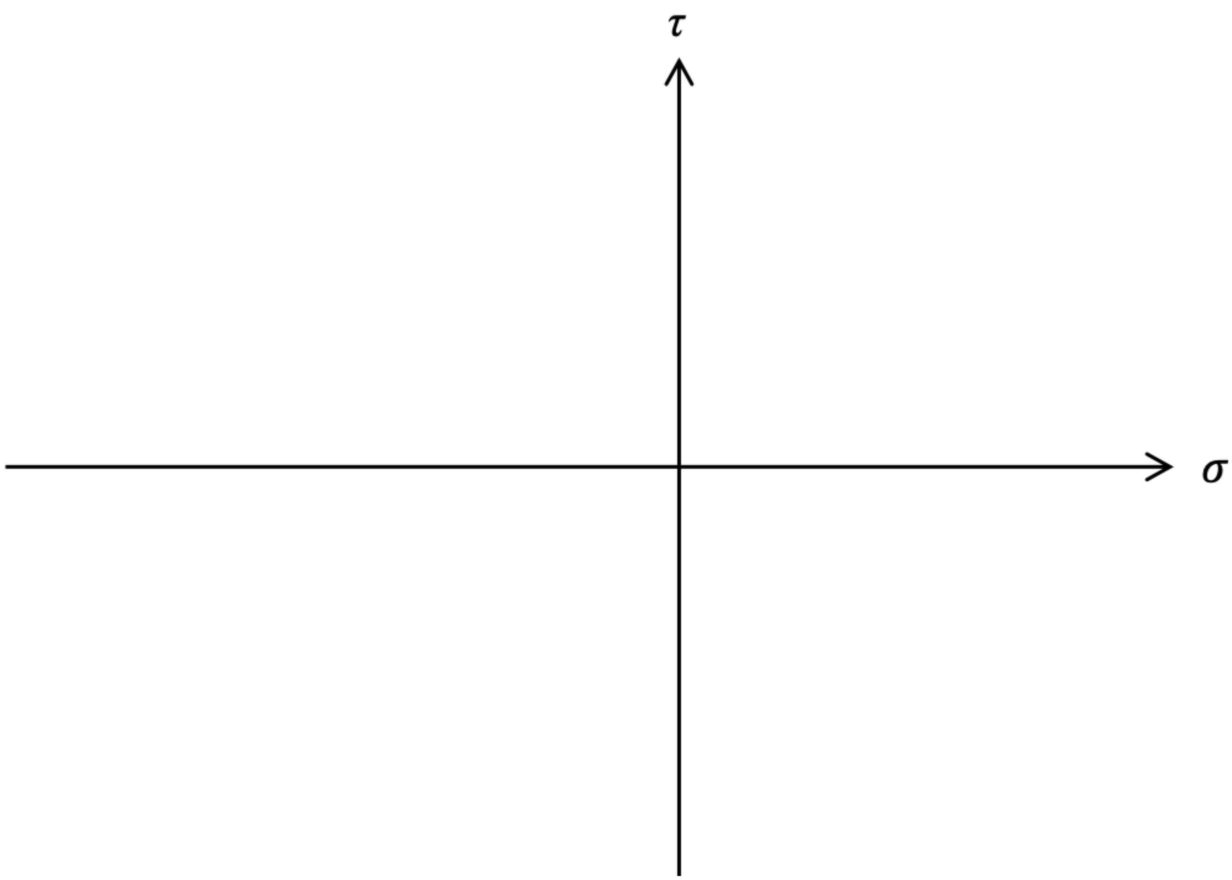


Figure 5: The Ductile Coulomb-Mohr theory is based on two simple tests.

The failure criterion for the DCM theory is:

$$\frac{1}{n} = \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}$$

The failure criterion for the DCM theory can be visualized using a **yield envelope** for a plane stress state (see Figure 6).

— For $\sigma_A \geqslant \sigma_B \geqslant 0$: $\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$ and $\sigma_3 = 0$.

$$n = \frac{S_{yt}}{\sigma_1} = \frac{S_{yt}}{\sigma_A}$$

— For $\sigma_A \geqslant 0 \geqslant \sigma_B$: $\sigma_1 = \sigma_A$, $\sigma_2 = 0$ and $\sigma_3 = \sigma_B$.

$$\frac{1}{n} = \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} = \frac{\sigma_A}{S_{yt}} - \frac{\sigma_B}{S_{yc}}$$

— For $0 \geqslant \sigma_A \geqslant \sigma_B$: $\sigma_1 = 0$, $\sigma_2 = \sigma_A$ and $\sigma_3 = \sigma_B$.

$$n = \frac{S_{yc}}{-\sigma_3} = \frac{S_{yc}}{-\sigma_B}$$

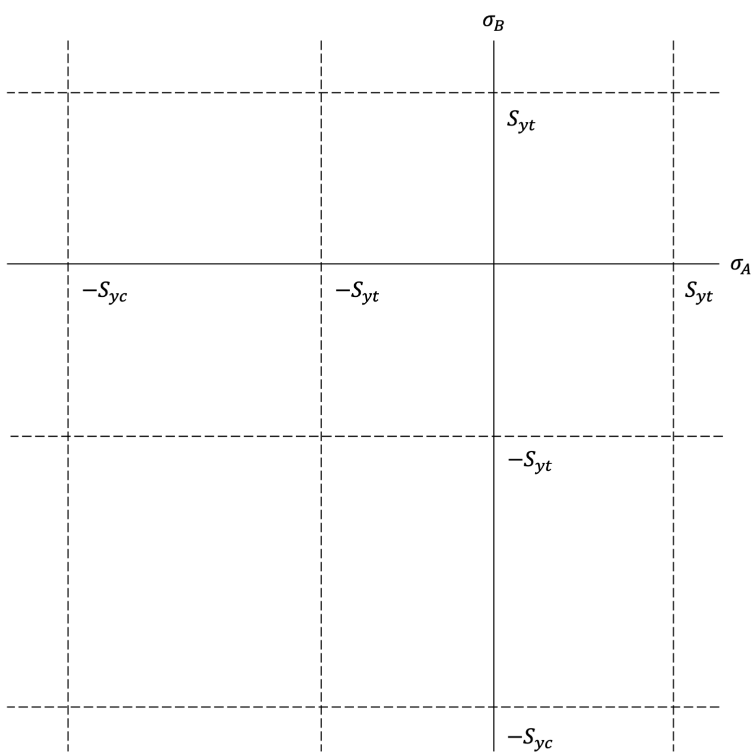


Figure 6: Yield envelope for the DCM theory.

2.8 Maximum Shear Stress theory for ductile materials

The **Maximum Shear Stress (MSS)** theory states that a component fails (yields) when the maximum shear stress at any location in the component to is greater than, or equal to, the maximum shear stress in a tension test specimen when that specimen starts to yield.

The failure criterion for the MSS theory is:

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{S_y}{2\tau_{max}}$$

The yield envelope for the MSS theory is shown in Figure 7 for a plane stress state.

— For $\sigma_A \geqslant \sigma_B \geqslant 0$: $\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$ and $\sigma_3 = 0$.

$$n = \frac{S_y}{\sigma_1} = \frac{S_y}{\sigma_A}$$

— For $\sigma_A \geqslant 0 \geqslant \sigma_B$: $\sigma_1 = \sigma_A$, $\sigma_2 = 0$ and $\sigma_3 = \sigma_B$.

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{S_y}{\sigma_A - \sigma_B}$$

— For $0 \geqslant \sigma_A \geqslant \sigma_B$: $\sigma_1 = 0$, $\sigma_2 = \sigma_A$ and $\sigma_3 = \sigma_B$.

$$n = \frac{S_y}{-\sigma_3} = \frac{S_y}{-\sigma_B}$$

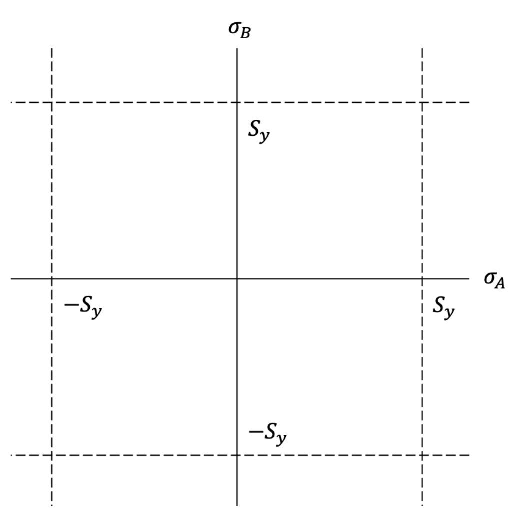


Figure 7: Yield envelope for the MSS theory.

2.9 Distortion Energy theory for ductile materials

The **Distortion Energy (DE)** theory states that a component fails (yields) when the distortion strain energy per unit volume in the component is greater than, or equal to, the distortion strain energy per unit volume in a tension test specimen when that specimen starts to yield.

Consider a stress element subjected to principal stresses σ_1, σ_2, and σ_3.

The principal stresses can be separated into two components (see Figure 8).

— The hydrostatic stress (σ_{avg}) causes only volume change.

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

— The distortional component causes only angular distortion.

The DE theory suggests that it is the distortional component that leads to failure.

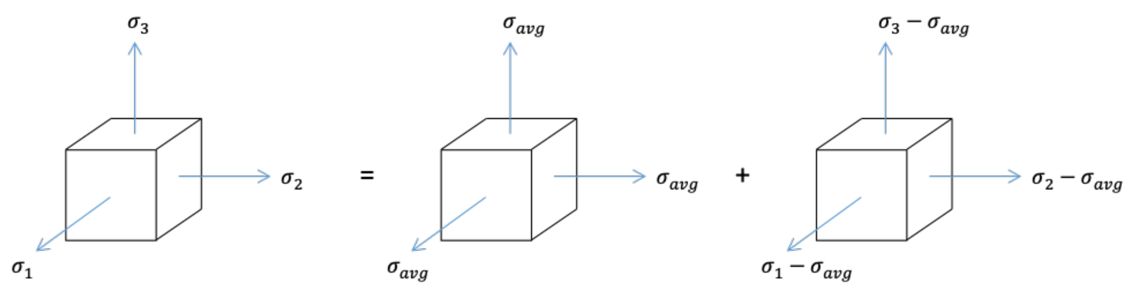


Figure 8: Principal stresses include a hydrostatic stress and a distortional component.

The failure criterion for the DE theory is:

$$n = \frac{S_y}{\sigma'}$$

The **von Mises stress** (σ') is:

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$\sigma' = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}{2}}$$

For a plane stress state, the von Mises stress is:

$$\sigma' = \sqrt{\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2}$$

A comparison of yield envelopes for the MSS theory and the DE theory is seen in Figure 9.

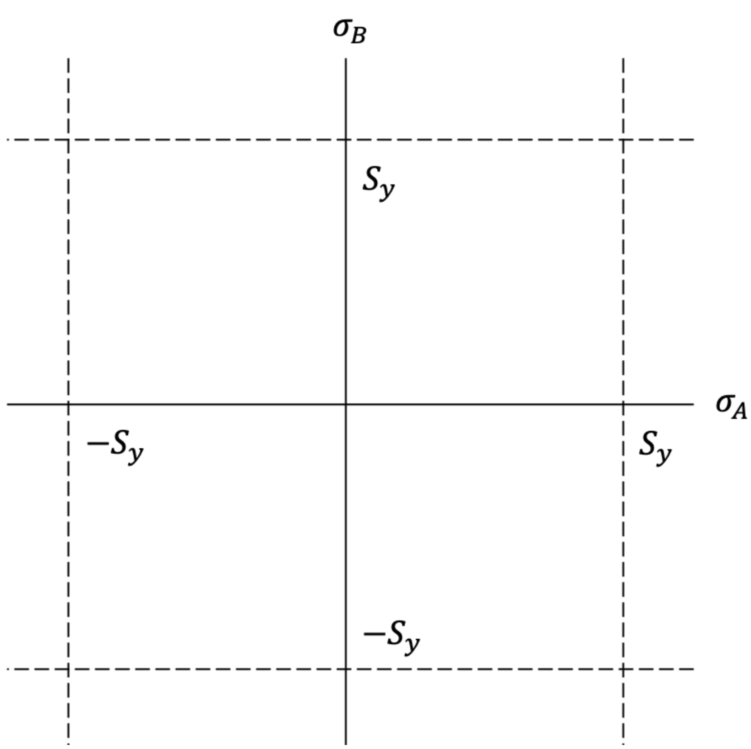


Figure 9: Comparison of yield envelopes for the MSS and DE theories.

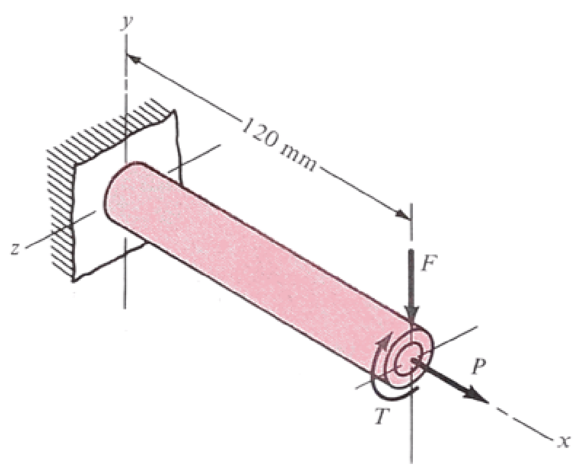
2.10 Example Problems: Static Failure Theories for Ductile Materials

Example 6 A cantilevered tube is made of an aluminum alloy with $S_y = 276$ MPa.

The applied loads are $F = 1.75$ kN, $P = 9.0$ kN, and $T = 72$ Nm.

The tube's outer diameter is 50 mm and the inner diameter is 45 mm.

The cross-sectional area is 3.73 cm^2, $I = 10.55$ cm^4, and $J = 21.1$ cm^4.



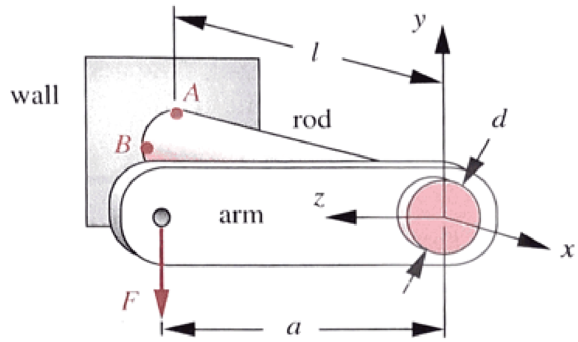
Determine the following.

(a) The factor of safety using the MSS theory.

(b) The factor of safety using the DE theory.

Example 7 The bracket is made of 2024-T4 aluminum with a yield strength of 47 ksi.

The rod length (l) is 6 inches and the arm length (a) is 8 inches. The rod diameter (d) is 1.5 inches. The applied load (F) is 1000 lbf.



Determine the following.

(a) The factor of safety using the MSS theory.

(b) The factor of safety using the DE theory.

Example 8 A part is made of AISI 1040 steel. The part has been annealed.

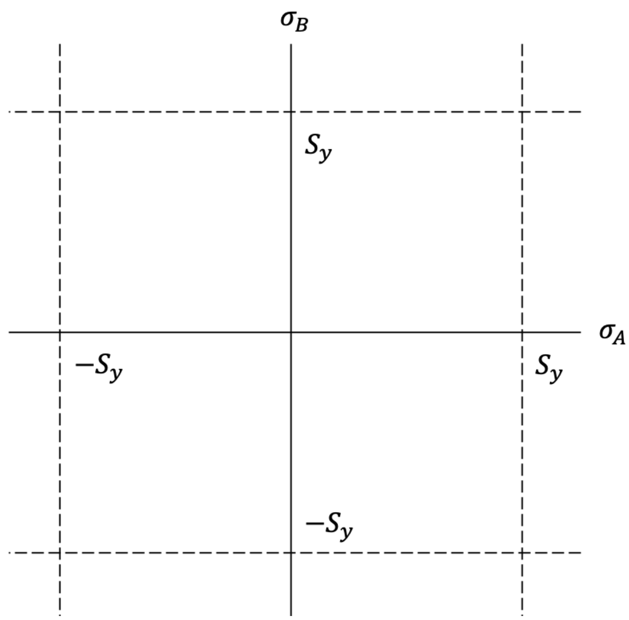
A part is subjected to static loading. At a location in the part, the part is subject to a state of plane stress given by $\sigma_x = 50$ MPa, $\sigma_y = 75$ MPa, and $\tau_{xy} = 50$ MPa.

Determine the following.

(a) Sketch the MSS and DE failure envelopes on the figure below. Plot the stress state.

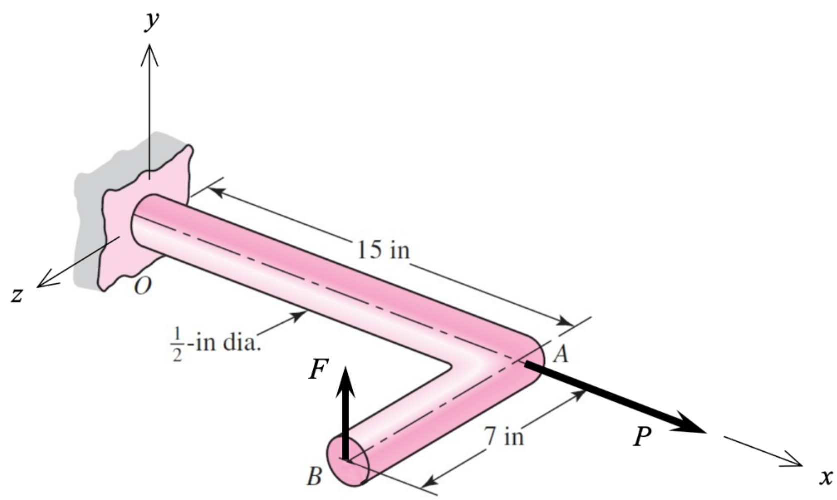
(b) The factor of safety using the MSS theory.

(c) The factor of safety using the DE theory.



Example 9 The cantilevered bar is subjected to two constant loads. Force $P = 20$ lbf acts at point A and is in the $+x-$direction. Force F acts at point B and is in the $+y-$direction.

The bar is a solid rod with a constant 1/2-in diameter.



Determine the following.

(a) For the bar made of AISI 1040 steel that has been quenched and tempered at 400°F, find the force F allowed to achieve a factor of safety of 2.0. Compare your answers using the MSS and DE theories.

2.11 Coulomb-Mohr theory for brittle materials

The **Brittle Coulomb-Mohr (BCM)** theory is identical to the DCM theory, except the material's ultimate tensile strength (S_{ut}) and ultimate compressive strength (S_{uc}) are used instead of the tensile yield strength (S_{yt}) and compressive yield strength (S_{yc}).

The failure envelope for the BCM theory is seen in Figure 10 for a plane stress state.

— For $\sigma_A \geqslant \sigma_B \geqslant 0$, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$.

$$n = \frac{S_{ut}}{\sigma_1} = \frac{S_{ut}}{\sigma_A}$$

— For $\sigma_A \geqslant 0 \geqslant \sigma_B$, $\sigma_1 = \sigma_A$ and $\sigma_3 = \sigma_B$.

$$\frac{1}{n} = \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}}$$

— For $0 \geqslant \sigma_A \geqslant \sigma_B$, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$.

$$n = \frac{S_{uc}}{-\sigma_3} = \frac{S_{uc}}{-\sigma_B}$$

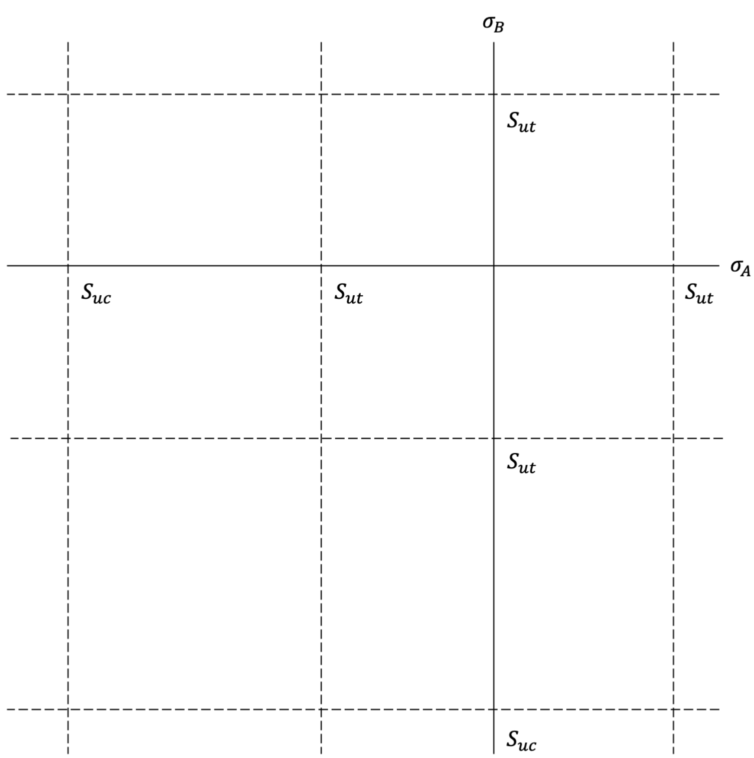


Figure 10: Failure envelope for the BCM theory.

2.12 Modified Mohr theory for brittle materials

The **Modified Mohr (MM)** theory was proposed based on observed data.

For a plane stress state, the MM theory is identical to the BCM theory in the first and third quadrants.

— For $\sigma_A \geqslant \sigma_B \geqslant 0$, $\sigma_1 = \sigma_A$ and $\sigma_3 = 0$.

$$n = \frac{S_{ut}}{\sigma_1} = \frac{S_{ut}}{\sigma_A}$$

— For $0 \geqslant \sigma_A \geqslant \sigma_B$, $\sigma_1 = 0$ and $\sigma_3 = \sigma_B$.

$$n = \frac{S_{uc}}{-\sigma_3} = \frac{S_{uc}}{-\sigma_B}$$

The MM theory differs from the BCM theory in the second and fourth quadrants.

— For $\sigma_A \geqslant 0 \geqslant \sigma_B$ and $|\frac{\sigma_B}{\sigma_A}| \leqslant 1$

$$n = \frac{S_{ut}}{\sigma_1} = \frac{S_{ut}}{\sigma_A}$$

— For $\sigma_A \geqslant 0 \geqslant \sigma_B$ and $|\frac{\sigma_B}{\sigma_A}| > 1$

$$\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{(S_{uc} - S_{ut})\sigma_1}{S_{uc}S_{ut}} - \frac{\sigma_3}{S_{uc}}$$

2.13 Example Problems: Static Failure Theories for Brittle Materials

Example 10 A part is made of ASTM 30 gray cast iron.

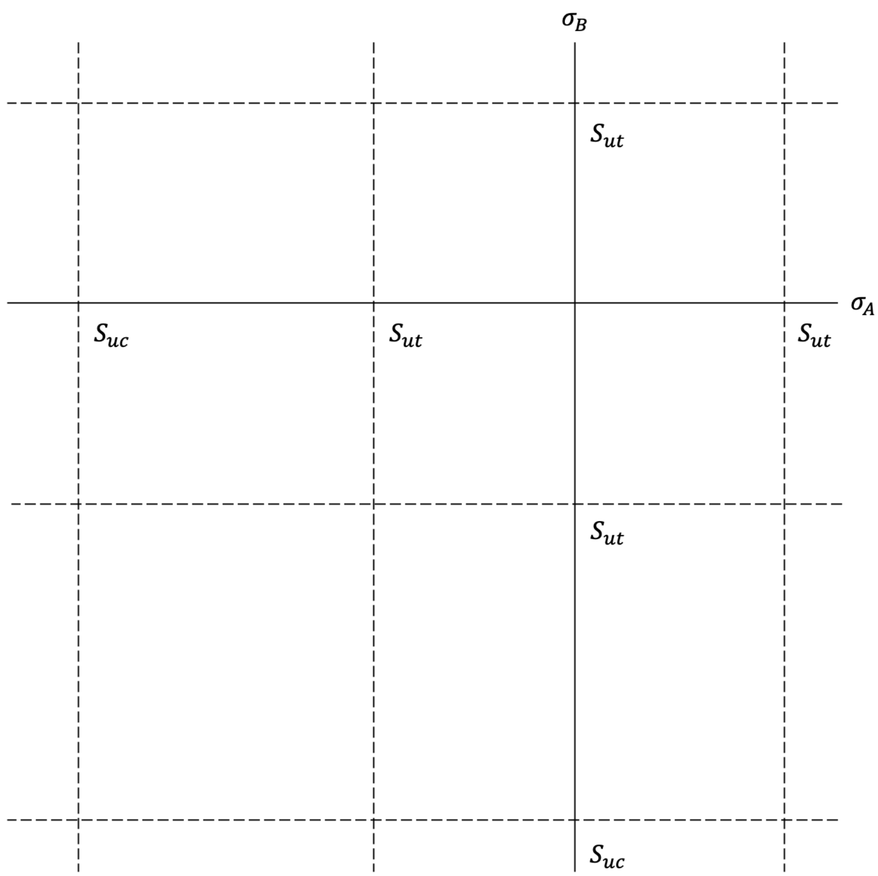
At a location in the part, the part is subject to a state of plane stress given by $\sigma_x = -15$ kpsi, $\sigma_y = 10$ kpsi, and $\tau_{xy} = -15$ kpsi.

Determine the following.

(a) Sketch the MM and BCM failure envelopes on the figure below. Plot the stress state.

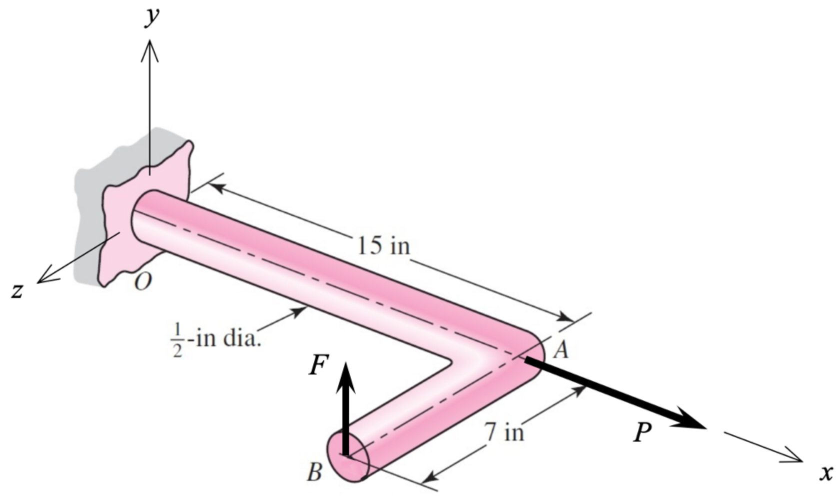
(b) The factor of safety using the MM theory.

(c) The factor of safety using the BCM theory.



Example 11 The cantilevered bar is subjected to two constant loads. Force $P = 20$ lbf acts at point A and is in the $+x-$direction. Force F acts at point B and is in the $+y-$direction.

The bar is a solid rod with a constant 1/2-in diameter.



Determine the following.

(a) For the bar made of gray cast iron with compressive strength of 12 kpsi and tensile strength of 5 kpsi, find the force F allowed to achieve a factor of safety of 2.0. Compare your answers using the MM and BCM theories.

Example 12 At a certain location, a machine component is subjected to a state of plane stress, where $\sigma_y = -20$ kpsi and $\tau_{xy} = 24$ kpsi.

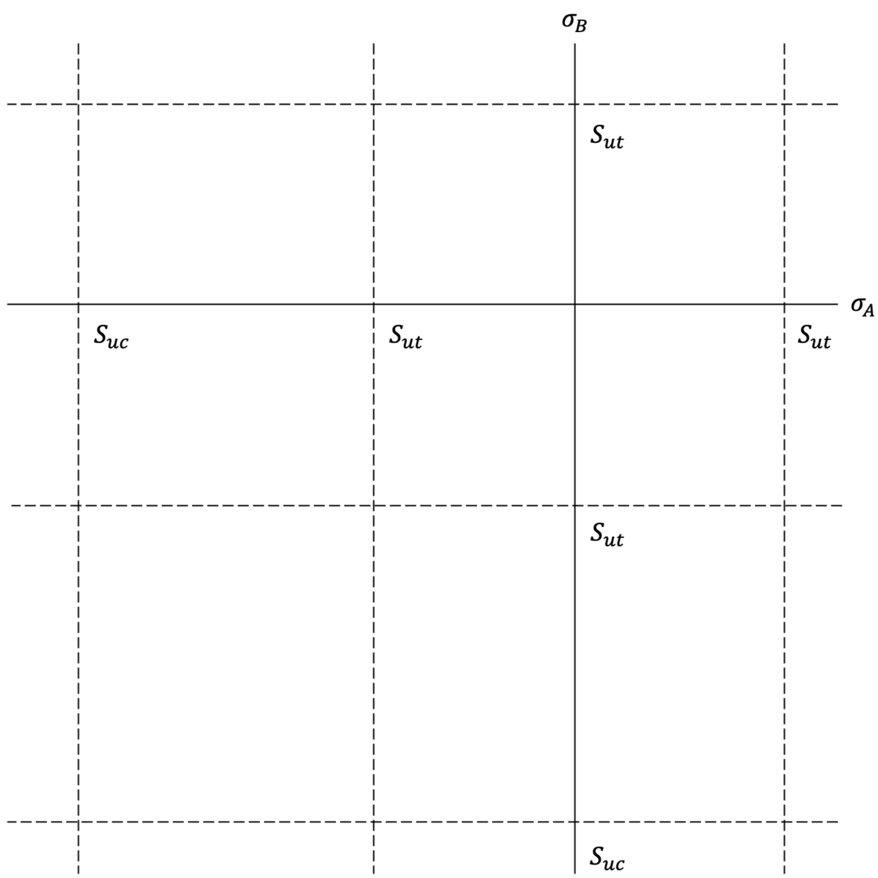
The material is ASTM 30 gray cast iron, a brittle material with ultimate tensile strength 31 kpsi and ultimate compressive strength 109 kpsi.

Determine the following.

(a) Sketch the MM and BCM failure envelopes on the figure below. Plot the stress state.

(b) The factor of safety using the MM theory.

(c) The factor of safety using the BCM theory.



3 Fatigue Failure Analysis

3.1 The Industrial Revolution and Engineering Design

The advances made during the Industrial Revolution led to the study of fatigue failure.

The Industrial Revolution began in Britain in the 18th century.

— Transformed an agricultural and handicraft economy to an economy dominated by industry and machine manufacturing

— Increased the application of science to industry

Table 2 compares considerations of engineering design prior to and necessitated by the Industrial Revolution.

Table 2: Considerations of engineering design.

	Engineering design prior to the Industrial Revolution	**Engineering design necessitated by the Industrial Revolution**
Power source(s)		
Cycles per day		
Materials		
Speeds		
Loads		
Factor of safety		

The understanding of fatigue failure, including the ability to predict fatigue failure, continues to advance.

— Increased understanding of the underlying mechanics.

- Fatigue failure is not due to a change in mechanical properties.
- The surfaces of a fractured part show the progression from crack initiation to crack propagation to catastrophic failure.

— Development of modeling techniques for practical use

— The study of fatigue failures including, but not limited to, the following:

- Versailles railroad axle (1842)
- Liberty ships (1943)
- de Havilland Comet crashes (1954)
- Kielland oil platform collapse (1980)
- Aloha B737 accident (1988)
- DC10 Sioux City accident (1989)
- MD-88 Pensacola engine failure (1996)
- Eschede railway accident (1998)
- GE CF6 engine failure (2016)

3.2 Fatigue Analysis Methods

The goal of fatigue analysis is to predict

— the number of cycles until failure occurs, or

— if a machine component is expected to have infinite life

Three methods can be used for fatigue analysis, and are compared in Table 3.

Table 3: Fatigue analysis methods.

	Linear-Elastic Fracture Mechanics	**Strain-Life Method**	**Stress-Life Method**
Design goal			
Application			
Advantage(s)			
Disadvantage(s)			
Works best when:			

TRUE · FALSE · IT DEPENDS

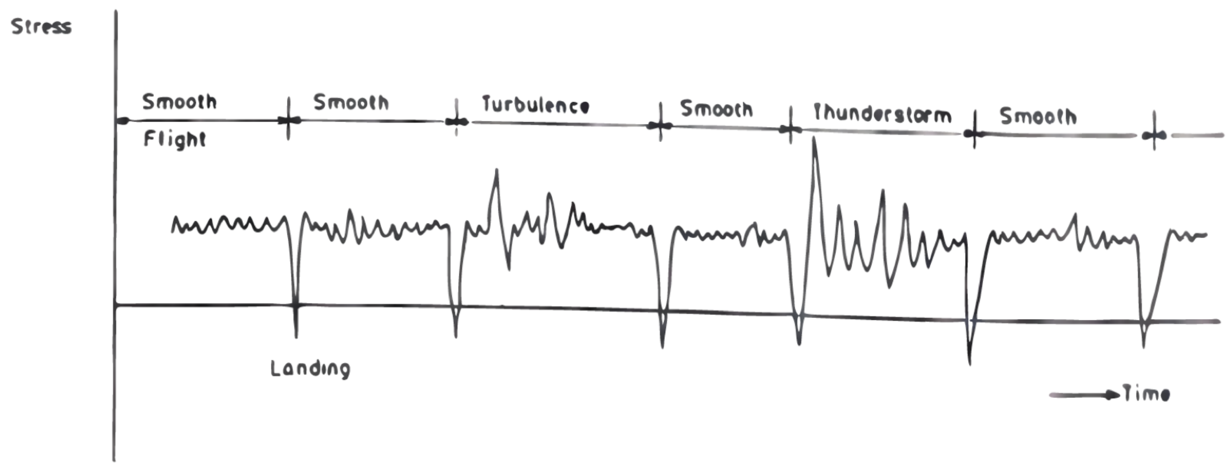
For each of the following scenarios, determine the most applicable fatigue method(s).

The driveshaft of a typical automated production machine rotates at 100 rpm. Assume one-shift operation.

◯ Stress-life ◯ Strain-life ◯ Linear elastic fracture mechanics

A commercial aircraft is subjected to the stress-time pattern shown below.

◯ Stress-life ◯ Strain-life ◯ Linear elastic fracture mechanics



Gas-turbine rotor blades, which operate under high stresses at high temperatures and go through thermal cycles at start-up and shut-down.

◯ Stress-life ◯ Strain-life ◯ Linear elastic fracture mechanics

3.3 Stress-Life Method

The **stress-life method** is the most mature method to perform fatigue analysis. It is not great at characterizing low-cycle fatigue.

To obtain data for the stress-life method, test specimens are subjected to **completely reversed stress** cycles until failure occurs.

— A completely reversed stress alternates between equal magnitudes of tension and compression.

— The frequency of completely reversed stress cycles is typically measured in Hertz (Hz). It is generally accepted that the fatigue behavior of metals is independent of frequency for frequencies less than 200 Hz.

TRUE · FALSE · IT DEPENDS

Sketch a completely reversed stress cycle.

Describe a test method that would achieve a normal stress that alternates between equal magnitudes of tension and compression?

3.4 Rotating Bending Fatigue Test

Wöhler's design of a rotating bending test machine uses a motor to rotate a test specimen with a load at its free end (see Figure 11).

The R.R. Moore design of a rotating bending test machine uses a static weight to induce four-point bending (see Figure 12).

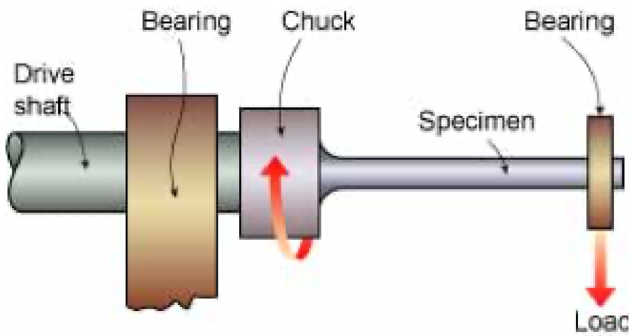


Figure 11: Wöhler's design of a rotating bending test machine.

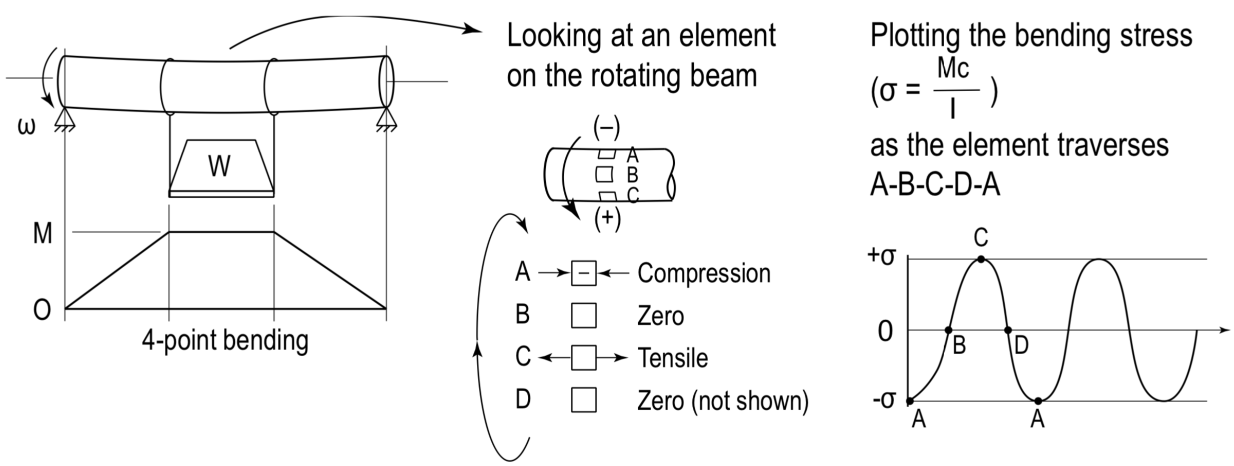


Figure 12: R.R. Moore design of a rotating bending test machine

TRUE · FALSE · IT DEPENDS

It is not uncommon for fatigue tests to test specimens up to 10^9 stress cycles.

Assuming continuous operation of the tester, estimate the time required to achieve 10^9 stress cycles using each of the following testers.

1. A traditional servohydraulic testing machine

2. A high-speed servohydraulic testing machine

3. A rotating beam fatigue testing system

TRUE · FALSE · IT DEPENDS

Flexure tests are used to measure the flexural strength of a material. Flexural strength is the maximum stress at the outer surface of the specimen corresponding to the peak applied force.

Flexural strength is not considered a material property due to the nonuniformity of the stress state. Measured values of flexural strength can differ by 10-20% from the compression strength, measured using a standardized compression test method.

The two most common types of flexure test are three- and four-point bending tests.

Why would a three-point bending test be preferred to a four-point bending test? Or vice-versa?

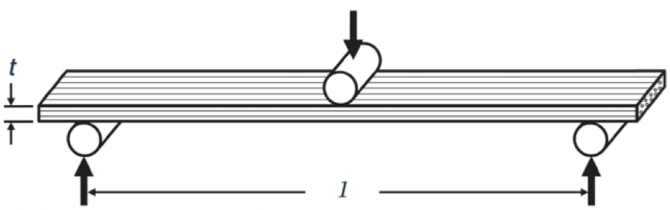


Fig. 1a: Three-point configuration.

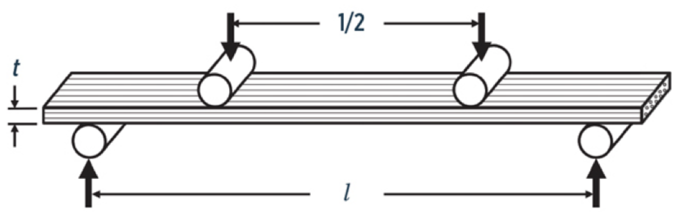


Fig. 1b: Four-point configuration.

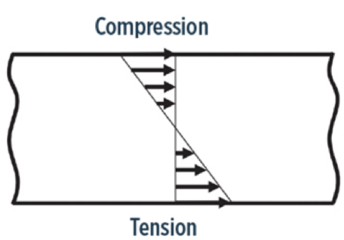


Fig. 1c: Through-the-thickness bending stress distribution.

3.5 The S-N Diagram

The magnitude of the completely reversed stress when failure occurs at N stress cycles is the material's fatigue strength (S_f).

A plot of material strength (S) as a function of stress cycles (N) is known as a **Wöhler curve**, a **stress-life diagram**, or an **S-N diagram**.

A typical S-N diagram for steel is shown in Figure 13.

The **endurance limit** (S_e) of a material is the magnitude of a completely reversed stress cycle that can be applied to the material indefinitely without failure.

— Ferrous and titanium alloys typically have endurance limits.

— Aluminum and copper do not have endurance limits and can experience fatigue failure at very low stresses.

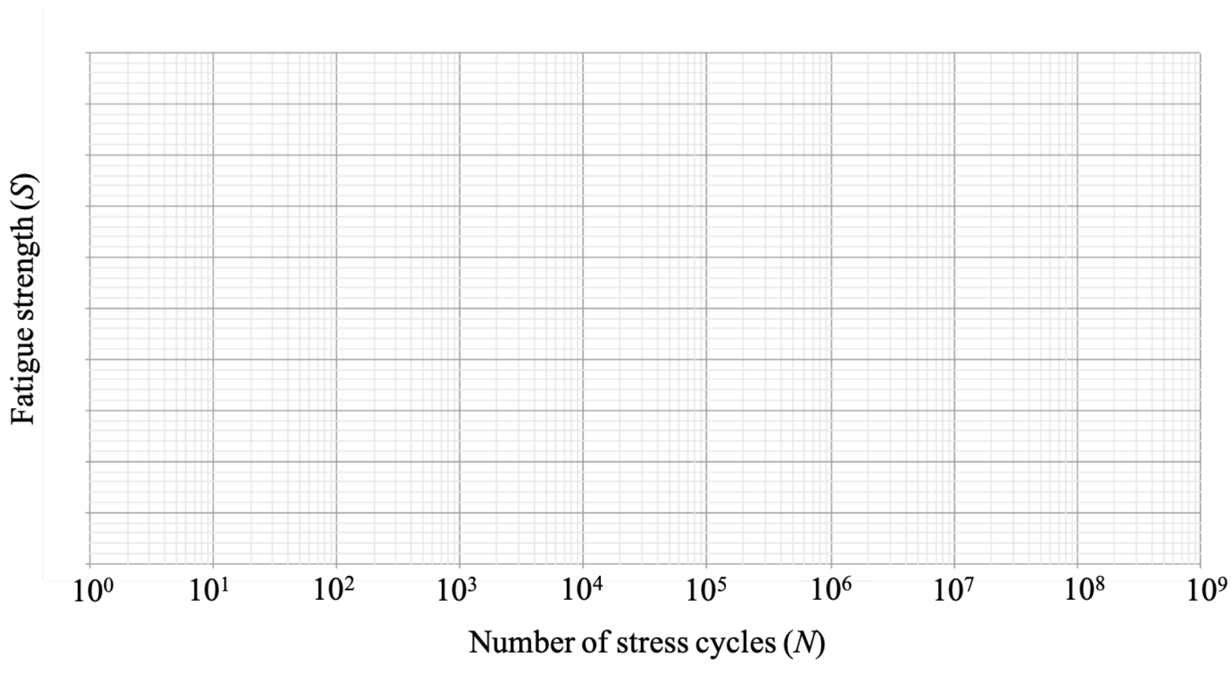


Figure 13: Idealized S-N diagram for steels.

TRUE · FALSE · IT DEPENDS

A part is subjected to cyclic loading at a frequency of 100 Hz.

The part is required to be designed for infinite life.

Which material should be used?

◯ AISI 1035 steel

◯ A6061 aluminum alloy

◯ Either AISI 1035 steel or A6061 aluminum alloy can be used.

◯ Neither AISI 1035 steel nor A6061 aluminum alloy should be used.

3.6 Marin Factors

The idealized S-N diagram shown in Figure 13 is generated from repeated experiments on identical test specimens.

The standard test specimen used in rotating bending fatigue test machine is shown in Figure 14.

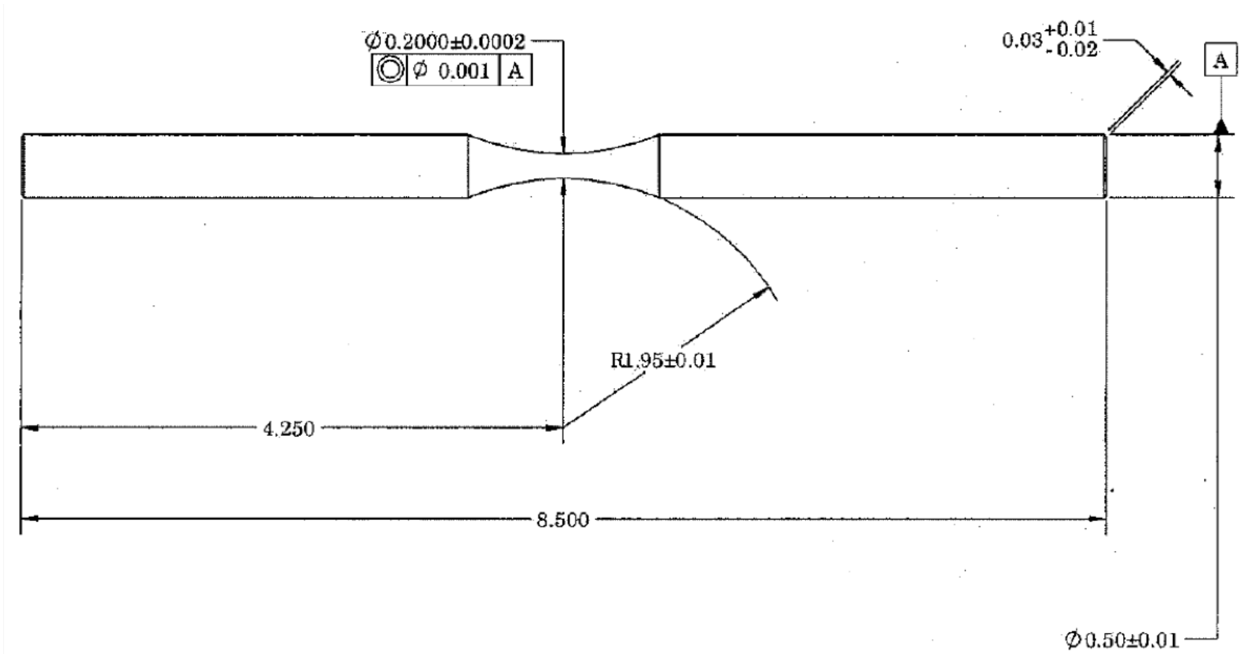


Figure 14: Test specimen subjected to completely reversed stress cycles.

For a real part, the endurance limit must be modified to reflect a real part operating in a real environment.

$$S_e = k_a k_b k_c k_d k_e S'_e$$

S_e is the endurance limit for a real part operating in a real environment.

S'_e is the endurance limit obtained from controlled experiments on the test specimen.

k_a, k_b, k_c, k_d, and k_e are **Marin factors** used to modify S'_e to become S_e.

— k_a: surface factor

— k_b: size factor

— k_c: load factor

— k_d: temperature factor

— k_e: reliability factor

See the Road Map at the end of Chapter 6 in the textbook for the procedure to obtain S_e.

TRUE · FALSE · IT DEPENDS

A material has an ultimate tensile strength of 1500 MPa. Identify the most likely value for the fully corrected endurance limit, S_e.

- ◯ 750 MPa
- ◯ 700 MPa
- ◯ 400 MPa
- ◯ 40 MPa
- ◯ None of these.

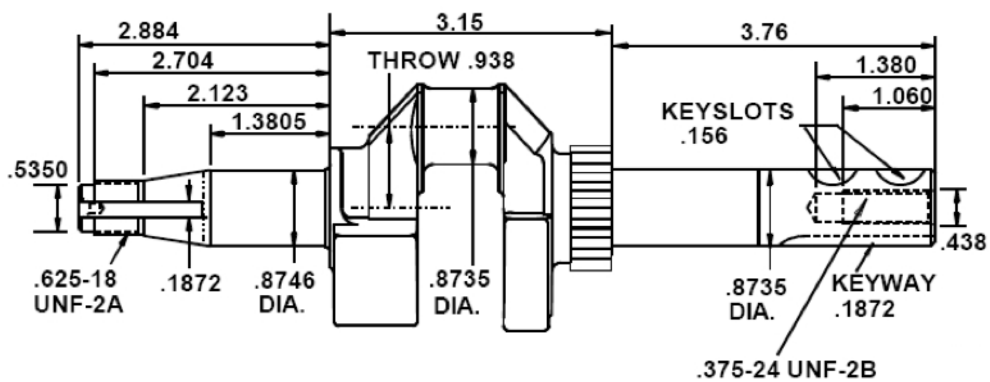
TRUE · FALSE · IT DEPENDS

A crankshaft for a small, single-cylinder engine is made of forged steel with yield strength of 625 MPa and ultimate tensile strength of 800 MPa.

The crankshaft's dimensions are shown in the diagram below.

The critical cross-section of the crankshaft is located in the crank pin (i.e., the portion of the crankshaft that is labeled as having a diameter of 0.8735 inches in the figure).

Determine the endurance limit at the critical cross-section.



3.7 Example Problems: Endurance Limits

Example 13 A 1.5-in-diameter AISI 1040 steel rod has a machined finish and a tensile strength of 110 kpsi.

Determine the following.

(a) The endurance limit if the rod is loaded in rotating bending.

(b) The endurance limit if the rod is subjected to repeated bending, but is not rotating (i.e., a four-point bend test).

Example 14 Two steels are being considered for manufacture of as-forged connecting rods subjected to bending loads.

One is AISI 4340 Cr-Mo-Ni steel capable of being heat-treated to a tensile strength of 260 kpsi.

The other is a plain carbon steel AISI 1040 with an attainable S_{ut} of 113 kpsi.

Each rod is to have a size giving an equivalent diameter of $d_e = 0.75$ in.

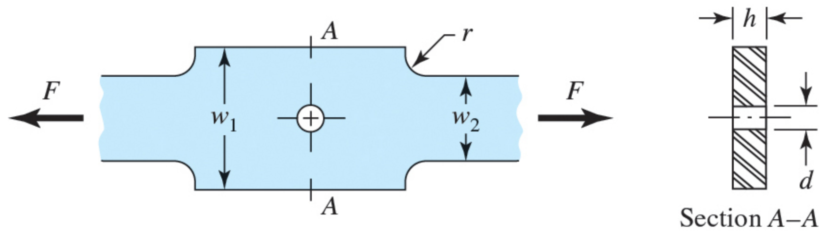
Determine the endurance limit for each material.

Example 15 A segment of a part is subjected to an axial load that varies between $F_{max} = +5$ kips and $F_{min} = -16$ kips.

The part has dimensions $r = 0.25$ in, $d = 0.40$ in, $h = 0.50$ in, $w_1 = 3.50$ in, and $w_2 = 3.0$ in.

The part is made of cold-drawn AISI 1018 steel.

Determine the fully corrected endurance limit.



Example 16 A solid round bar with diameter of 2 inches has a groove cut to a diameter is 1.8 inches, with a radius of 0.1 inch.

The bar is not rotating.

The bar is loaded with a repeated bending load that causes the bending moment at the groove to fluctuate between 0 and 25,000 lbf·in.

The bar is hot-rolled AISI 1095, but the groove has been machined.

Determine the fully corrected endurance limit.

Example 17 A 1-in-diameter solid round bar has a 0.1-in deep groove with a 0.1-in radius machined into it. The bar is AISI 1020 CD steel and is subjected to a completely reversed torque of 1800 lbf-in.

Determine the fully corrected endurance limit.

Example 18 A 4" square steel bar has a machined finish and is subjected to a completely reversed axial load at a temperature of 600^{o}F. The desired reliability is 99%. The ultimate tensile strength of the steel is 90 kpsi at room temperature.

Determine the following.

(a) The uncorrected endurance limit (S'_e).

(b) The corrected endurance limit (S_e).

(c) Estimate the fatigue strength at 10^3 cycles.

(d) Sketch and label the S-N diagram.

3.8 Stress Concentration and Notch Sensitivity

Any irregularity or discontinuity in a part alters the stress distribution in the immediate vicinity of the irregularity or discontinuity.

The irregularity or discontinuity is called a **stress raiser**. Stress raisers may also be referred to as **notches**.

The region in which a stress raiser occurs is called an area of **stress concentration**.

Stress concentration factors K_t and K_{ts} relate the actual maximum stress at the stress raiser (σ_{max} or τ_{max}) to the nominal stress (σ_0 or τ_0).

$$K_t = \frac{\sigma_{max}}{\sigma_0}$$

$$K_{ts} = \frac{\tau_{max}}{\tau_0}$$

K_t and K_{ts} are found in Figure A-15 for a variety of geometries and for loads of axial tension/compression, bending, and torsion.

Fatigue stress concentration factors, K_f and K_{fs} are used when a part is subjected to cyclic loading.

K_f and K_{fs} account for a material's notch sensitivity, q.

$$K_f = 1 + q(K_t - 1) = \frac{\sigma_{max}}{\sigma_0}$$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = \frac{\tau_{max}}{\tau_0}$$

A **fully notch sensitive** material will have $q = 1$ and $q_s = 1$.

For steels and aluminum alloys that are not fully notch sensitive, q is found from Figure 6-26 and q_s is found from Figure 6-27.

3.9 Example Problems: Stress Concentration and Notch Sensitivity

Example 19 A segment of a part is subjected to an axial load $F = 5$ kips.

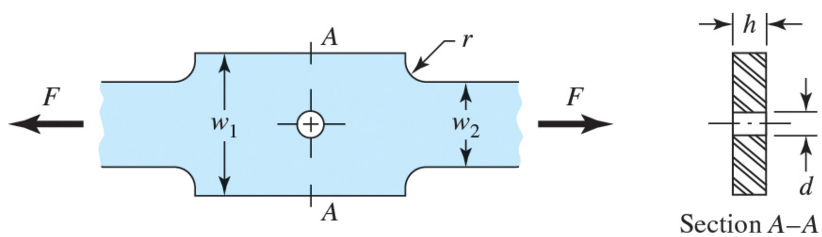
The part has dimensions $r = 0.25$ in, $d = 0.40$ in, $h = 0.50$ in, $w_1 = 3.50$ in, and $w_2 = 3.0$ in.

Determine the following.

(a) The part is made of a ductile material. Is a crack more likely to initiate at the hole or at the fillet?

(b) The part is made of the same material, but has been heat treated and quenched (HT&Q) to increase its strength. The material lost its ductility due to the HT&Q.

Is a crack now more likely to initiate at the hole or at the fillet?

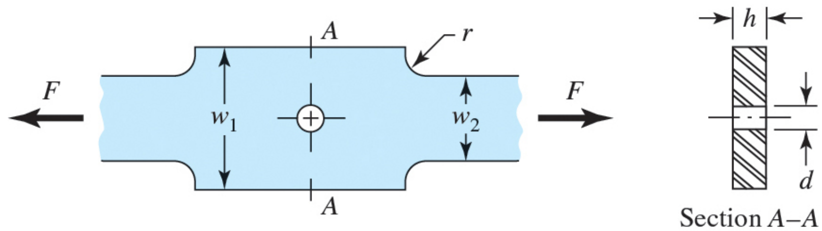


Example 20 A segment of a part is subjected to an axial load that varies between $F_{max} = +5$ kips and $F_{min} = -16$ kips.

The part has dimensions $r = 0.25$ in, $d = 0.40$ in, $h = 0.50$ in, $w_1 = 3.50$ in, and $w_2 = 3.0$ in.

The part is made of cold-drawn AISI 1018 steel.

Determine the fatigue stress concentration factor.



Example 21 A solid round bar with diameter of 2 inches has a groove cut to a diameter is 1.8 inches, with a radius of 0.1 inch.

The bar is not rotating.

The bar is loaded with a repeated bending load that causes the bending moment at the groove to fluctuate between 0 and 25,000 lbf·in.

The bar is hot-rolled AISI 1095, but the groove has been machined.

Determine the fatigue stress concentration factor.

Example 22 A 1-in-diameter solid round bar has a 0.1-in deep groove with a 0.1-in radius machined into it. The bar is AISI 1020 CD steel and is subjected to a completely reversed torque of 1800 lbf-in.

Determine the fatigue stress concentration factor.

3.10 Characterizing Cyclic Loading

It is often necessary to simplify real loading patterns in order to apply the stress-life method.

Cyclic loads (F, M, or T) or stresses (σ or τ) can be characterized by the maximum, minimum, amplitude (or alternating component), and mean (or midrange component) values.

For example, a time-varying force that cycles between a maximum of F_{max} and a minimum of F_{min} is characterized by F_a and F_m.

$$F_a = \frac{|F_{max} - F_{min}|}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

TRUE · FALSE · IT DEPENDS

For the vertical force shown below (the dashed line), estimate the following.

(a) $F_{min} =$

(b) $F_{max} =$

(c) $F_a =$

(d) $F_m =$

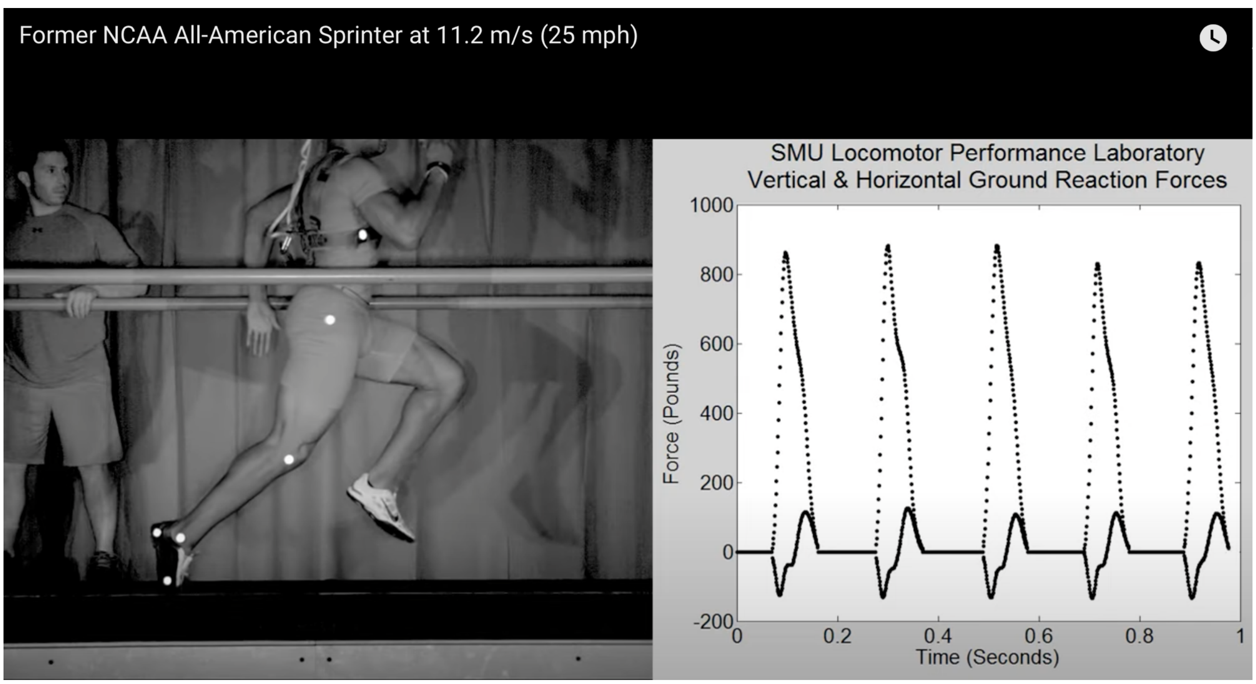
For the horizontal force shown below (the solid line), estimate the following.

(a) $F_{min} =$

(b) $F_{max} =$

(c) $F_a =$

(d) $F_m =$



TRUE · FALSE · IT DEPENDS

For the vertical force shown below in blue, estimate the following.

(a) $F_{min} =$

(b) $F_{max} =$

(c) $F_a =$

(d) $F_m =$

For the anterior-posterior (front-back) force shown below in red, estimate the following.

(a) $F_{min} =$

(b) $F_{max} =$

(c) $F_a =$

(d) $F_m =$

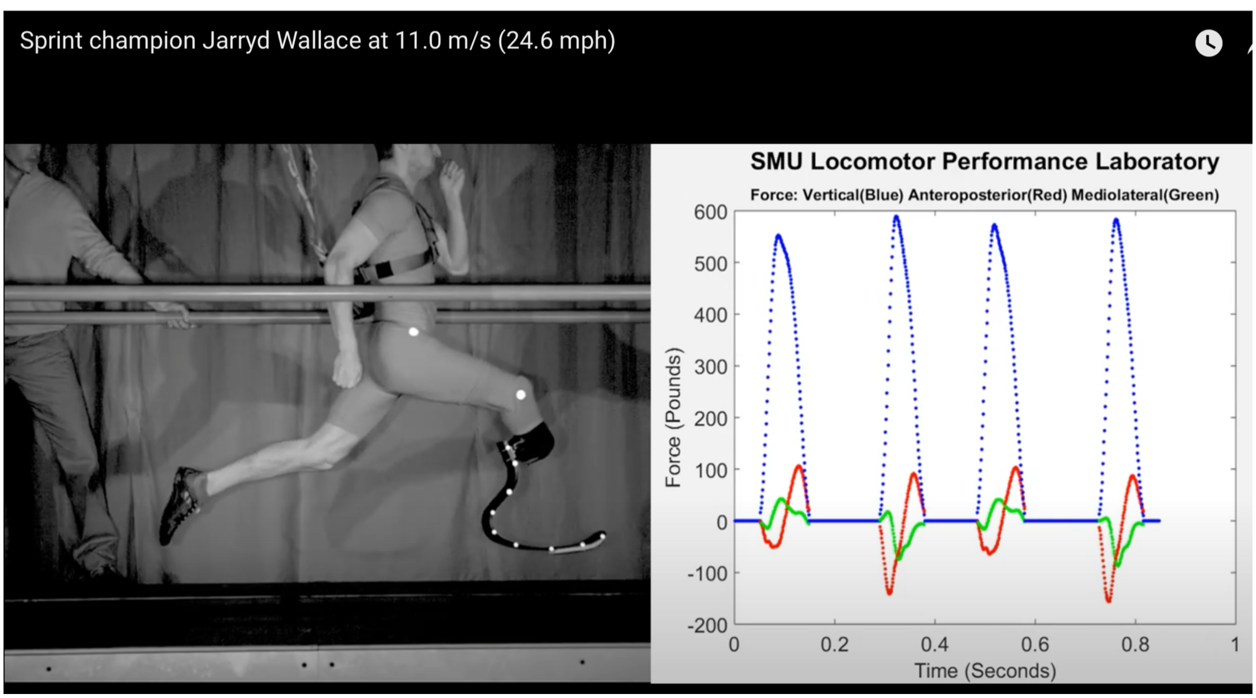
For the medial-lateral (left-right) force shown below in green, estimate the following.

(a) $F_{min} =$

(b) $F_{max} =$

(c) $F_a =$

(d) $F_m =$



TRUE · FALSE · IT DEPENDS

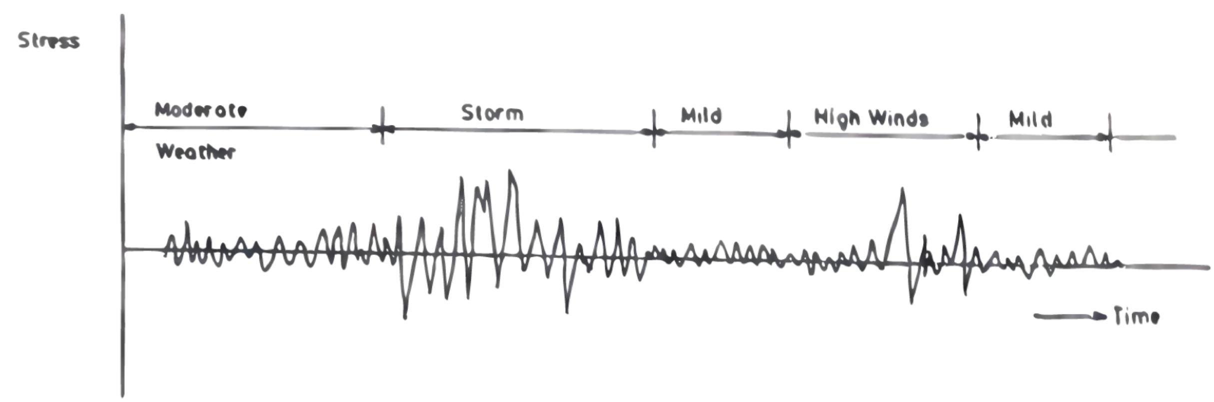
For the cyclic loads shown below, show the following stresses on the diagrams.

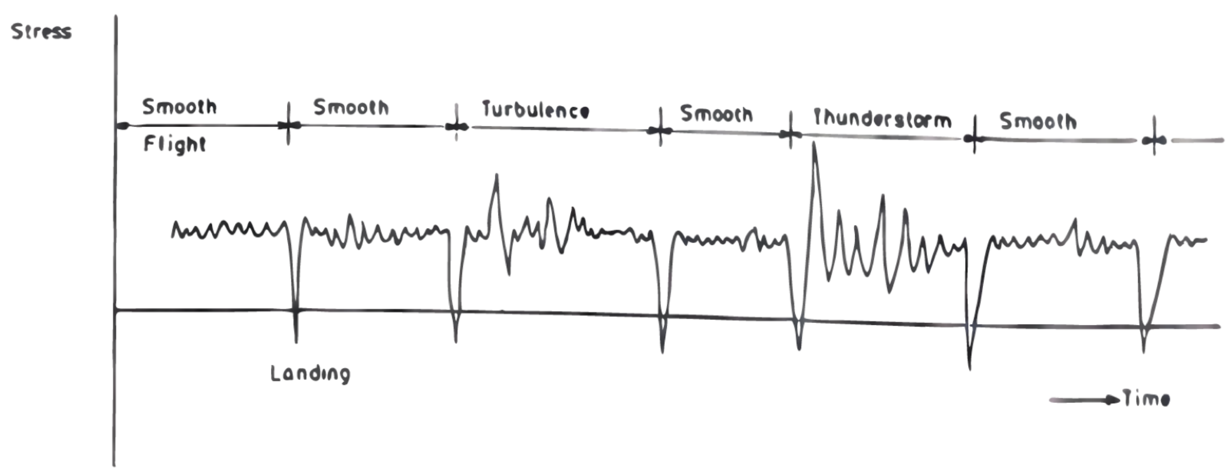
(a) σ_{min}

(b) σ_{max}

(c) σ_a

(d) σ_m





TRUE · FALSE · IT DEPENDS

A **completely reversed load** is characterized by which of the following?

◯ $\sigma_{min} = 0$

◯ $\sigma_{max} = 0$

◯ $\sigma_m = 0$

◯ $\sigma_a = 0$

A **repeating load** is characterized by which of the following?

◯ $\sigma_{min} = 0$

◯ $\sigma_{max} = 0$

◯ $\sigma_m = 0$

◯ $\sigma_a = 0$

A **static load** is characterized by which of the following?

◯ $\sigma_{min} = 0$

◯ $\sigma_{max} = 0$

◯ $\sigma_m = 0$

◯ $\sigma_a = 0$

3.11 Fatigue Failure Criteria for the Stress-Life Method

Figure 15 shows several theories can be used to represent the infinite life boundary.

— For completely reversed loading ($\sigma_m = 0$), failure is predicted at the endurance limit (S_e).

— For static loading ($\sigma_a = 0$), failure is predicted at the material's yield strength (S_y), ultimate tensile strength S_{ut}, or true fracture strength ($\tilde{\sigma}_f$).

Note that even when infinite life is predicted, it is possible for the part to fail due to **first-cycle yielding**.

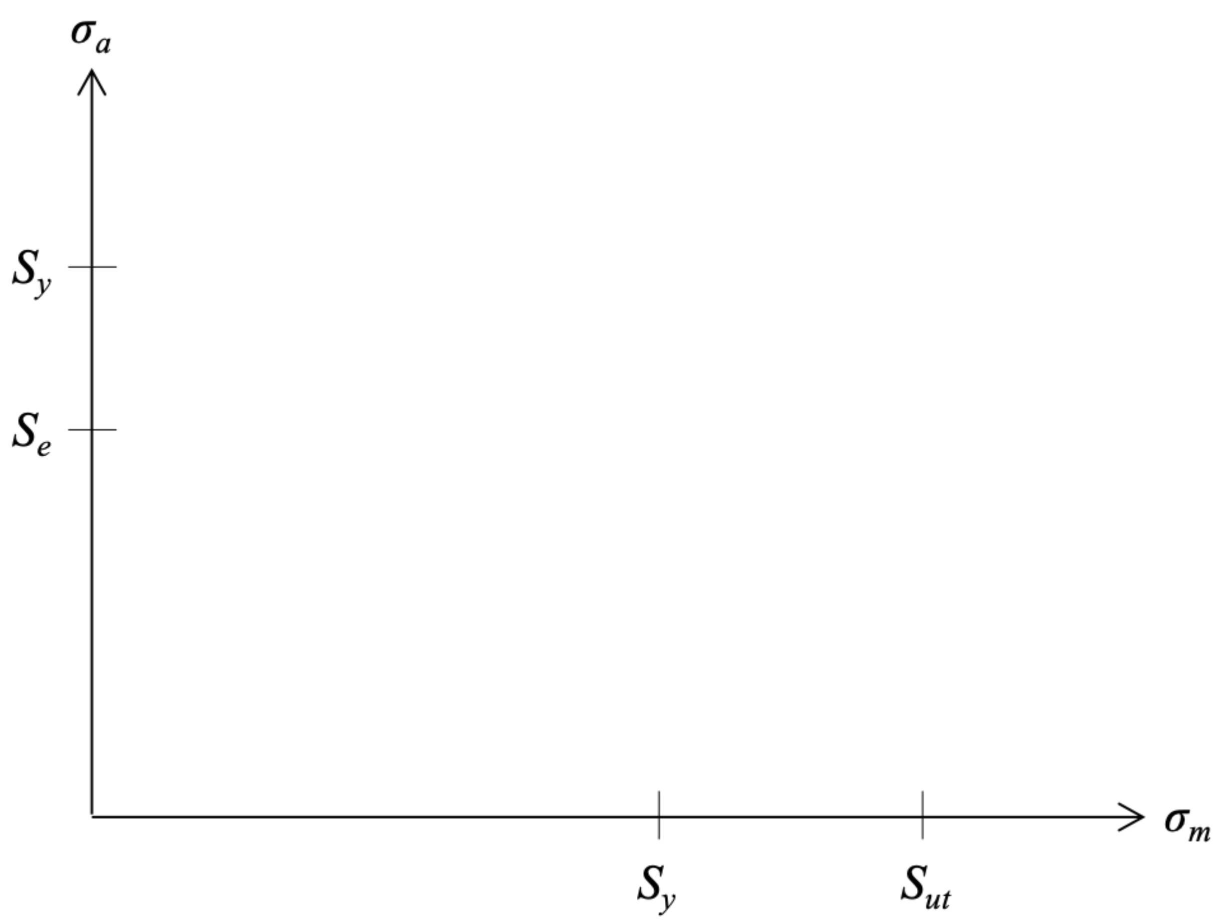


Figure 15: Comparison of fatigue failure theories.

3.12 Fatigue Failure Analysis

The recipe to perform fatigue failure analysis is:

1. Determine the life of the component.
   (a) Is infinite life predicted? Find the factor of safety for infinite life, n_f.
   (b) If $n_f < 1$, infinite life is not predicted. Find the number of cycles until the part fails.
2. Check for first-cycle yielding.

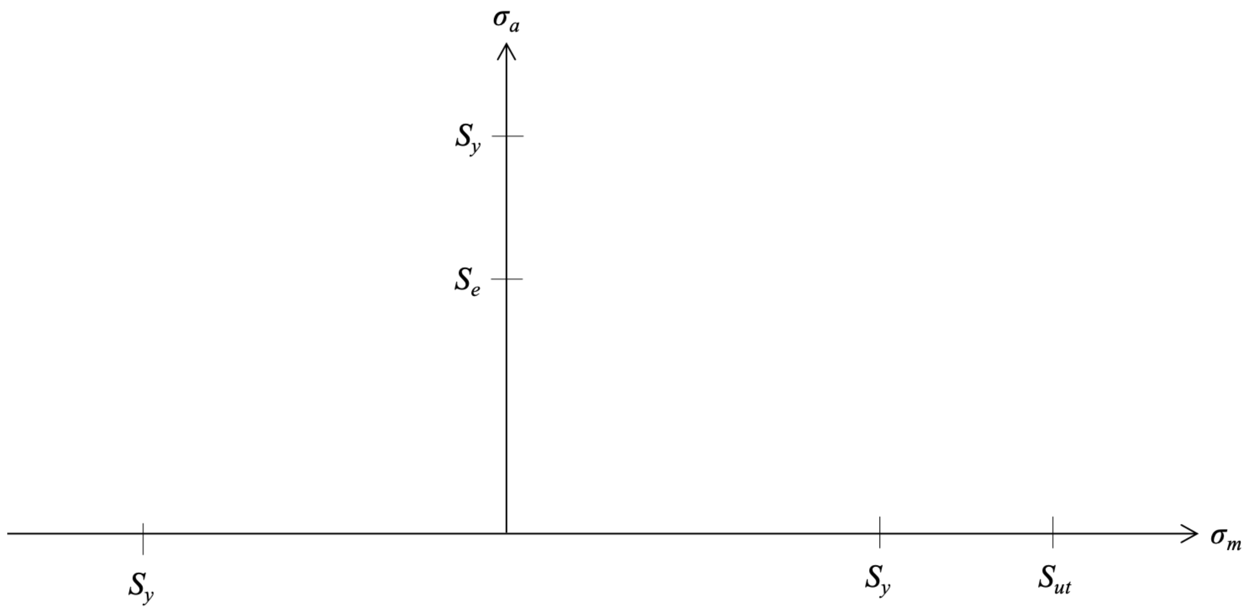


Figure 16: Fluctuating-stress diagram for the Goodman criterion with zones for infinite life, finite life, and first-cycle yielding.

3.12.1 Factor of safety for infinite life using the Goodman criterion

From Figure 15, the straight line between $(\sigma_m, \sigma_a) = (0, S_e)$ and $(\sigma_m, \sigma_a) = (S_{ut}, 0)$ is the **Goodman line**.

The Goodman line is a commonly used design criterion to conservatively represent the infinite life boundary.

For $\sigma_m > 0$, the factor of safety for infinite life using the Goodman line is:

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

For $\sigma_m < 0$, experiments have shown that a compressive mean stress is not detrimental to fatigue life (see Figure **??**).

For $\sigma_m < 0$, the factor of safety for infinite life is:

$$n_f = \frac{S_e}{\sigma_a}$$

3.12.2 Number of cycles until failure

For finite-life, first find the equivalent completely reversed stress σ_{ar}.

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/S_{ut}}$$

The finite life (N) with factor of safety n is:

$$N = \left(\frac{\sigma_{ar}/n}{a}\right)^{1/b}$$

Constants a and b are functions of S_{ut} and S_e. The coefficient f is found from Figure 6-23 in the text.

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

3.12.3 Check for first-cycle yielding

The factor of safety for first-cycle yielding is:

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_a + |\sigma_m|}$$

3.13 Example Problems: Fatigue Failure Analysis

Example 23 A segment of a part is subjected to an axial load that varies between $F_{max} = +5$ kips and $F_{min} = -16$ kips.

The part has dimensions $r = 0.25$ in, $d = 0.40$ in, $h = 0.50$ in, $w_1 = 3.50$ in, and $w_2 = 3.0$ in.

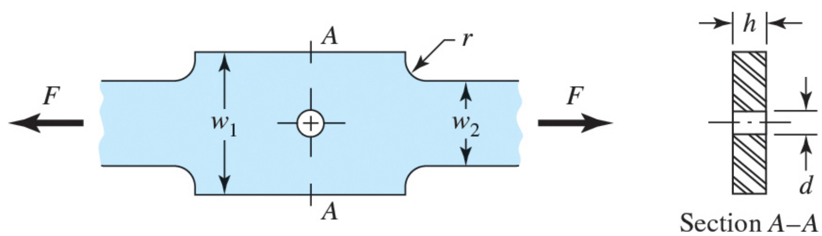
The part is made of cold-drawn AISI 1018 steel.

The fully corrected endurance limit was found in Example 16 and the fatigue stress concentration factor was found in Example 21.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

(b) Check for first-cycle yielding.



Example 24 A solid round bar with diameter of 2 inches has a groove cut to a diameter is 1.8 inches, with a radius of 0.1 inch.

The bar is not rotating.

The bar is loaded with a repeated bending load that causes the bending moment at the groove to fluctuate between 0 and 25,000 lbf·in.

The bar is hot-rolled AISI 1095, but the groove has been machined.

The fully corrected endurance limit was found in Example 15 and the fatigue stress concentration factor was found in Example 20.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

(b) Check for first-cycle yielding.

3.14 Predicting Fatigue Failure for Pure Shear

For the case of pure torsion, consider the following.

— Replace σ_m and σ_a with τ_m and τ_a.

  - The factor of safety for infinite life.
  - The equivalent completely reversed stress and the factor of safety for finite life.
  - The check for first-cycle yielding.

— For Marin factor k_c, use $k_c = 0.59$ when calculating S_e.

— Replace S_y with $S_{sy} = 0.577S_y$.

— Replace S_{ut} with S_{su}, where $S_{su} = 0.67S_{ut}$.

3.15 Example Problems: Fatigue Failure for Pure Shear

Example 25 A 1-in-diameter solid round bar has a 0.1-in deep groove with a 0.1-in radius machined into it. The bar is AISI 1020 CD steel and is subjected to a completely reversed torque of 1800 lbf-in.

The fully corrected endurance limit was found in Example 17 and the fatigue stress concentration factor was found in Example 22.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

(b) Check for first-cycle yielding.

3.16 Predicting Fatigue Failure for Combined Loading

For **combined loads**, the von Mises stresses (σ'_a and σ'_m) are calculated.

$$\sigma'_a = \sqrt{[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial}(\sigma_a)_{axial}]^2 + 3[(K_{fs})_{torsion}(\tau_a)_{torsion}]^2}$$

$$\sigma'_m = \sqrt{[(K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axial}(\sigma_m)_{axial}]^2 + 3[(K_{fs})_{torsion}(\tau_m)_{torsion}]^2}$$

The values for σ'_a and σ'_m are used to calculate:

— The factor of safety for infinite life using the Goodman line.

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}$$

— The equivalent completely reversed stress and the factor of safety for finite life.

$$\sigma'_{ar} = \frac{\sigma'_a}{1 - \sigma'_m/S_{ut}}$$

$$N = \left(\frac{\sigma'_{ar}/n}{a}\right)^{1/b}$$

— The check for first-cycle yielding.

$$n_y = \frac{S_y}{\sigma'_{max}} \approx \frac{S_y}{\sigma'_a + \sigma'_m}$$

Some notes for calculating fatigue life for cases of combined loading.

— For Marin factor k_b, there could be a different value for each load type. Choose the lowest value of k_b.

— For Marin factor k_c, use $k_c = 1$.

— If two or more stress components have different frequencies, the problem is complicated.

3.17 Example Problems: Combined Loading

Example 26 A steel bar has $S_e = 40$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 80$ kpsi.

The bar is subjected to a steady torsional stress of 15 kpsi and an alternating bending stress of 25 kpsi.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

(b) Check for first-cycle yielding.

Example 27 A solid square rod is cantilevered at one end.

The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ± 2 kN.

The material is AISI 1080 hot-rolled steel.

Neglect any stress concentrations.

Determine the following.

(a) The dimension of the square cross-section if the rod must support the load for 10^5 cycles with a design factor of 1.5.

(b) Using the dimension found in part (a), predict the number of cycles to failure if the rod supports a steady 1 kN axial load in addition to the completely reversing transverse load.

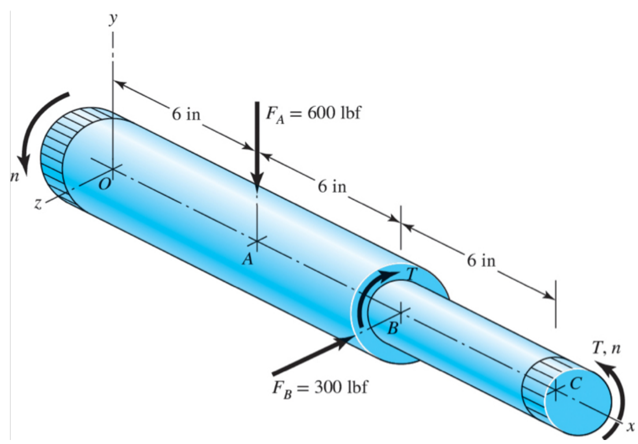
Example 28 The rotating stepped shaft is steel with $S_{ut} = 91$ kpsi, $S_y = 77$ kpsi, and a fully corrected endurance limit of $S_e = 40$ kpsi.

The diameter of the larger section is 2.5 inches and the diameter of the smaller section is 1.5 inches.

The shaft is simply supported by bearings at O and C.

The shaft is loaded with constant forces F_A and F_B and a torque that alternates between 0 and 1800 lbf-in.

The stress concentration factors at the step are $K_f = 2.1$ and $K_{fs} = 1.7$.



Determine the following.

(a) Determine the reaction forces at bearings O and C.

(b) Draw the bending moment and torque diagrams.

(c) You plan to use the Goodman criterion to find the factor of safety for infinite life. Complete the following table for critical cross-sections at A and at B.

	Location A	Location B
K_f		
K_{fs}		
M_{min}		
M_{max}		
M_a		
M_m		
T_{min}		
T_{max}		
T_a		
T_m		

4 Shaft design

A **shaft** transmits rotary motion, torque, and power from a source to a load.

Shafts have circular cross-sections. They are either solid or hollow tubes.

Shafts carry gears, pulleys, and/or sprockets to transmit rotary motion, torque, and power.

— Gears mate with gears

— Pulleys mate with belts

— Sprockets mate with chains

Couplings connect shafts with other shafts.

Shafts are supported by bearings.

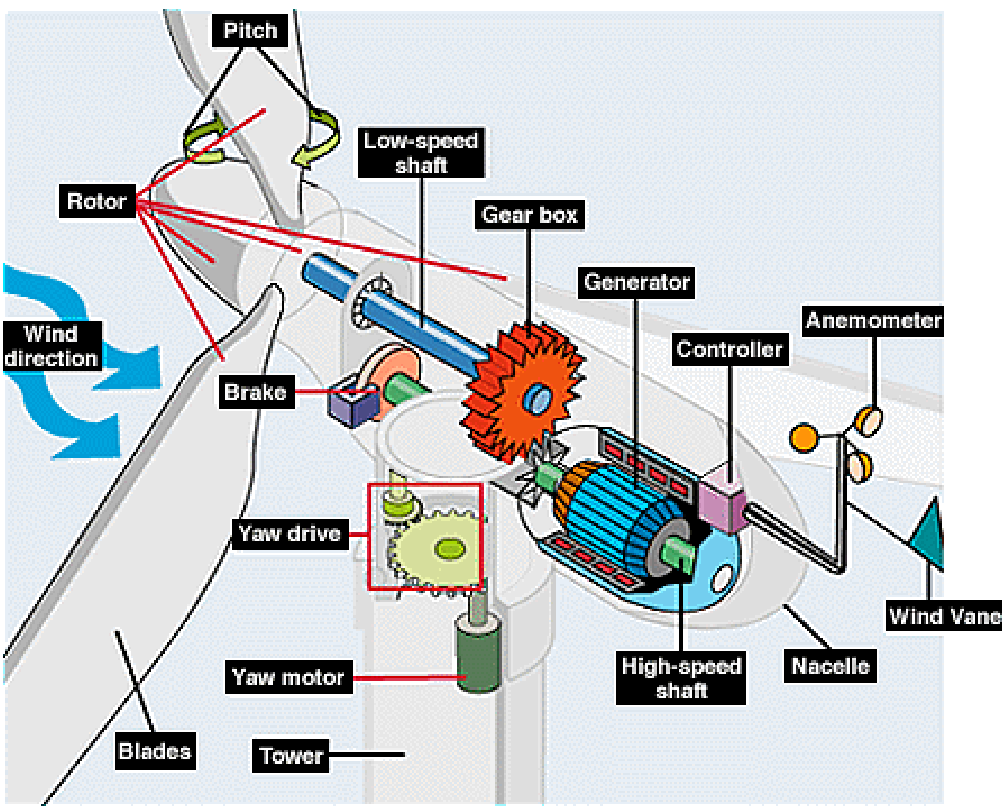


Figure 17: Components of a horizontal-axis wind turbine.

TRUE · FALSE · IT DEPENDS

Two designs are being considered for a rotating shaft that will support a transverse load.

Shaft design A is twice the length of shaft design B.

Which shaft design will have the smaller deflection?
◯ Design A
◯ Design B

Which shaft design will have the smaller stress?
◯ Design A
◯ Design B

TRUE · FALSE · IT DEPENDS

For the same load, the same cross-section, and the same length, which beam design will have the larger deflection?

◯ A cantilever beam
◯ A simply-supported beam

TRUE · FALSE · IT DEPENDS

A shaft is to be designed where the primary concern is to minimize deflection.

Which steel should be used?

◯ A low carbon steel
◯ A high carbon steel

4.1 Best practices for shaft design

1. To minimize both deflections and stresses, the shaft length should be kept as short as possible and overhangs minimized.

2. A cantilever beam will have a larger deflection than a simply supported (straddle mounted) one for the same length, load, and cross-section, so straddle mounting should be used unless a cantilever shaft is dictated by design constraints.

   Pulleys are often overhung, for example, so that the belt can be changed without disassembling the shaft and bearings.

3. A hollow shaft has a better stiffness/mass ratio (specific stiffness) and higher natural frequencies than a comparably stiff or strong solid shaft, but will be more expensive and larger in diameter.

4. Try to locate stress raisers away from regions of large bending moments if possible and minimize their effects with generous radii and reliefs.

5. If minimizing deflection is the primary concern, then low-carbon steel may be the preferred material since its stiffness is as high as that of higher strength steels and a shaft designed for low deflection will tend to have low stresses.

6. Deflections at gears carried on the shaft should not exceed about 0.005 in and the relative slope between gear axes should be less than about 0.03^o.

7. If plain (sleeve) bearings are used, the shaft deflection across the bearing length should be less than the oil-film thickness of the bearing.

8. If non-self-aligning rolling element bearings are used, the shaft angular deflection should be kept less than about 0.03^o at the bearing.

9. If axial thrust loads are present, they should be taken to ground through a single thrust bearing per load direction. Do not split axial loads between thrust bearings as thermal expansion of the shaft can overload the bearing.

10. The first natural frequency of the shaft should be at least 3x the highest forcing frequency expected in service, and preferably much more. A factor of 10x or more is ideal, but this is often difficult to achieve in mechanical systems.

4.2 Critical speeds of rotating shafts

All machine components have mass. A moving mass will store kinetic energy.

All machine components are made of elastic materials. Elastic materials act as springs. Springs store potential energy.

A machine, or a machine component, vibrates when energy is repeatedly transferred between kinetic and potential.

A bell rings because it vibrates at its **natural frequency** (ω_n) after it is struck. This **free vibration** will continue until it dies out due to the damping in the system.

$$\omega_n = \sqrt{\frac{k}{m}}$$

Systems will also vibrate if acted on by a time-varying load (also known as a **driving function** or a **forcing function**). The system will vibrate at the driving function's **forcing frequency**.

Resonance will occur if the forcing frequency coincides with the system's natural frequency.

Two design strategies can be used to avoid resonance.

1. Keep all forcing, or self-exciting, frequencies below the system's first critical frequency.
2. Accelerate the system rapidly through the resonance, before the vibrations have a chance to build up amplitude.

4.3 Coupling hubs to shafts

4.3.1 Keys

A **key** is used to transmit torque between a shaft and a shaft-supported element (see Figure 18).

Keys are available in several styles. The most commonly used keys are straight keys. Straight keys are also known as parallel keys or machine keys.

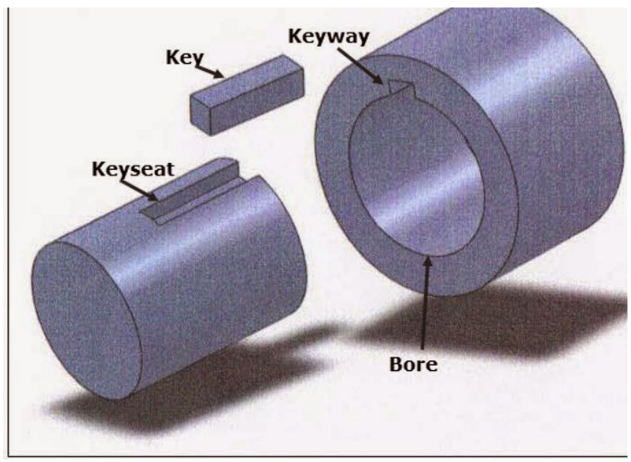


Figure 18: A key transmits torque between two rotating elements.

TRUE · FALSE · IT DEPENDS

The table below summarizes safety factors for three different designs.

	Design A	**Design B**	**Design C**
Keyseat	3.1	3.5	3.92
Key	1.8	4.1	3.90
Keyway	2.8	3.2	3.91

Which of the designs is most appropriate?

◯ Design A
◯ Design B
◯ Design C

Stress concentration factors for the keyseat should be considered when designing the shaft.

Suggestions for stress concentration factors are summarized in Table 7-1 of the Shigley text.

Parallel keys may be negatively toleranced or positively toleranced.

— When **negatively toleranced**, the key will never be larger than the nominal size.
  - The keyseat can be cut with a standard milling cutter at the nominal size.
  - Key will fit in the keyseat with a slight clearance.

— When **positively toleranced**, the key will be slightly larger than the nominal size.
  - Used when a closer fit between the key and keyseat is desired.
  - May require additional machining of the key.

Backlash may occur when the loading torque changes direction.

— Any clearance between the key and the keyseat will result in impact and high stresses.

To avoid backlash...

— Use a positively toleranced key.

— Use a setscrew to secure a negatively toleranced key.

TRUE · FALSE · IT DEPENDS

When using oversized (positively toleranced) keys, the engineer needs to be concerned with backlash when torque-loads cycle between -10 N-m to 50 N-m.

◯ True

◯ False

Key cross-section is dictated by the shaft diameter (see Table 7-6 in the Shigley text).

Key length is chosen to carry the torsional load.

— Limited by the hub length of the attached element.

— Should not be longer than $\approx$ 1.5 times the shaft diameter.

TRUE · FALSE · IT DEPENDS

Describe the failure modes for a key.

Keys fail in two ways.

— The key is sheared across its width at the interface between the hub and the shaft.

— The key is crushed due to a compressive load.

Factors of safety for static loading of keys are found as follows.

1. Shear failure due to static loading.

   The factor of safety compares the shear stress and the material's shear yield strength.

$$n = \frac{S_{sy}}{\tau} = \frac{0.577 S_y}{\tau}$$

   The shear stress is the shear force divided by the area being cut.

$$\tau = \frac{F}{A_{shear}} = \frac{T/r}{w \cdot l}$$

2. Bearing failure due to a compressive load.

   The factor of safety compares the normal stress and the material's compressive yield strength.

$$n = \frac{S_{yc}}{\sigma}$$

   The normal stress is the crushing force divided by the area being crushed.

$$\sigma = \frac{F}{A_{crush}} = \frac{T/r}{\frac{h}{2} \cdot l}$$

TRUE · FALSE · IT DEPENDS

Consider a keyed shaft subjected to cyclic loads.

How would you determine the factor of safety for infinite life?

4.3.2 Example Problems: Key Design

Example 29 A 2500 in-lbf torque is transmitted between a shaft and a gear.

The shaft diameter is 2 inches.

Choose an appropriate key, where the key is made of 1020 cold-drawn steel.

Example 30 The figure below shows a gear keyed to a simply-supported shaft.

The shaft is steel with $S_{ut} = 108$ kpsi and $S_y = 62$ kpsi and has a 2-in diameter.

The shaft has a machined finish, is at room temperature, and the reliability is 90%.

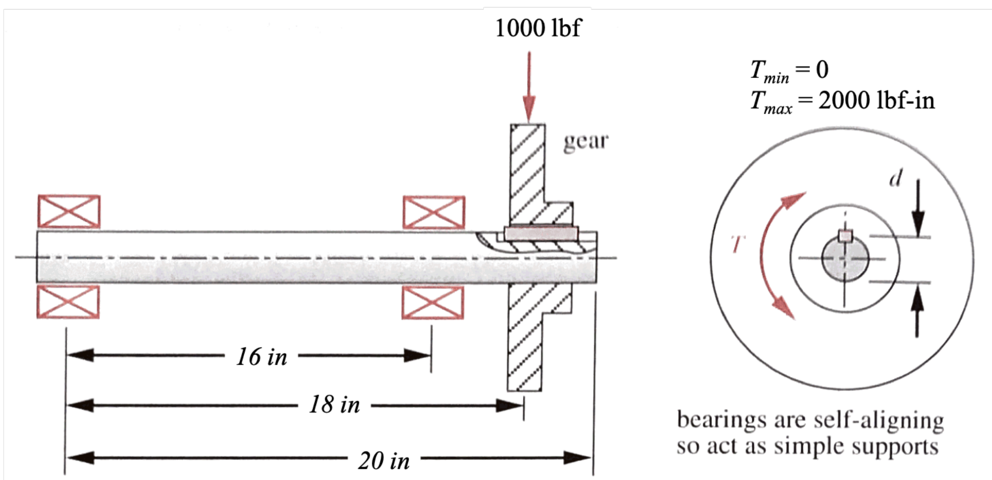
The key is to be chosen from the selection of Machine Keys available on the McMaster-Carr website (`https://www.mcmaster.com`).

Determine the following.

(a) For the shaft, find the factor of safety for infinite life (using the modified Goodman criterion) or the expected life of the part.

(b) Select an appropriate key for the gear, with a factor of safety of 2. Consider crushing and failure by shear. Failure by shear should consider both static and fatigue failures.

Note that the 1000 lbf applied load does not act on the key; the key transmits torque between the gear and the shaft, not the radial load.



4.3.3 Splines

Splines are formed by contouring the outside of the shaft and the inside of the hub, as seen in Figure 19.

Splines have a couple of advantages as compared to keys.

— Splines can transmit larger loads.

— Splines can accommodate large axial motions between the shaft and the hub.

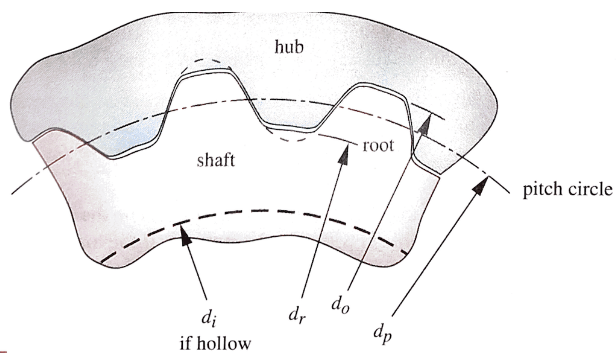


Figure 19: Involute spline geometry.

4.3.4 Interference fits

An **interference fit**, also known as a **press fit** or a **shrink fit**, can also be used to couple a shaft and a hub.

For these fits, the hole in the hub is slightly smaller than the shaft diameter. The elastic deformation of both the shaft and the hub creates large normal and frictional forces between the parts.

- In a press fit, two parts are forced together slowly in a press.
- In a shrink fit, the hub is heated to expand its diameter.
- In an expansion fit, the shaft is cooled to reduce its diameter.

The amount of interference needed varies with shaft diameter.

- A common guideline is 0.001 in of interference for diameters up to 1 inch, and 0.002 in of interference for diameters up to 4 inches.
- Preferred fits are also standardized.

See Table 7-9 in the Shigley text for descriptions of preferred fits using the basic hole system.

— The uppercase letter in the symbol characterizes the hole.

— The lowercase letter in the symbol characterizes the shaft.

The **basic size** (D or d) is the **nominal diameter**.

The maximum hole diameter is:

$$D_{max} = D + \Delta D$$

ΔD is the **tolerance grade** for the hole. The tolerance grade for the hole is specified by the number before the slash in the symbol.

— Clearance fit H11/c11 has tolerance grade IT11.

— Interference fit H7/u6 has tolerance grade IT7.

For basic size D, the tolerance grade is found from Table A-11 for metric sizes or Table A-13 for inch sizes.

The minimum hole diameter is:

$$D_{min} = D$$

For shafts, the maximum shaft diameter is:

$$d_{max} = d + \delta_F$$

δ_F is the **fundamental deviation**. The fundamental deviation for the shaft is specified by the letter after the slash in the symbol.

— Clearance fit H11/c11 has fundamental deviation c.

— Interference fit H7/u6 has fundamental deviation u.

For basic size d, the fundamental deviation is found from Table A-12 for metric sizes or Table A-14 for inch sizes.

For shafts with clearance fits (c, d, f, g, and h), the minimum shaft diameter is:

$$d_{min} = d + \delta_F - \Delta d$$

For shafts with transition or interference fits (k, n, p, s, and u), the minimum shaft diameter is:

$$d_{min} = d + \delta_F + \Delta d$$

Δd is the tolerance grade for the shaft. The tolerance grade for the shaft is specified by the number after the slash in the symbol.

— Clearance fit H11/c11 has tolerance grade IT11.

— Interference fit H7/u6 has tolerance grade IT6.

For basic size d, the tolerance grade is found from Table A-11 for metric sizes or Table A-13 for inch sizes.

The maximum clearance is:

$$c_{max} = \frac{D_{max} - d_{min}}{2}$$

The minimum clearance is:

$$c_{min} = \frac{D_{min} - d_{max}}{2}$$

The average clearance is:

$$c_{avg} = \frac{c_{min} + c_{max}}{2}$$

4.3.5 Example Problems: Fits

Example 31 Consider a H7/p6 interference fit with $D = 60$ mm.

Determine the following.

(a) The maximum and minimum hole diameters.

(b) The maximum and minimum shaft diameters.

Example 32 Find the maximum and minimum clearances for a H8/f7 clearance fit with $D = 15$ mm.

5 Materials and Material Processing

5.1 Numbering, Processing, and Heat Treatment of Steels

Steel is an alloy of iron and carbon.

— In its pure form, iron is soft and generally not useful as an engineering material.

The mechanical properties of steel can be changed.

— Add carbon

  - 0.05-2% carbon by weight
  - Cast iron is steel with more than 2% carbon by weight

— Alloying

  - Manganese greater than 1.65%, silicon over 0.5%, copper above 0.6%, or other minimum quantities of alloying elements such as chromium, nickel, molybdenum, vanadium, or tungsten are present.

— Heat treatment

Table 4: Types of Steel.

	Low-carbon	**Medium-carbon**	**High-carbon**	**Ultra-high-carbon**
% carbon (by weight)	0.05-0.25%	0.3-0.5%	0.6-1.0%	1.25-2%
Also called:				
Ductility				
Machinability				
Weldable				
Tensile strength				
Can be hardened or strengthened with heat treatment				
Cost				
How formed				
Applications	Structural steel Signs Automobiles Furniture Decorations Wire Fencing Nails	Shafts Axles Gears Crankshafts Couplings Forgings	Springs Edged tools High-strength wire	Knives Punches

Tables A-20 and A-21 tabulate material properties for some steels.

5.1.1 How are steels numbered?

The Society of Automotive Engineers (**SAE**) published the Unified Numbering System for Metals and Alloys (**UNS**) in 1975. The American Iron and Steel Institute (**AISI**) adopted a similar system.

The UNS numbering system includes six characters.

— The letter prefix designates the material.

- **G**: carbon and alloy steels
- **A**: aluminum alloys
- **C**: copper-base alloys
- **S**: stainless or corrosion-resistant steels

— The first pair of digits gives the composition, excluding the carbon content.

- **G10**: plain carbon
- **G41**: chromium-molybdenum
- **G43**: nickel-chromium-molybdenum
- See Section 2-8 in the Shigley text for a full listing.

— The second pair of digits gives the approximate carbon content.

— The last digit is for special situations.

The SAE and/or AISI numbering system is the middle four digits of the UNS number.

TRUE · FALSE · IT DEPENDS

What is the composition of the following steels?

UNS No. G10180 (SAE and/or AISI No. 1018)

AISI No. 4140

5.1.2 What are CD and HR?

During **hot working** processes, the metal is heated above its recrystallization temperature.

- Hot rolling (**HR**) creates a bar of a particular shape or dimension.
  - See Figure 2-19 for common shapes made by hot rolling.
- Extrusion is applying large pressure to heated metal, forcing it to flow through an orifice.
- Forging is modern-day blacksmithing, using hammers, presses, or forging machines.

During **cold working** processes, the metal is formed at room temperature.

- Spinning works sheet metal around a rotating form into a circular shape.
- Stamping includes punch-press operations of blanking, coining, forming, and shallow drawing.
- Cold rolling is used to produce wide flats and sheets.
- Cold drawing (**CD**) is the process where hot rolled bars are cleaned and then drawn through a die that reduces the size 1/16 or 1/32 inch.
  - Cold drawn steel usually needs to be drawn multiple times through different dies to achieve the right size, leading to higher production costs.

Compared to hot worked parts, cold worked parts have a bright new finish, are more accurate, and require less machining.

TRUE · FALSE · IT DEPENDS

Using the data in Table A-20 for AISI 1035 steel, sketch the stress-strain diagrams for HR and CD.

TRUE · FALSE · IT DEPENDS

For the following applications, is HR steel or CD steel more appropriate?

Railroad tracks

Metal furniture

Structural I-beams

Machine keys

5.1.3 What about hardness?

Hardness is resistance to penetration by a pointed tool.

Hardness testing is nondestructive in most cases.

— The hardness number (R_B or R_C) is read directly from a dial in Rockwell hardness tests.

— in Brinell hardness tests, the hardness (H_B) is the applied load divided by the spherical surface area of the indentation.

- For many materials, the relationship between S_{ut} and H_B is roughly linear.

TRUE · FALSE · IT DEPENDS

Using the data in Table A-20, what is the relationship between H_B and S_{ut} for steels?

TRUE · FALSE · IT DEPENDS

A steel member has a Brinell of $H_B = 275$.

Estimate the ultimate strength of the steel in MPa.

5.1.4 What about heat treatment?

Heat treatments are time- and temperature-controlled proocesses.

Steels are heat treated for a number of reasons.

— To relieve residual stresses.
  - **Residual stresses** are stresses that remain in a part in the absence of external loading or thermal gradients.

— To modify material properties including hardness (strength), ductility, and toughness (the material's ability to absorb energy).

Annealing is used to soften a material, make it more ductile, relieve residual stresses, and refine the grain structure.

— Material is heated to 100°F above the critical temperature.

— Held at this temperature until carbon dissolves and diffuses through the material.

— Cools slowly, usually in the furnace where it was heated.

Normalizing is a subset of annealing.

— Heated to a slightly higher temperature.

— Cools more quickly.

— Material is harder than fully annealed steel.

Quenching is a controlled cooling rate.

— After being quenched, the metal is very hard, but it is brittle.

Tempering reheats the steel to a temperature lower than the critical temperature, and then cools the steel in still air.

— Adjusts ductility, hardness, and toughness to the desired levels.

TRUE · FALSE · IT DEPENDS

A steel shaft experiences high torque starts and stops. During running, the shaft experiences moderate shocks.

Which is the best choice for heat treating the shaft?

◯ Annealing

◯ Quenching

◯ Tempering

◯ Case Hardening

5.2 Material Selection

1. Use what has been used before for a similar application.

2. Use a systematic approach.

3. Use materials selection charts.

   — See the Ashby charts, Figures 2-23 through 2-27 in the Shigley text.

TRUE · FALSE · IT DEPENDS

You are designing a part with the following material requirements.

— Density (ρ) between 4 and 10 Mg/m^3

— Young's modulus (E) between 60 and 250 GPa

— Strength (S) spanning 25 to 1050 MPa

Select the material(s) that meet the design requirements.

☐ Aluminum alloys

☐ Copper alloys

☐ Lead alloys

☐ Magnesium alloys

☐ Nickel alloys

☐ Carbon steels

☐ Stainless steels

☐ Titanium alloys

☐ Zinc alloys

☐ None of the above

6 Surface Failure

Recall that machines, and machine components, can fail in three ways.

1. Become obsolete
2. Wear out
3. Break (or permanently deform)

Wear encompasses many types of failures, all of which involve changes to the part **surface**.

— Adhesive wear

— Abrasive wear

— Erosion

— Corrosion wear

— Surface fatigue

The wear motions of interest are:

— Sliding

— Rolling

— A combination of sliding and rolling

6.1 Surface Geometry

Even an apparently smooth surface will have microscopic irregularities. The microscopic mountain peaks on a surface are called **asperities**.

When two surfaces are pressed together, the contact area depends on the asperities and is difficult to determine (see Figure 20).

- The tips of the asperities will initially contact the mating part and the initial area will be very small.
- As the mating force increases, the asperity tips yield and spread until the contact area is sufficient to support the load.

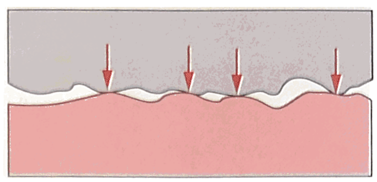


Figure 20: The actual contact between surfaces occurs only at the asperity tips.

6.2 Friction

Understanding asperities and surface contact gives insight into friction forces.

Coulomb friction is the model used to calculate the force of dry friction.

Pressing two surfaces together with a normal force creates:

— Elastic deformations

— Adhesions

The Coulomb friction force is the force necessary to shear the adhered and elastically interlocked asperities in order to allow a sliding motion.

The coefficient of friction can vary greatly, depending on:

— Surface cleanliness

— Surface roughness

— Sliding velocity

— Relative motion (sliding vs. rolling)

— Lubrication

6.3 Adhesive Wear

When clean surfaces are pressed together, asperities will tend to adhere to one another due to the attractive forces between the surface atoms of the two materials.

Scoring or **scuffing** may occur when a particle of one material breaks free and scratches the surface of both parts.

Adhesive wear can be avoided/minimized by pairing appropriate materials (see Figure 21).

— **Identical** and **metallurgically compatible** materials should <u>not</u> be run together in unlubricated sliding contact.

- If mating materials are metals, are compatible, and are extremely clean, the adhesive forces will be high and the sliding friction can generate enough localized heat to weld the asperities together. This is called **cold welding**.
- A lubricant film effectively isolates the two materials and can prevent adhesion even between identical materials.

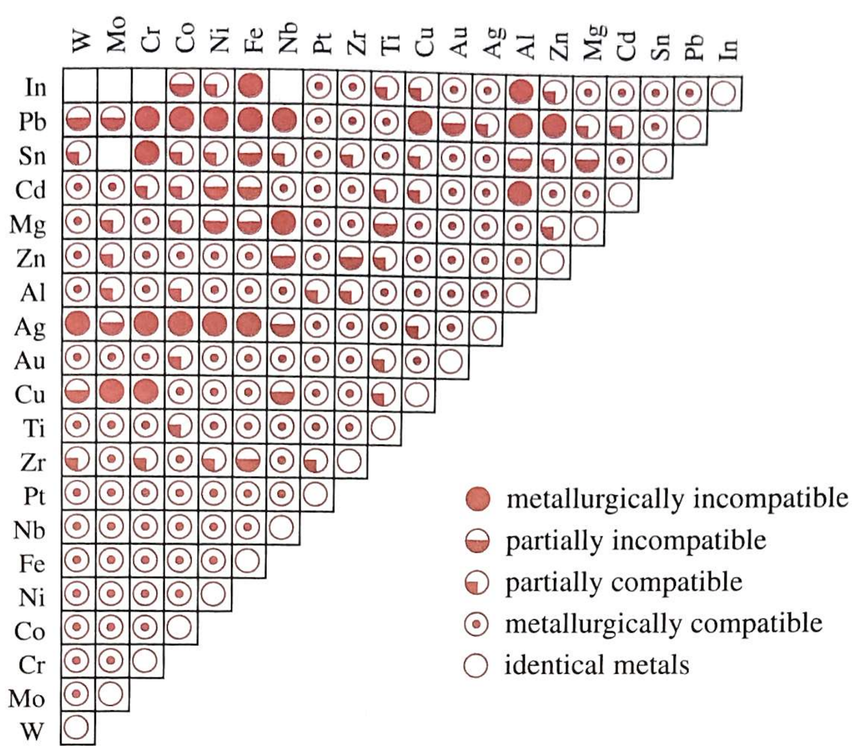


Figure 21: Metallurgically incompatible materials will have the best resistance to adhesive wear.

6.4 Abrasive Wear

Abrasion is a material removal process in which the affected surfaces lose mass.

— **Two-body abrasion** is a hard(er) material sliding against a soft(er) material.

— **Three-body abrasion** is the introduction of hard particles between two sliding surfaces.

Abrasion can be controlled or uncontrolled.

6.5 Corrosion

Corrosion occurs in normal environments and in nearly all materials.

Oxidation is the most common form of corrosion.

— Oxidation in aluminum is self-limiting.

— Oxidation in iron alloys is not self-limiting.

The rate of corrosion increases due to sliding or rolling contact.

— Surface contact can break up the oxide film and expose new material.

— Flakes of the oxide layer can contribute to abrasive wear.

Corrosion fatigue occurs when a part is cyclically stressed in a corrosive environment. Crack growth will be more rapid than due to each individual factor.

Fretting corrosion is significant loss of material from the interface of a press-fit or clamped joint. Fretting is some combination of abrasion, adhesion, and corrosion.

6.6 Surface Fatigue

TRUE · FALSE · IT DEPENDS

Consider a marble rolling over a smooth surface.

— What is the contact area?

— What is the normal stress at the contact patch?

Now consider a cylinder rolling over a smooth surface.

— What is the contact area?

— What is the normal stress at the contact patch?

Hertzian contact stresses are the localized stresses that develop as two curved surfaces come in contact.

— Materials deform to create a sufficient contact area to support the applied load.

— Deformation can be elastic or plastic.

The deformation depends on:

— The moduli of elasticity of the surfaces

— The radii of curvature of the surfaces

See Section 3-19 in the Shigley text.

For a rolling sphere or cylinder, Hertzian contact stresses are repeated at the rotation frequency. This cyclic loading leads to **surface fatigue failure**.

The progression of surface fatigue failure is shown in Figure 22.

1. Crack initiation
2. Pitting
3. Flaking
4. Spalling
5. Disintegration

Figure 22: The progression of surface fatigue failure shown on the inner raceway of a rolling element bearing.

6.7 Example Problems: Surface Failure

Example 33 A carbon steel ball with 25-mm diameter is pressed together with an aluminum ball with 40-mm diameter by a force of 10 N.

Determine the following.

(a) The maximum shear stress.

(b) The depth at which the maximum shear stress will occur for the aluminum ball.

Example 34 An aluminum alloy cylinder roller with 1.25-in diameter and 2-in length rolls on the inside of a cast-iron ring that is 2-in thick and has a 6-in inner diameter.

Determine the maximum contact force F that can be used if the shear stress is not to exceed 4000 psi.

7 Bearings

A **bearing** is a device that supports (bears) a load while allowing motion that is inherent to the machine.

Journal bearings consist of a journal (shaft) and a bearing (see Figure 23).

— The journal and bearing fit together with a high degree of geometrical conformity.

— The load is carried over a large area.

— The clearance between the journal and the bearing is usually 1/1000 of the shaft diameter.

— Pressure is built in the lubricant to support the load.

In **rolling-element bearings** (REB), the load is carried over a small area (see Figure 24).

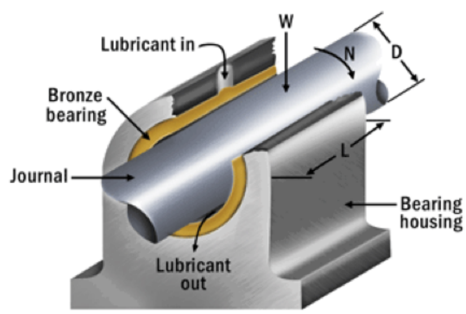


Figure 23: Components of a journal bearing.

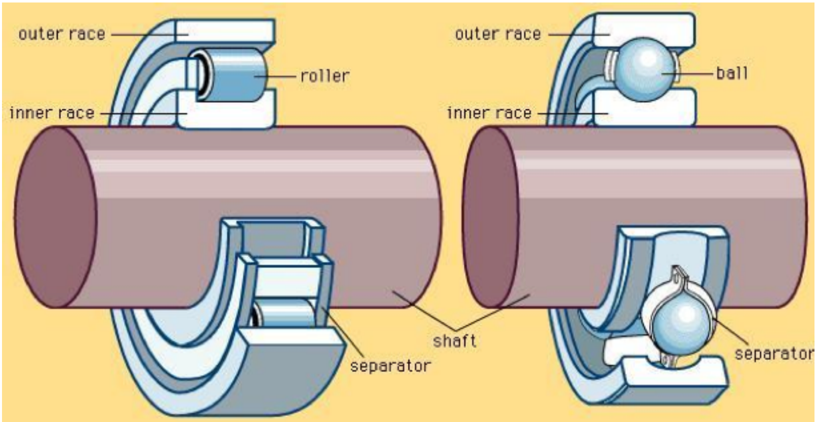


Figure 24: Components of a REB.

Table 5 compares journal bearings and REB.

Table 5: Considerations of engineering design.

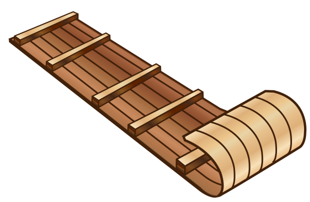
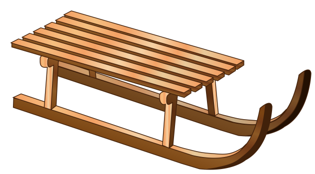
Consideration	**Journal Bearing**	**Rolling Element Bearing**
Motion		
Stresses		
Life		
Friction		
Operating speed		
Cost		
Noise		
Load capacity		
Maintenance		
Rotation accuracy		
Axial length		
Typical applications		

TRUE · FALSE · IT DEPENDS

You're planning to go sledding down Slayter Hill, and looking to up your sled game.

You are considering two bespoke sled designs (Design A and Design B).

The sleds are made of the same material.

Design A	Design B
	

Which design can carry more weight?

◯ Design A

◯ Design B

Assuming both sleds carry the same weight, which design will be faster?

◯ Design A

◯ Design B

Which design do you expect to cost less?

◯ Design A

◯ Design B

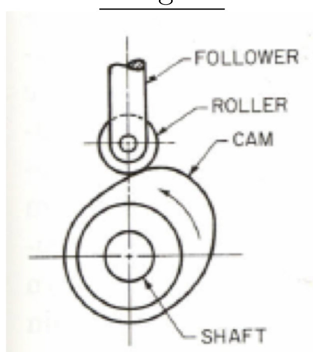
TRUE · FALSE · IT DEPENDS

A cam is a device to move a follower in a specific way.

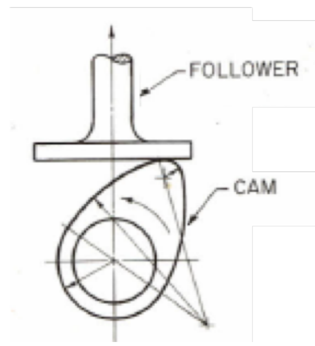
Design A shows a cam with a roller-follower and includes three components: the cam, the roller, and the follower. The roller rolls on the cam surface as the cam rotates about a shaft.

Design B shows a cam with a flat-face follower and includes two components: the cam and the follower. The follower slides on the cam surface as the cam rotates about a shaft.

Design A



Design B



Which design do you expect to be faster?

◯ Design A

◯ Design B

Which design do you expect to wear out faster?

◯ Design A

◯ Design B

Which design do you expect to be more expensive?

◯ Design A

◯ Design B

TRUE · FALSE · IT DEPENDS

You agreed to help a friend move a heavy piece of furniture. You are considering two designs to assist the move.

Design A is a furniture dolly, where the dolly rolls on four casters.

Design B is a set of furniture sliders. The sliders are smooth disks that the piece of furniture rests on.

Design A	Design B
	

Which design will make the piece of furniture easier to move? In other words, which design needs less horizontal force applied to the piece of furniture?

◯ Design A

◯ Design B

If the same horizontal force is applied to the piece of furniture, which design will allow the piece of furniture to move faster?

◯ Design A

◯ Design B

Which design do you expect to be more expensive?

◯ Design A

◯ Design B

8 Rolling-element bearings (REB)

REB manufacturers are an excellent resource for technical information.

— NSK (Tokyo, Japan)

— NTN (Osaka, Japan)

— Schaeffler (Herzogenaurach, Germany)

— SKF (Gothenburg, Sweden)

— Timkin (North Canton, Ohio)

A typical REB consists of an inner ring, an outer ring, rolling elements, and a cage (see Figure 25).

The primary purposes of the cage are:

— Separating the rolling elements to reduce the frictional heat generated in the bearing.

— Keeping the rolling elements evenly spaced to optimize load distribution.

— Guiding the rolling elements in the unloaded zone of the bearing.

— Retaining the rolling elements of separable bearings when one bearing ring is removed during mounting or dismounting.

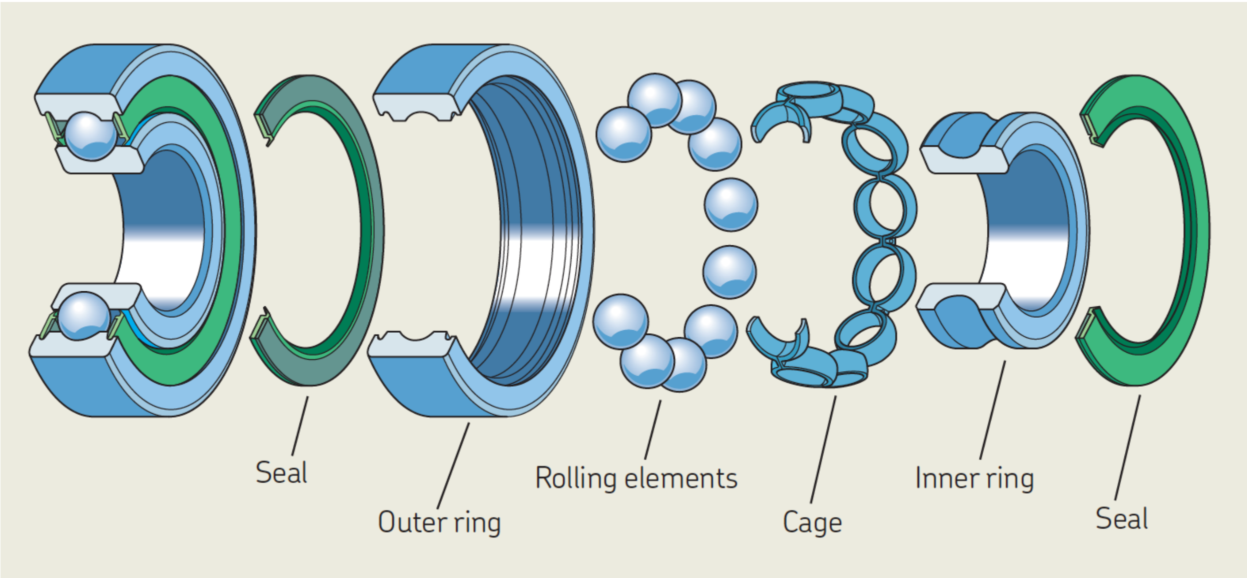


Figure 25: Typical parts of a REB.

8.1 Ball Bearings and Roller Bearings

There are two types of REB: **ball bearings** and **roller bearings**.

Ball bearings:

— Point contact with the ring raceways

— Small(er) contact area → low(er) rolling friction → high speeds, but limited load carrying capacity

Roller bearings:

— Line contact with the ring raceways

— Large(r) contact area → high(er) rolling friction → heavier loads, but at lower speeds

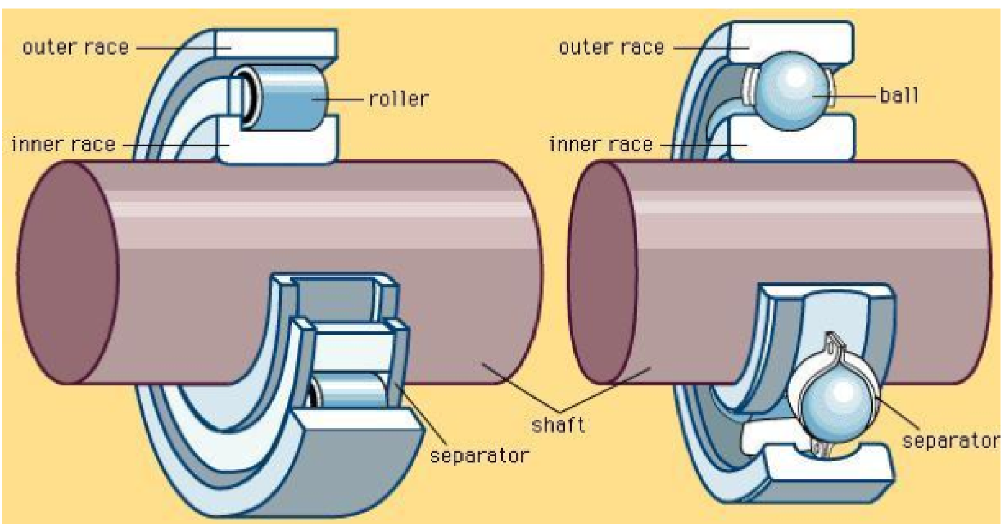


Figure 26: Components of a REB.

Roller bearings accommodate heavier loads than same-sized ball bearings, but the direction of the loads is the main factor in bearing selection.

— **Radial bearings** accommodate loads that act predominantly perpendicular to the shaft.

— **Thrust bearings** accommodate loads that act predominantly along the axis of the shaft.

The **contact angle** determines if a bearing is a radial bearing (contact angle $\leqslant 45°$) or a thrust bearing (contact angle $> 45°$).

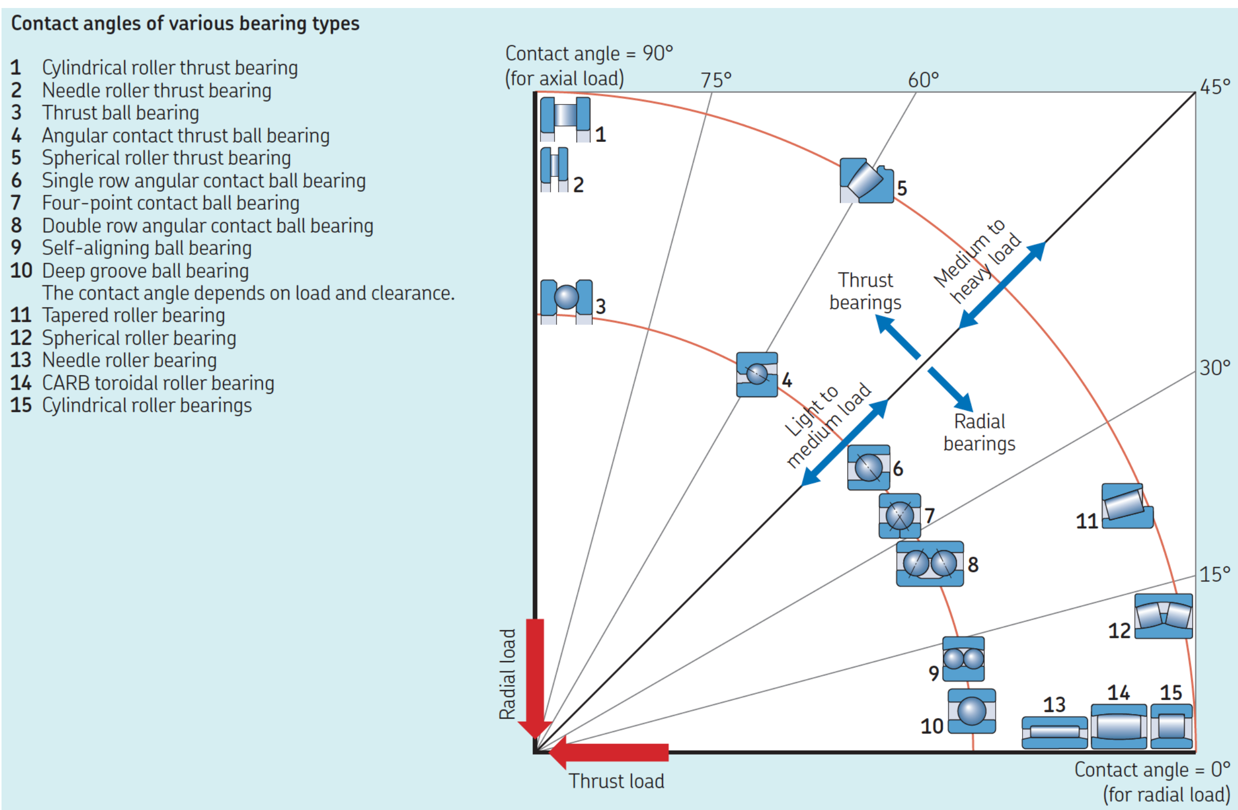


Figure 27: Contact angles of various bearing types.

8.2 REB Failure

REB fail when the the bearing exhibits the first pit. The raceway typically fails first.

Recall for Hertzian contact, τ_{max} occurs just below the surface.

— Crack initiates at the location of τ_{max}.

— The crack propagates to the surface.

— Lubricant enters the crack and eventually wedges a chip loose.

Bearing failure will be heard (growling) and/or felt (vibration).

It started with a whisper. It made a lot of noise when turning.

REB will continue to run after failure.

— The noise and/or vibration will increase.

— Eventually pitting will become spalling.

— Damage will spread to adjacent parts.

REB will not fail if the bearing is...

— Clean

— Properly lubricated

— Sealed

— Properly mounted

— Operated at a reasonable temperature

8.3 REB Life, Load, and Reliability

Bearing life is the number of revolutions (or the number of operating hours at a given speed) that the bearing is capable of enduring before the first sign of metal fatigue.

Tests on seemingly identical bearings under identical operating conditions result in a large variation of bearing life.

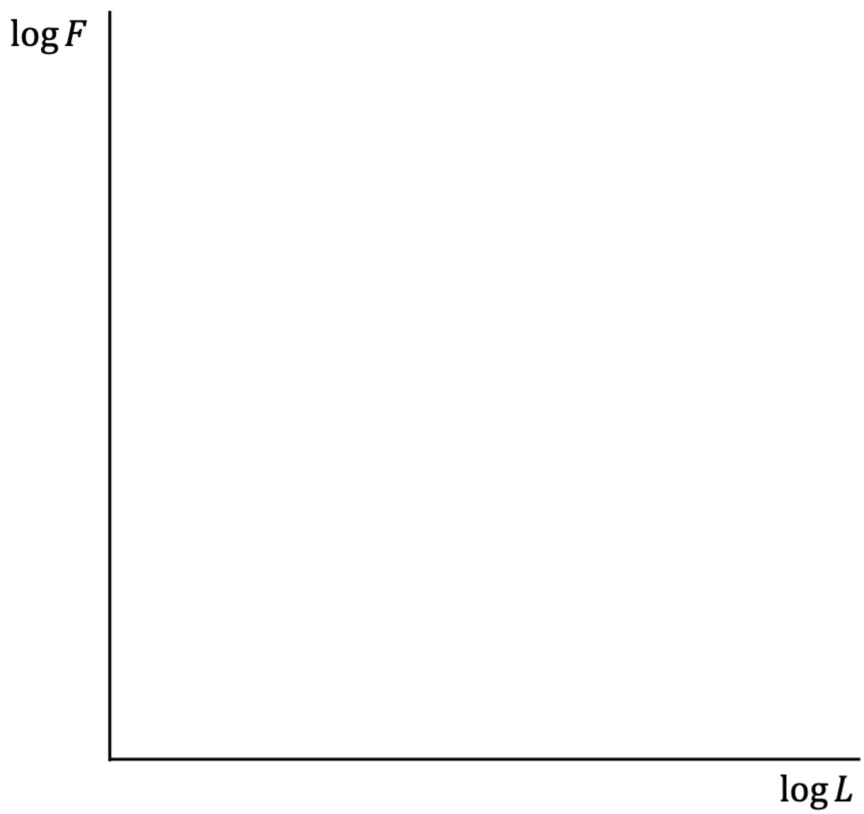


Figure 28: Load-life log-log curve.

To select a bearing, the bearing's load (F), life (L), and reliability requirements are related to the published catalog data. See Figure 29 for an excerpt from a bearing catalog.

$$a_1 F_R L_R^{1/a} = F_D L_D^{1/a}$$

The a_1 parameter is tabulated in the ISO 281 standard and accounts for the desired bearing reliability.

Reliability	a_1
90%	1
95%	0.64
96%	0.55
97%	0.47
98%	0.37
99%	0.25

F_R is the catalog rating load. The catalog rating load is in units of lbf or kN.

— Also referred to as the *Basic Dynamic Load Rating*.

— The rating load is the radial load that causes 10% of a group of bearings to fail at the bearing manufacturer's rating life.

— The catalog load rating is a reference value; it is not an actual load achieved by a bearing.

L_R is the catalog rating life in number of revolutions.

— The rating life is the life that 90% of a sufficiently large group of apparently identical bearings, operating under identical operating conditions, can be expected to attain or exceed.

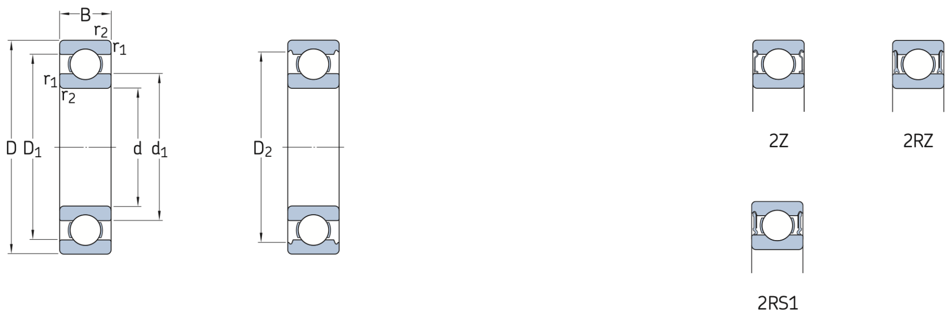
— L_R is usually 10^6 revolutions, but may vary between manufacturers.

F_D is the desired radial load. The desired radial load is in units of lbf or kN.

L_D is the desired life in revolutions.

The exponent a is 3 for ball bearings and 10/3 for roller bearings.

1.1 Single row deep groove ball bearings
d **75 – 80** mm



Principal dimensions			Basic load ratings		Fatigue load limit	Speed ratings		Mass	Designations	
			dynamic	static		Reference speed	Limiting speed[1)]		Bearing open or capped on both sides	capped on one side[1)]
d	D	B	C	C_0	P_u					
mm			kN		kN	r/min		kg	–	
75	95	10	12,5	10,8	0,585	–	4 000	0,15	▸ **61815-2RS1**	–
	95	10	12,5	10,8	0,585	14 000	7 000	0,15	▸ **61815-2RZ**	–
	95	10	12,5	10,8	0,585	14 000	8 500	0,15	▸ **61815**	–
	105	16	24,2	19,3	0,965	13 000	8 000	0,36	▸ **61915**	–

Figure 29: An excerpt from the SKF catalog.

8.4 REB Combined Loading

For REB supporting combined loads, the axial thrust load, F_a, and radial load, F_r, are combined into an equivalent radial load:

$$F_e = X_i V F_r + Y_i F_a$$

The rotation factor, V, is 1 when the inner ring rotates and 1.2 when the outer ring rotates.

X_i and Y_i are found in Table 11-1 of the Shigley text. These factors are tabulated for values of e, where e is correlated to the ratio of the axial thrust load, F_a, and the basic static load rating, C_0.

8.5 Example Problems: REB

Example 35 Determine if bearing A or bearing B can carry the higher load.

Bearing	C_{10} (kN)	**Rating system**
A	2.0	3000 hours at 500 rpm
B	7.0	10^6 cycles

Example 36 A deep groove ball bearing is to be chosen from Table 11-2.

The bearing must support a radial load of 8 kN, an axial load of 3 kN, and have a life of 10^8 revolutions. The outer bearing rotates.

Example 37 The shaft shown in Figure 55 is proposed to support the loads shown in Figure 31.

The shaft rotates freely at a constant angular velocity and is supported by rolling element bearings at O and C.

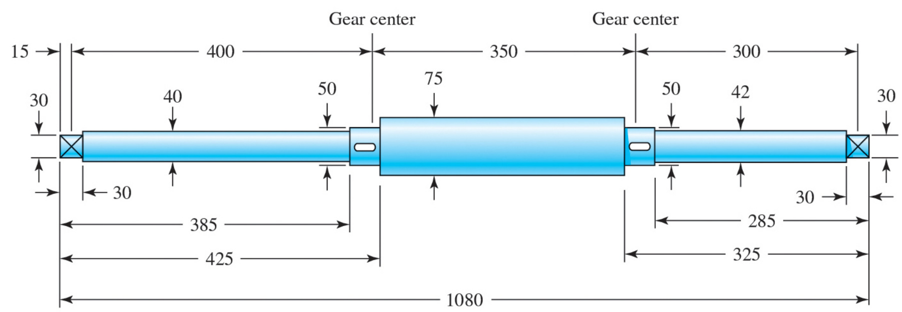


Figure 30: Proposed shaft geometry (units in mm).

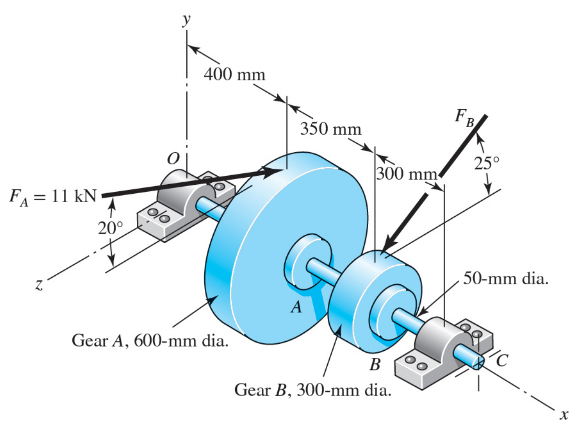


Figure 31: Shaft loading.

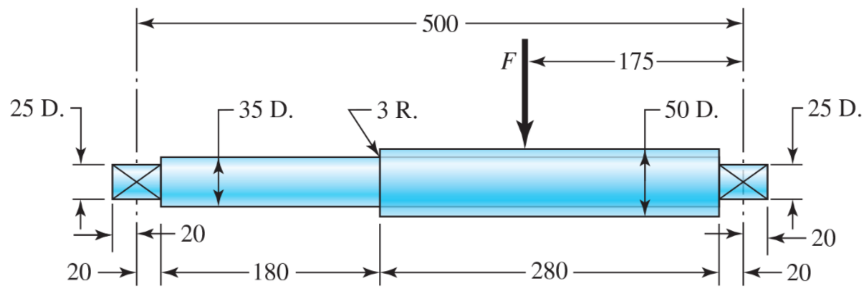
Determine the following.

(a) Determine the magnitudes of the reactions at O and C.

(b) For a bearing life of 10^7 revolutions, select bearings for O and C from the available bearings on McMaster-Carr.

Example 38 The rotating shaft shown below supports a load $F = 6$ kN.

Access the Timkin catalog for deep groove ball bearings:
`https://www.timken.com/resources/10857_deep-groove-ball-brgs-catalog/`



Determine the following.

(a) The L_{10} life for Timkin bearings from the online catalog. Where in the catalog was this information located?

(b) Choose deep groove ball bearings to support the rotating shaft from the bearings on Page 21. The bearings should support the rotating shaft for 10^8 cycles and for 99% reliability.

9 Lubrication

Three types of lubrication occur in sliding bearings.

1. **Boundary**: Surfaces physically contact and adhesive or abrasive wear may occur.
2. **Mixed-film**: Hydrodynamic fluid film begins to form, reducing asperity contact and friction.
3. **Full-film**: Surfaces are fully separated by a film of lubricant.
   (a) Hydrostatic
   (b) Hydrodynamic
   (c) Elastohydrodynamic (EHD)

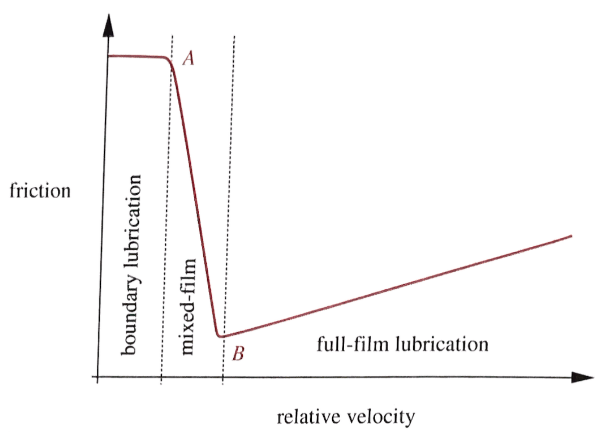


Figure 32: Change in friction with relative velocity in a sliding bearing.

TRUE · FALSE · IT DEPENDS

Consider driving on pavement when it begins to rain.

Describe your experience with the three types of lubrication.

Boundary

Mixed-film

Full-film

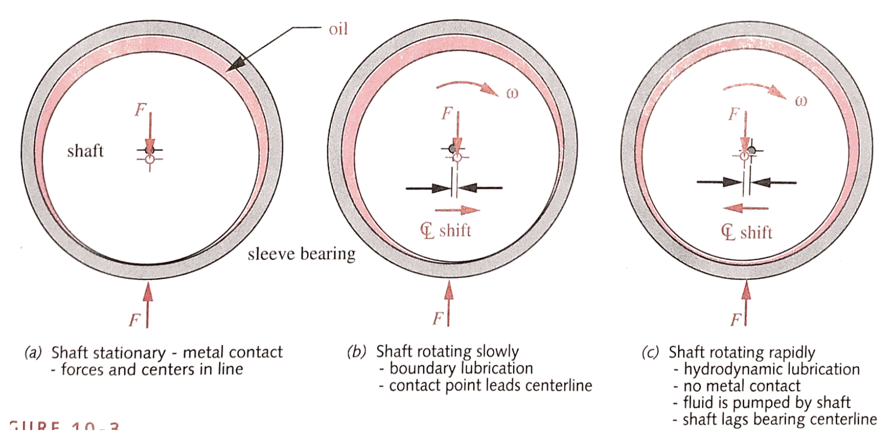


Figure 33: Boundary, mixed-film, and full-film hydrodynamic lubrication in journal bearings during start-up.

Hydrostatic lubrication is the continuous supply of high-pressure lubricant to the sliding interface.

— Requires a reservoir (sump) to hold the lubricant, a pump to pressurize the lubricant, and plumbing to distribute the lubricant.

— Coefficient of friction is 0.002 to 0.010.

— Examples: air hockey tables, air palettes, hovercrafts.

Hydrodynamic lubrication is the supply of sufficient lubricant to the sliding interface to allow the relative velocity of the mating surfaces to pump the lubricant within the gap. The coefficient of friction is 0.002 to 0.010.

Elastohydrodynamic (EHD) lubrication occurs when contacting surfaces are non-conforming (e.g., gear teeth, cam and follower, REB).

— Nonconforming surfaces tend to expel the lubricant.

— Small contact area is big enough of a flat surface to allow a full hydrodynamic film...

- if the sliding velocity is high enough.
- if the contact pressure is high enough in the contact zone to increase the fluid viscosity.

— EHD full-film thickness is 1 μm.

— Surface roughness must be no more than 1/2 to 1/3 of the film thickness.

10 Journal Bearings

Journal bearings are analyzed and designed by relating friction to velocity.

Consider a plate with area A. The plate is acted on by force F and has a velocity U relative to a fixed surface. The plate and surface are separated by a lubricant with thickness h and dynamic viscosity μ.

Now consider a shaft with radius r. The shaft rotates at a speed N within a bearing with length l. The clearance between the journal (shaft) and the bearing is c.

Now incorporate friction f and a supported load W into the analysis of the rotating shaft.

Finally, replace the supported load with the pressure in the lubricant.

Petroff's equation relates the coefficient of friction, f, to the clearance ratio, r/c. Note that Petroff's equation was derived assuming that the journal is concentric with the bearing.

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c}$$

Recall that the shaft speed, N, is in units of revolutions per second.

Pressure, P, is the radial load, W, divided by the projected area of the journal in the bearing.

$$P = \frac{F}{A} = \frac{W}{2rl}$$

Multiplying Petroff's equation by the clearance ratio gives an expression for the **Sommerfeld number**, S.

$$f\frac{r}{c} = 2\pi^2 \frac{\mu N}{P} \left(\frac{r}{c}\right)^2 = 2\pi^2 S$$

The dimensionless group $\frac{\mu N}{P}$ is a design constraint that designates film thickness.

— $\frac{\mu N}{P} < 1.7 \times 10^{-6}$: thin film (boundary or mixed film)

— $\frac{\mu N}{P} \geqslant 1.7 \times 10^{-6}$: thick film (full film)

TRUE · FALSE · IT DEPENDS

For both the thin film and thick film lubrication regimes, consider the progression of events that occurs when lubricant temperature increases.

Consideration	**Thin film**	**Thick film**
Temperature	$\uparrow$	$\uparrow$
Viscosity (μ)		
$\frac{\mu N}{P}$		
Friction (f)		
Heat generated		
Temperature		

10.1 Design of Journal Bearings

When designing/selecting a journal bearing for an application, the following variables are often specified by the overall design of the machine.

— The viscosity μ

— The load W or the nominal pressure P

— The speed N

— The bearing dimensions r, c, and l

Given these variables, the dependent variables can be determined,

— The coefficient of friction f (Figure 12-17)

— The temperature rise ΔT (Figure 12-23)

— The volume flow rate of oil Q (Figure 12-18)

— The minimum film thickness h_0 (Figure 12-15)

10.2 Example Problems: Journal Bearings

Example 39 A full journal bearing has a 25-mm journal diameter, a 25.03-mm bore diameter, and $l/d = 0.5$.

The bearing supports a 1.2 kN radial load and the journal runs at 1100 rpm.

The oil is SAE 30 and the lubricant temperature is 40°C.

Determine the following.

(a) The minimum film thickness, h_0, the position of the minimum film thickness, ϕ, the terminating position of the film, ϕ_0, and the eccentricity, e.

(b) The coefficient of friction, f.

(c) The lubricant total flow, Q, and the side flow, Q_s.

(d) The maximum pressure developed in the film.

(e) The temperature rise.

Example 40 A full journal bearing has a diameter of 3.003 in and $l/d = 1$.

The bearing supports a shaft with $d = 3$ in. The radial load is 675 lbf and the journal rotates at 10 rev/s.

SAE 40 is supplied with a steady-state temperature of 140°F.

Determine the following.

(a) The average film temperature

(b) The minimum film thickness

(c) The heat loss rate

(d) The side-flow rate

Example 41 The shaft has a 2-in diameter and rotates at 2000 rpm.

The shaft is to be supported by a journal bearing with clearance ratio $r/c = 600$ and $l/d = 1$.

The oil is SAE 10 and the operating temperature is 50°F.

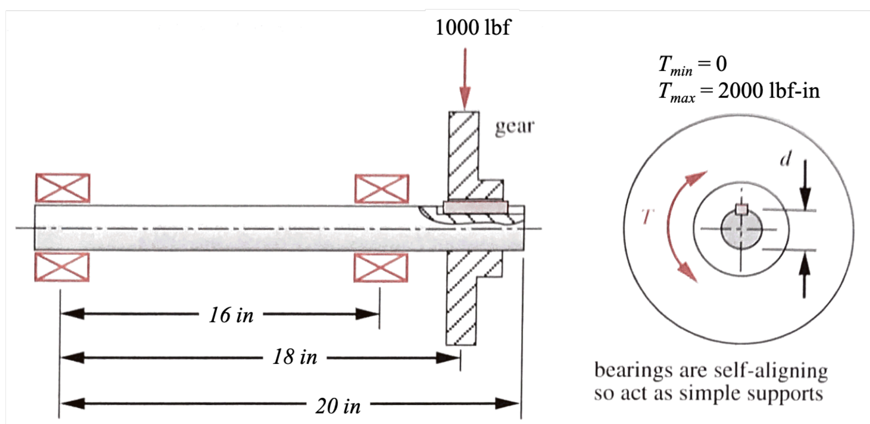
Determine the following for the right bearing only.

(a) The coefficient of friction f.

(b) The volume flow rate of oil Q.

(c) The minimum film thickness h_0.

(d) The power lost due to friction.



Example 42 A full journal bearing is 25 mm long with a l/d ratio of 0.5.

The bushing bore has a diameter of 50.05 mm. The load is 1.5 kN and the journal speed is 800 rev/min. The operating temperature is 50ºC and SAE 30 lubricating oil is used.

Determine the following.

(a) The Sommerfeld number.

(b) The minimum oil-film thickness in mm.

(c) The coefficient of friction.

(d) The power loss to friction in W.

11 Gears

Rolling contact is the simplest way to transfer motion from one rotating part to another.

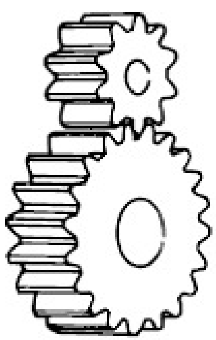
The rotating parts will not slip until the applied torque exceeds the frictional torque.

A **gear set** is formed when meshing teeth are added to the rotating parts. Gear sets are machines that exchange torque for velocity.

— The **pinion** is the smaller gear in the gear set.

— The **gear** is the larger gear in the gear set.

TRUE · FALSE · IT DEPENDS



For the spur gear set shown:

◯ The pinion rotates faster than the gear.
◯ The pinion rotates slower than the gear.

◯ More torque acts on the pinion than on the gear.
◯ Less torque acts on the pinion than on the gear.

The **fundamental law of gearing** states that the angular velocity ratio, m_V, between the gears of a gear set must remain constant throughout the mesh.

$$m_V = \frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}}$$

The **torque ratio**, or **mechanical advantage**, is the reciprocal of the angular velocity ratio.

$$m_A = \frac{1}{m_V} = \frac{\omega_{in}}{\omega_{out}} = \pm\frac{r_{out}}{r_{in}}$$

The $\pm$ in m_A and m_V accounts for internal or external gear sets.

The **gear ratio** is the magnitude of either m_V or m_A.

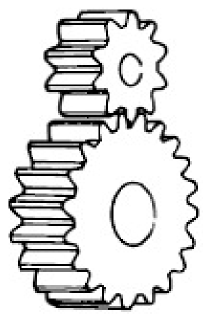
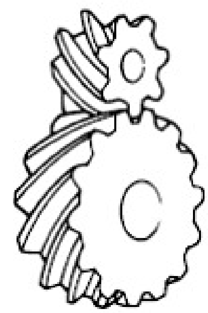
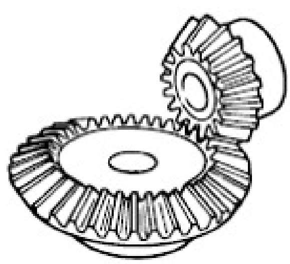
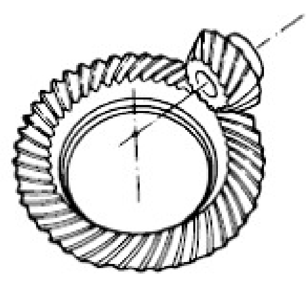
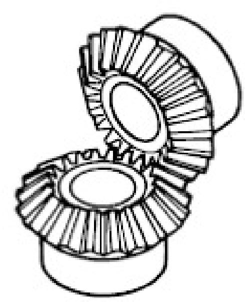
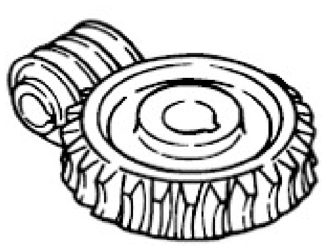
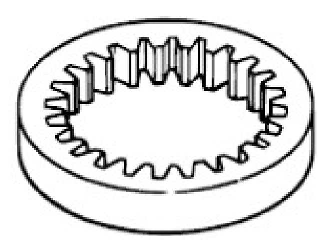
— The gear ratio is usually positive.

— The gear ratio is usually greater than 1.

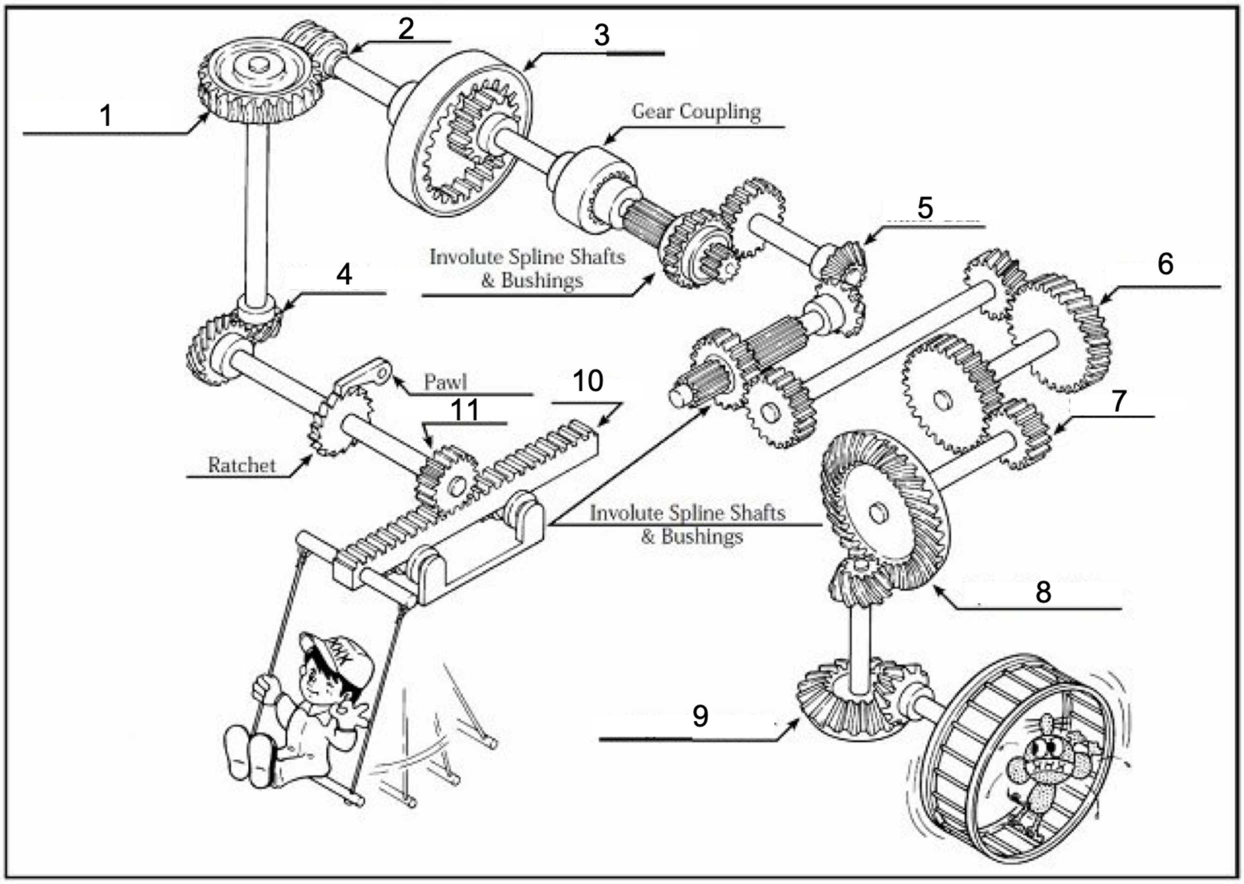
— The gear ratio may be expressed as an integer or as a ratio.

— A gear ratio of up to about 10:1 can be achieved with one pair of gears.

11.1 Gear types

 **Spur gear**	 **Helical gear**	
 **Bevel gear**	 **Spiral bevel gear**	 **Miter gear**
 **Worm gear**	 **Screw gear**	 **Internal gear**

TRUE · FALSE · IT DEPENDS



Match each of the terms with the correct gear in the figure above.

______ Helical gear	______ Rack	______ Straight bevel gear
______ Internal gear	______ Screw gear	______ Worm
______ Miter gear	______ Spiral bevel gear	
______ Pinion	______ Spur gear	______ Worm wheel

11.2 Spur gear terminology

See Figure 13-5 in the Shigley text for spur gear nomenclature.

The **pitch circle** is a theoretical circle that is used for calculations.

The **pitch diameter** (d) is the pitch circle diameter and has units of inches or millimeters.

The pitch circles of two meshing gears are tangent at the **pitch point**.

When two gears are in mesh, their pitch circles roll on one another without slipping.

The **pitch-line velocity** (V) is tangent to the pitch circles and has magnitude:

$$V = |\omega_{pinion} r_{pinion}| = |\omega_{gear} r_{gear}|$$

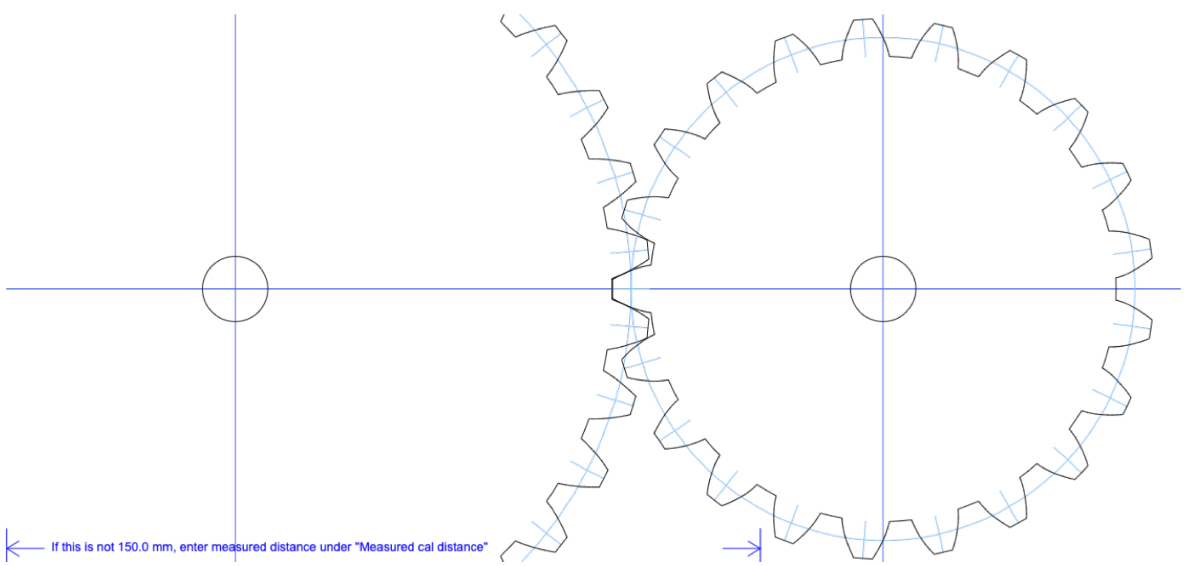


Figure 34: The pitch circles of two meshing gears are tangent at the pitch point.

The **circular pitch** (p) is the distance measured along the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.

- The circular pitch is the sum of the **tooth thickness** and the **width of space**.
- The circular pitch has units of inches or millimeters.

$$p = \frac{\pi d}{N}$$

The **diametral pitch** (P) is the ratio of the gear's number of teeth (N) and the pitch diameter. The diametral pitch is used only in U.S. units and has units of teeth per inch.

$$P = \frac{N}{d} = \frac{\pi}{p}$$

The **module** (m) is used only in SI units and has units of millimeters. The module is the reciprocal of P.

$$m = \frac{d}{N} = \frac{p}{\pi}$$

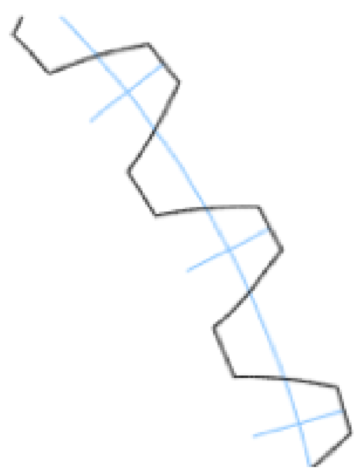


Figure 35: Circular pitch is the sum of the tooth thickness and the width of space.

11.3 Gear tooth profiles

Gear tooth profiles are almost exclusively **involutes** of a circle.

The involute of a circle is the spiraling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle called the base circle.

— The base circle, like the pitch circle, is a theoretical circle that is used for calculations.

As a gearset rotates, the curved surface of a pinion tooth presses against the curved surface of a gear tooth.

— The point of contact is where the two surfaces are tangent to each other.

— The contact force is perpendicular to both tooth surfaces.

The contact force acts along the **line of action**. For involute gear tooth geometry, the line of action is also tangent to the base circles of the pinion and of the gear.

The **pressure angle** (ϕ) is the angle between the line of action and the direction of the pitch-line velocity. Standard values of ϕ are 14.5° and 20° (and sometimes 25°).

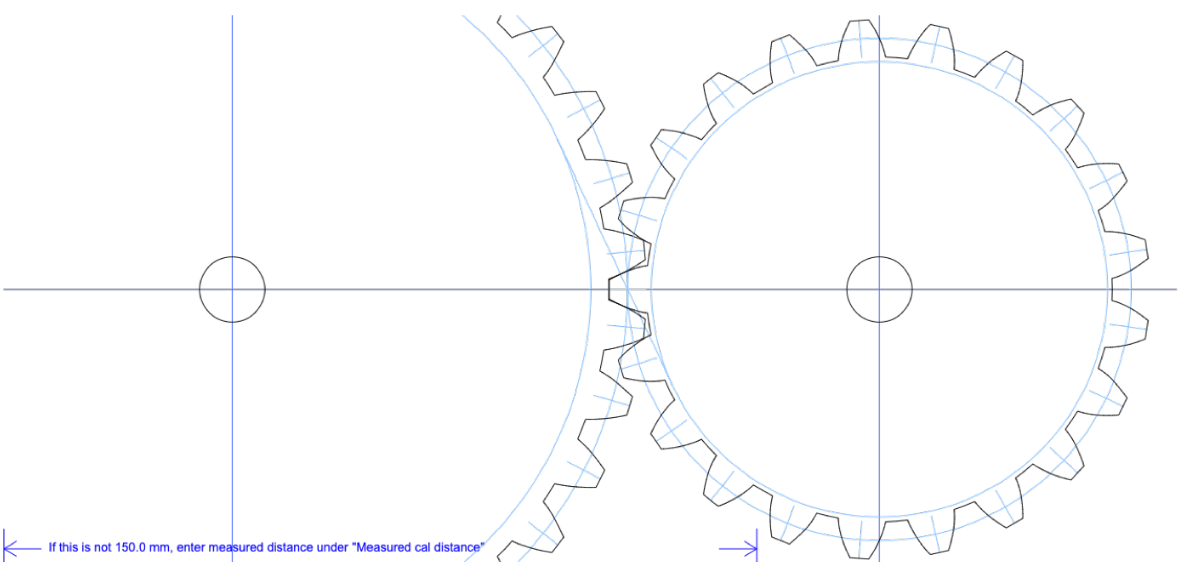


Figure 36: The line of action is tangent to the base circles of the two meshing gears.

11.4 Mesh considerations

The relative motion between two gears in mesh is:

- Pure rolling when the point of contact is the pitch point.
- A combination of rolling and sliding when the point of contact is not the pitch point.

The **contact ratio** (m_c) is the average number of pairs of teeth in contact.

- Continuous motion transfer requires two pairs of teeth in contact as the mesh moves from one pair of teeth to the next.
- Gears should not be designed with $m_c < 1.2$

The contact ratio is calculated from the length of the line of action between the addenda circles of the pinion and the gear (L_{ab}):

$$m_c = \frac{L_{ab}}{p \cos \phi}$$

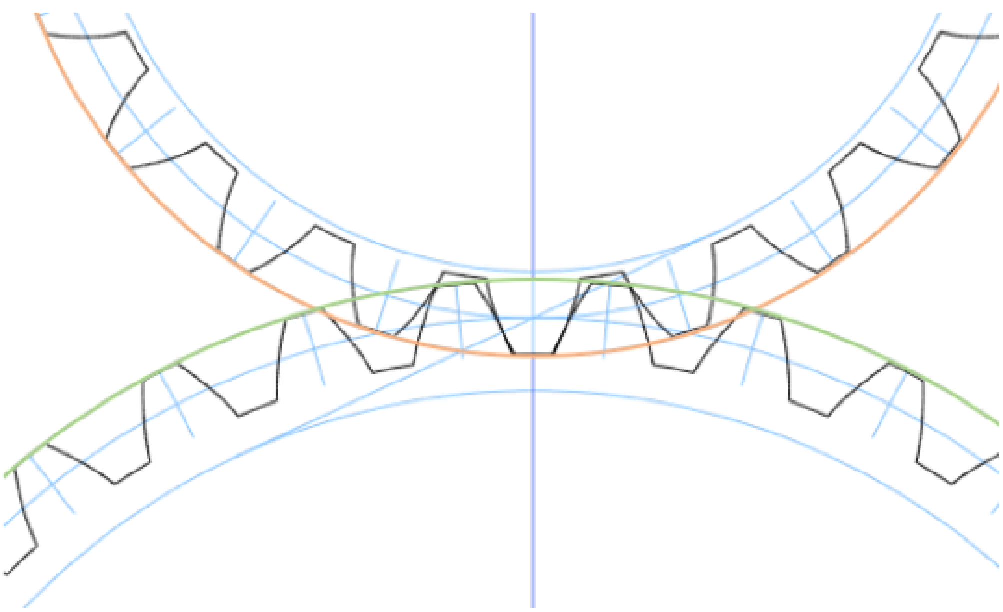


Figure 37: Determination of L_{ab}.

Backlash is the difference between the tooth thickness and the width of space.

— A small amount of backlash is required to prevent gears binding.

— Too much backlash is problematic if the pinion rotation reverses.

Interference occurs when the tooth profiles of meshing gears are not conjugates.

— Contact occurs in the noninvolute portion of the gear tooth.

Interference can be corrected by **undercutting** the gear teeth.

— A cutting tool removes the interfering portions of the teeth.

— Weakens the tooth.

Interference can be avoided by proper choice of gear teeth and pressure angle. See the equations in Section 13-7 of the Shigley text.

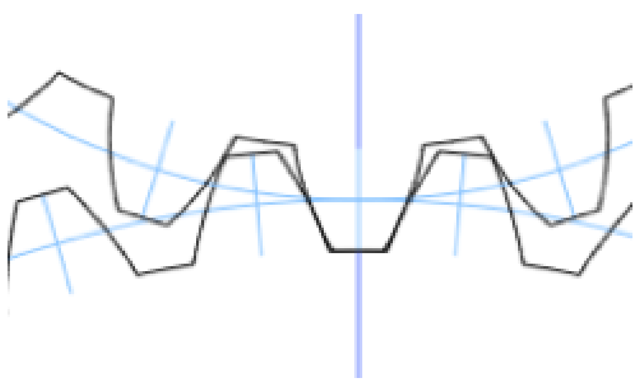
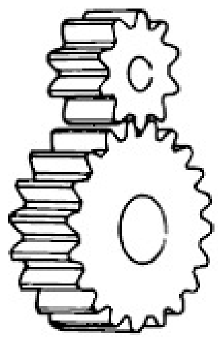


Figure 38: Undercutting gear teeth corrects interference.

11.5 Spur gear force analysis

TRUE · FALSE · IT DEPENDS



For the spur gear set shown, answer the following.

The transmitted load is conserved.
◯ True
◯ False

Torque is conserved.
◯ True
◯ False

Power is conserved.
◯ True
◯ False

Free body diagrams for a pinion meshing with a gear are shown in Figure 39. The pinion and gear have pressure angle ϕ.

Recall that the contact force between meshing gear teeth acts along the line of action.

— The **transmitted load** (W^t) is the tangential component of the contact force.

— The radial component of the contact force serves no useful purpose.

Torque is related to transmitted load by gear radius.

$$T_{pinion} = r_{pinion} W^t$$

$$T_{gear} = r_{gear} W^t$$

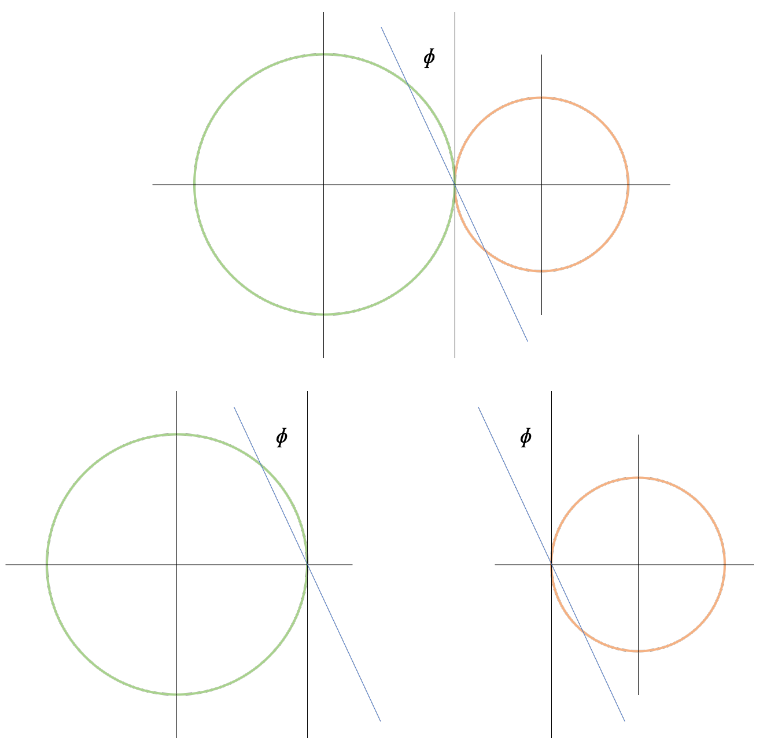


Figure 39: Free body diagrams of spur gears.

11.6 Gear failure

Gears fail due to one of two potential reasons.

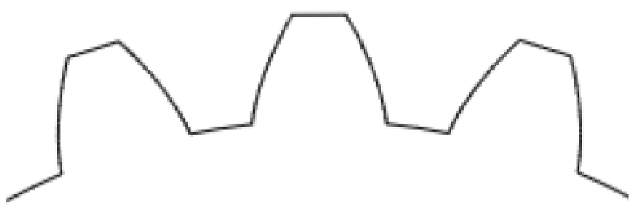
1. Surface fatigue.
2. Fatigue due to fluctuating bending stresses at the root of the gear tooth.

TRUE · FALSE · IT DEPENDS

A gear tooth can be modeled as a cantilevered beam.

Derive an expression for the bending stress at the base of the gear tooth.

The gear tooth has face width F, is t wide at the root, and the transmitted load W^t acts a distance l from the tooth root.



11.7 The Lewis equation

The bending stress at the base of the tooth is the basis for the original Lewis equation.

$$\sigma = \frac{Mc}{I} = \frac{6W^t l}{Ft^2}$$

The original Lewis equation has since been extended to include a velocity factor K_v and a form factor Y.

$$\sigma = \frac{K_v W^t P}{FY}$$

K_v is the velocity factor and accounts for dynamic effects. The velocity factor is a function of the pitch line velocity (V), material, and tooth profile.

W^t is the transmitted load.

P is the diametral pitch.

F is the face width.

Y is the Lewis Form Factor. The Lewis Form Factor is a function of number of teeth and is tabulated in Table 14-2 in the Shigley text for a pressure angle $\phi = 20°$.

The Lewis equation includes the following assumptions.

— Gear teeth do not share the load.

— The greatest force is exerted at the tip of the tooth.

11.8 AGMA Stress and Strength Equations

Road maps for the American Gear Manufacturers Association (AGMA) approaches to spur gear analysis are found in the Shigley text.

— Figure 14-17: Spur gear bending

— Figure 14-18: Spur gear wear

11.9 Example Problems: Spur Gears

Example 43 A simple gear train contains a pinion, an idler gear, and a gear.

The pinion shaft passes 20 hp at 2500 rpm.

The train ratio is 3.5:1.

The pinion has 14 teeth, a 25° pressure angle, and diametral pitch $P = 6$ teeth/inch.

The idler has 17 teeth.

Determine the following.

(a) The gear diameters in inches.

(b) The transmitted load.

Example 44 A 15-tooth spur pinion has a module of 3 mm and runs at a speed of 1600 rpm.

The driven gear has 60 teeth.

Determine the following.

(a) The speed of the driven gear.

(b) The circular pitch.

(c) The theoretical center-to-center distance.

(d) Is the gearset susceptible to interference?

Example 45 A spur gearset consists of a 21-tooth pinion and a 33-tooth gear.

The pitch-line velocity is $V_t = 2000$ ft/min.

Both the pinion and the gear are manufactured to a quality level of 10.

It is proposed that standard 25°, full-depth teeth are used, with both pinion and gear hobbed from an AISI 4140 grade 2 nitrided steel that is through-hardened to 300 Brinell.

A pinion life of 10^5 cycles is desired with a 96% reliability.

Complete the following table in order to use the AGMA equations.

Variable	**Pinion**	**Gear**
W^t	2800 lbf	2800 lbf
K_o	1	1
K_v		
K_s		
P_d	6 teeth/inch	6 teeth/inch
F	2 inches	2 inches
K_m	1.7	1.7
K_B		
J		
S_t		
Y_N		
K_T		
K_R		
S_F		
C_p		
d_P		
C_f		
I	0.117	0.117
S_c	163 ksi	163 ksi
Z_N		
C_H		
S_H		

11.10 Gear Trains

TRUE · FALSE · IT DEPENDS

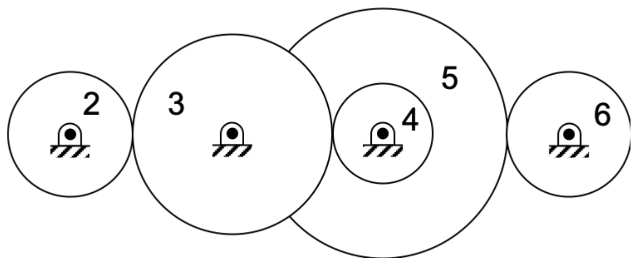
The pinion in the gear train below has angular speed ω_2. Gear 6 is the gear train output.

Gears 4 and 5 are on a common shaft.

Determine the following.

(a) The gear ratio for the gear train.

(b) Identify the gear that functions as an idler.



A pinion (gear 2) has N_2 teeth, pitch diameter d_2, and rotates at n_2 rpm.

The pinion drives gear 3. Gear 3 has N_3 teeth, pitch diameter d_3, and rotates at n_3 rpm.

The speed of the driven gear (gear 3) is:

$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right|$$

A gear ratio of up to about 10:1 can be achieved with a single pair of gears.

— Larger gear ratios can be achieved by combining more gears in a gear train.

The **train value** of a gear train is:

$$e = \pm \frac{product\ of\ driving\ tooth\ numbers}{product\ of\ driven\ tooth\ numbers}$$

e is positive if the first and last gears rotate in the same direction and negative if they rotate in opposite directions.

— Count the number of meshes. e is negative for an odd number of meshes and positive for an even number of meshes.

11.10.1 Compound Gear Trains

A **compound gear train** has two, or more, gears attached to the same shaft.

— Large(r) train ratios while minimizing footprint.

The recipe for designing a compound gear train is as follows.

1. Determine/choose the number of stages to attain the desired gear ratio.
2. Divide the overall ratio into the portions for each stage.
   — Keep portions as evenly divided between stages as possible.
     - For a two-stage compound gear train, assign the square root of the overall train value to each stage.
     - For an exact train value, factor the value into integer components.
3. Pick the number of pinion teeth in order to avoid interference.
4. Pick the number of teeth for the mating gear.

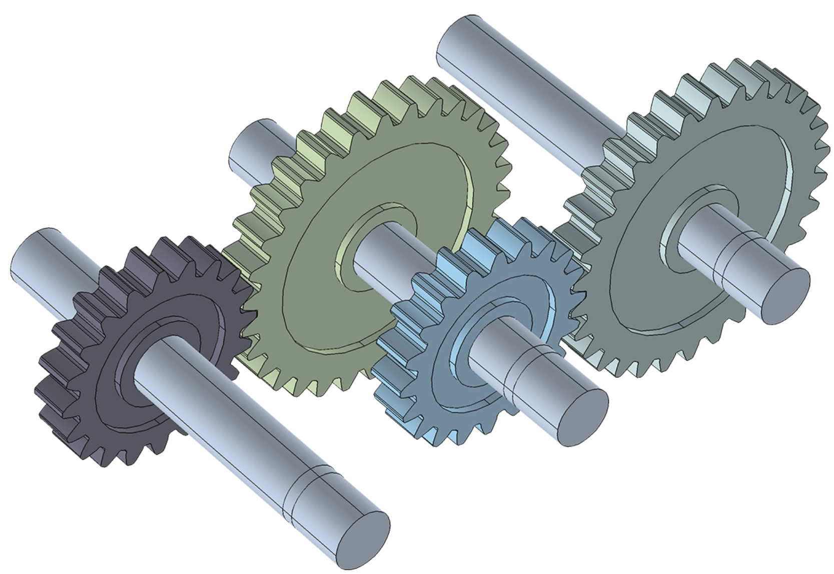


Figure 40: A two-stage compound gear train.

A **compound reverted gear train** is a compound gear train where the input and output shafts are algined.

This requires the distances between the shafts to be the same for both stages of the gear train.

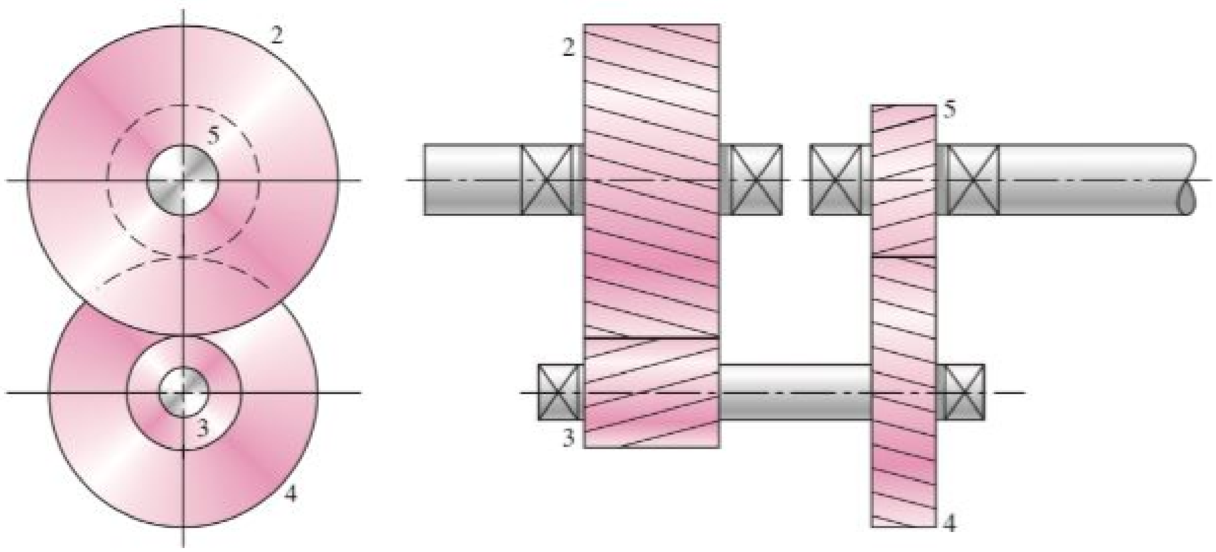


Figure 41: A two-stage compound reverted gear train.

11.10.2 Epicyclic Gear Trains

TRUE · FALSE · IT DEPENDS

Planetary gearsets consist of ring, planet(s), and sun gears.

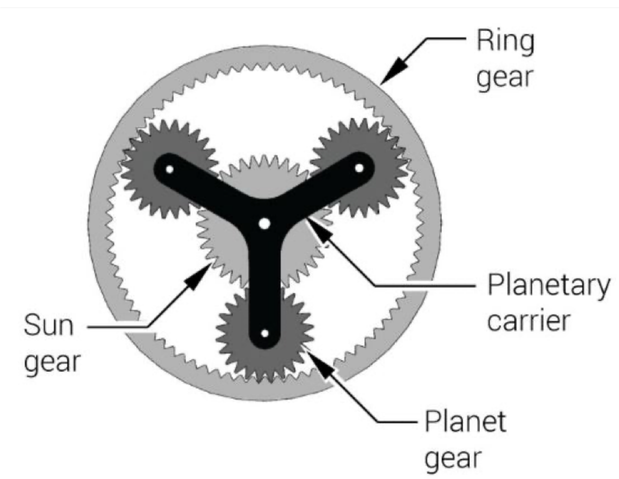
The planetary carrier, or arm, connects the planets.

Of the ring, planet, and sun gears, one is the input, one is the output, and one is fixed.

Two equations relate the gears' number of teeth to the gears' angular velocities.

$$N_s\omega_s + N_p\omega_p - (N_s + N_p)\omega_c = 0$$

$$N_r\omega_r - N_p\omega_p - (N_r - N_p)\omega_c = 0$$



Derive the gear ratio if the sun is the fixed gear and the ring is the output.

11.11 Example Problems: Gear Trains

Example 46 Design a three-stage compound spur gear train for an overall ratio of approximately 592:1. Specify tooth numbers for each gear in the train.

Example 47 A compound reverted gear train is to be designed to provide the appropriate ratio between the minute hand and the hour hand of the clock.

The hour hand is attached to the output shaft such that the hour hand will rotate with gear 5.

The minute hand rotates with gear 2.

Specify appropriate numbers of teeth, while minimizing the gearbox size and avoiding interference in the teeth. Use spur gears with a 20° pressure angle.

Example 48 Design a planetary gear train for an overall velocity ratio of exactly 0.2 if the sun gear is the input, the arm is the output, and the ring gear is stationary. Specify tooth numbers for each gear in the train.

11.12 Helical Gears

Helical gears are very similar to spur gears.

The teeth are angled with respect to the axis of rotation at a **helix angle** ψ. The helix angle is typically 10 to 45°.

Advantages of helical gears over spur gears include the following.

— Helical gears run quieter and with less vibration due to gradual tooth contact.

— Helical gears have stronger teeth for the same normal pitch, pitch diameter, and number of teeth.

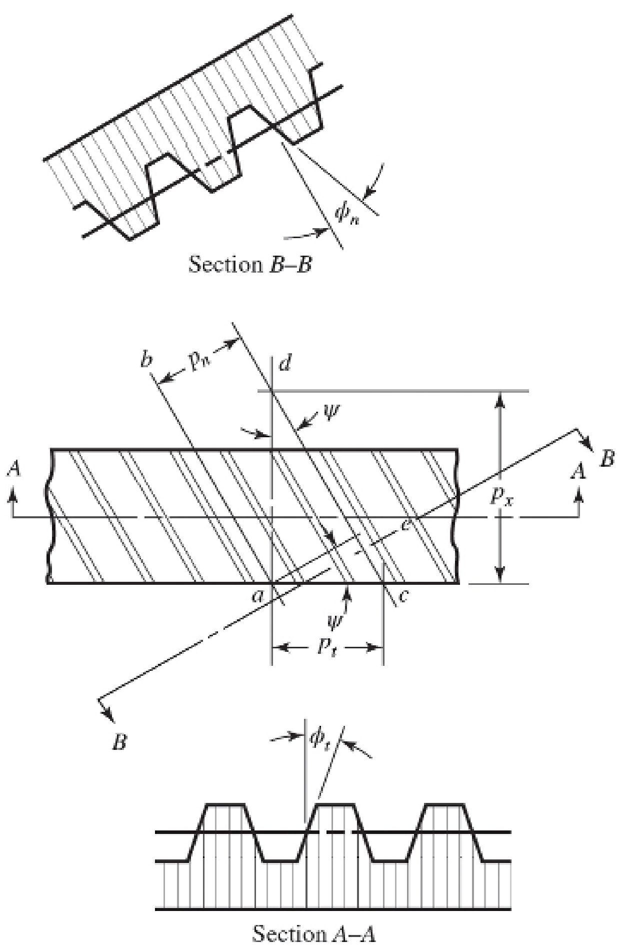


Figure 42: Helical gear nomenclature. Section A-A is the transverse plane and Section B-B is the normal plane.

TRUE · FALSE · IT DEPENDS

You are designing a transmission for a race car. Should you use spur gears or helical gear in the gearset?

Consideration	**Spur gears**	**Helical gears**
Noise		
Load		
Smoothness		
Axial load		
Friction		
Efficiency		

The pressure angles in the two planes are related by:

$$\tan\phi_t = \tan\phi = \tan\phi_n / \cos\psi$$

The resultant force W acting on a tooth is at a compound angle defined by ϕ and ψ.

The transmitted load W^t is found the same way as for spur gears, and can be found from the torque applied to the gear or to the pinion.

$$W^t = \frac{T_p}{r_p} = \frac{T_g}{r_g}$$

In addition to the radial component W_r due to the pressure angle, an axial component W_a acts to separate the gears axially.

$$W_r = W^t \tan\phi$$

$$W_a = W_t \tan\psi$$

$$W = \frac{W^t}{\cos\psi\cos\phi_n}$$

The AGMA equations for bending stress and for wear in spur gears are also used for helical gears.

— Geometry factors I and J must account for ψ.

For a real application, see the standard **ANSI/AGMA 2001-D04 (R2016): Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth**.

11.13 Bevel Gears

Bevel gears are cut on mating cones rather than the mating cylinders of spur or helical gears.

— **Straight bevel gears** have teeth cut parallel to the cone axis, analogous to spur gears.

— **Spiral bevel gears** have teeth cut at spiral angle ψ to the cone axis, analogous to helical gears.

Advantages of spiral bevel gears over straight bevel gears are similar to the advantages of helical gears over spur gears.

— Spiral bevels run quieter and smoother than straight bevels.

— Spiral bevels can be smaller in diameter for the same load capacity.

The AGMA equations for bending stress and for wear are summarized in Figures 15-14 and 15-15 in the Shigley text.

For a real application, see the standard **ANSI/AGMA 2005-D03 (R2008): Design Manual For Bevel Gears**.

11.14 Worm Gears

A **wormset** consists of a **worm** and a **worm gear**.

— The worm is analogous to a screw thread, and the worm gear is analogous to a nut.

The worm is a a helical gear with a helix angle so large that a single tooth wraps continuously around its circumference.

— The gear ratio is the number of teeth on the worm gear.

Advantages of wormsets include the following.

— Large gear ratios (up to 360:1).

  – Used to greatly increase torque and/or greatly reduce speed.

— Can be self-locking.

  – Depends on geometry, friction, surface finish, lubrication, and vibration.

The tooth forms for worms and worm gears are not involutes.

— Worms and worm gears are not interchangeable; they are made and replaced as sets.

— Relative motion between worm and worm gear is sliding.

Only a few materials are suitable for wormsets.

— The worm is highly stressed and requires a hardened steel.

— The wormgear needs to be made from a material soft and compliant enough to conform to the hard worm under high-sliding conditions.

For a real application, see the standard **ANSI/AGMA 6022: Design Manual For Cylindrical Wormgearing**.

11.15 Example Problems: Helical, Bevel, and Worm Gears

Example 49 A helical gearset is to transmit 5 hp with an input pinion speed of 300 rpm.

The helical gears have a 20° normal pitch angle and a 30° helix angle and a normal diametral pitch of 6 teeth/inch.

The pinion has 16 teeth and drives a 48 tooth gear.

The gears are to be grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards. The gears will be uncrowned and centered on their shafts between bearings.

A pinion life of 10^8 cycles is desired with a 90% reliability.

Determine the AGMA factors of safety for bending and contact stresses.

Example 50 A catalog of stock bevel gears lists a power rating of 5.2 hp at 1200 rpm pinion speed for a straight-bevel gearset consisting of a 20-tooth pinion driving a 40-tooth gear.

The gear pair has a 20° normal pressure angle, a face width of 0.71 in, a diametral pitch of 10 teeth/in, and is through-hardened to 300 BHN.

Assume the gears are for general industrial use, are generated to a transmission accuracy number of 5, and are uncrowned.

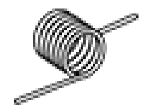
Also assume the gears are rated for a life of 3×10^6 revolutions with a 99% reliability.

Given these data, comment on the stated catalog power rating.

12 Mechanical Springs

A **spring** is a machine component that is intentionally significantly more compliant from the other parts in a load bearing path.

Springs are designed to provide a push, a pull, or a twist force or to store/absorb energy.

 **Compression Spring**	 **Extension Spring**	 **Torsion Spring**

The spring rate k is the slope of the load-deflection curve.

$$k = \frac{F}{y}$$

$$k = \frac{M}{\theta}$$

Spring rates are constant for linear springs and vary for nonlinear springs. Linear springs are often desired in order to control loading.

Springs can be combined in series or parallel.

— When combined in <u>series</u>, the same force passes through all springs and each spring contributes to the total deflection.

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + ... + \frac{1}{k_n}$$

— When combined in <u>parallel</u>, all springs have the same deflection and the force splits among the individual springs.

$$k_{total} = k_1 + k_2 + ... + k_n$$

12.1 Helical Compression Springs: Geometry

The **wire diameter** is d and the **mean coil diameter** is D.

The **inner diameter** (D_i) and **outer diameter** (D_o) are calculated from D and d.

$$D_i = D - d$$
$$D_o = D + d$$

The **spring index**, C, is the ratio of D and d.

$$C = \frac{D}{d}$$

For a helical compression spring, C should be between 4 and 12. A spring with $C < 4$ is hard to manufacture and a spring with $C > 12$ is prone to buckling and/or tangling.

The **pitch**, p, is the axial length between adjacent coils.

The **free length** of the spring, L_0, is the overall spring length in the unloaded condition.

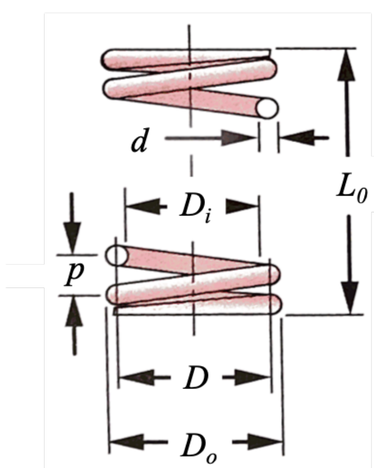


Figure 43: Helical compression spring dimensional parameters.

The **assembled length** L_a is the length after installation to its initial deflection $y_{initial}$.

— The initial deflection and the spring rate k determines the **preload** at assembly.

The **working load** compresses the spring through its **working deflection** $y_{working}$.

The **minimum working length** L_m is the shortest dimension to which the spring is compressed in service.

The **shut height** or **solid height** L_s is the length when compressed such that all coils are in contact.

— The spring can support loads up to the compressive strength of the wire when compressed to L_s.

A minimum clash allowance of 10-15% is recommended to avoid reaching the shut height in service with out-of-tolerance springs, or with excessive deflections. The clash allowance is:

$$\frac{L_m - L_s}{y_{working}} = \frac{y_{clash}}{y_{working}}$$

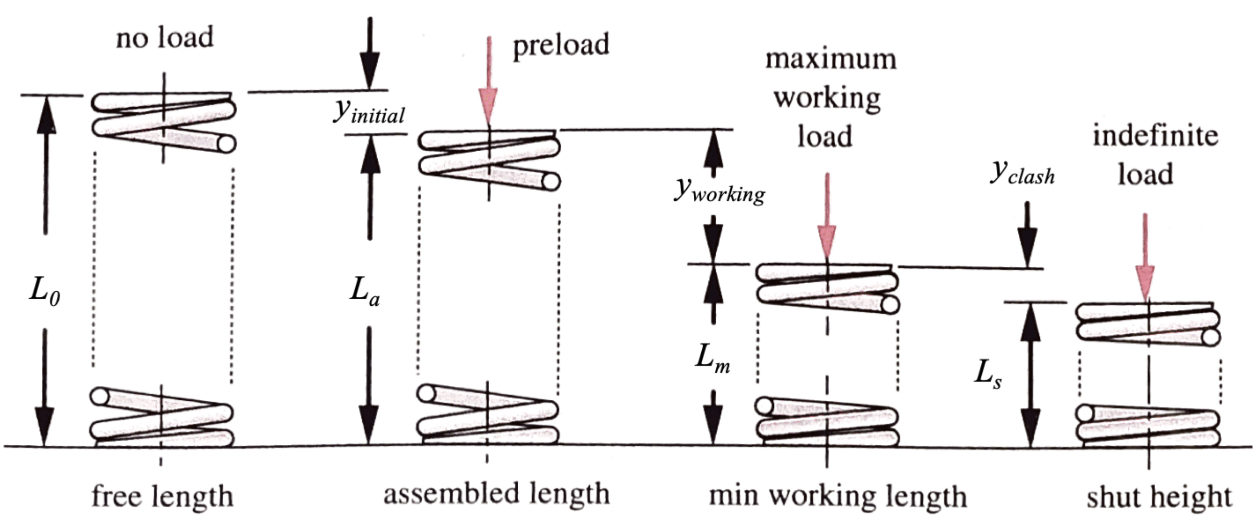


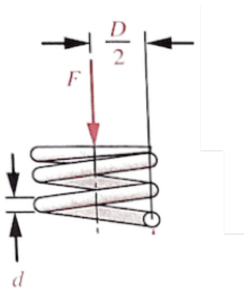
Figure 44: Lengths of a helical compression spring in use.

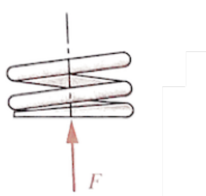
12.2 Helical Compression Springs: Stress

TRUE · FALSE · IT DEPENDS

A helical compression spring, with mean coil diameter D and wire diameter d, is compressed by force F.

The internal loads acting in the wire are (select all that apply):
◯ Axial tension
◯ Axial compression
◯ Torsion
◯ Bending
◯ Transverse shear





TRUE · FALSE · IT DEPENDS

A helical compression spring, with mean coil diameter D and wire diameter d, is compressed by force F.

Sketch the stress distribution due to torsion and transverse shear.

Derive an expression for τ in terms of d, D, and F.

The exact solution for the combination of torsion and transverse shear is:

$$\tau = \frac{Tr}{J} + \frac{F}{A} = \frac{F\frac{D}{2}\frac{d}{2}}{\frac{\pi d^4}{32}} + \frac{F}{\frac{\pi}{4}d^2} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Incorporating the spring constant C gives:

$$\tau = \frac{2C}{2C} \cdot \frac{8FD}{\pi d^3} + \frac{8FD}{2C\pi d^3} = \left(\frac{2C+1}{2C}\right) \cdot \frac{8FD}{\pi d^3} = K_s \frac{8FD}{\pi d^3}$$

The coefficient K_s acts as a stress concentration factor.

Now consider the effect of wrapping the wire into a helix. Doing so increases the stress at the inner fiber, as illustrated in Figure 45.

Figure 45: Stress distribution due to torsion, transverse shear, and stress concentration.

The stress concentration factor K_s can be replaced by another K factor that also accounts for curvature.

Two options are the Wahl factor, K_W, and the Bergsträsser factor, K_B.

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$K_B = \frac{4C+2}{4C-3}$$

TRUE · FALSE · IT DEPENDS

Within the range of recommended values of the spring index C, determine the maximum and minimum percentage difference between the Wahl factor, K_W, and the Bergsträsser factor, K_B.

12.3 Helical Compression Springs: Spring Rate

The spring rate k is the slope of the force-deflection curve.

$$k = \frac{F}{y}$$

The deflection, y, is found using Castigliano's theorem:

$$y = \frac{\partial U}{\partial F}$$

The total strain energy includes a torsional component and a shear component.

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Substituting parameters relevant to helical compression springs gives:

$$U = \frac{4F^2 D^3 N_a}{d^4 G} + \frac{2F^2 D N_a}{d^2 G}$$

Now using Castigliano's theorem, the deflection is:

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N_a}{d^4 G} + \frac{4FDN_a}{d^2 G}$$

Including the spring index C gives:

$$y = \frac{8FD^3 N_a}{d^4 G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3 N_a}{d^4 G}$$

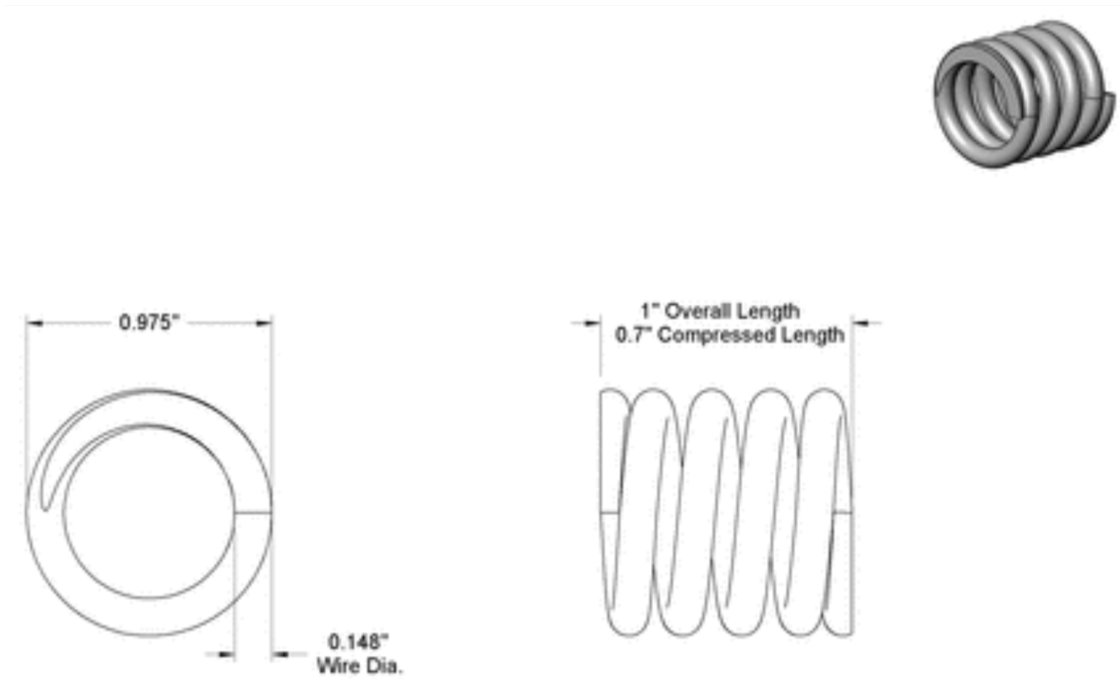
The spring rate k is;

$$k = \frac{F}{y} \approx \frac{d^4 G}{8D^3 N_a}$$

TRUE · FALSE · IT DEPENDS

Consider the helical compression spring shown.

Suggest three ways to increase the spring's stiffness, k.



12.4 Helical Compression Springs: Materials

The ideal spring material would have:

— High ultimate strength

— High yield strength

— Low modulus of elasticity

The most commonly used spring materials are summarized in Table 10-3 of the Shigley text.

For spring materials, the tensile strength varies with wire diameter. As the wire diameter gets smaller and smaller, the tensile strength strength approaches the theoretical strength of atomic bonds.

$$S_{ut} = \frac{A}{d^m}$$

The constants A and m are found in Table 10-4 of the Shigley text.

The yield strength, S_y, and torsional yield strength, S_{sy}, are found in Table 10-5 of the Shigley text.

12.5 Helical Compression Springs: Static Failure

12.5.1 Buckling

A compression spring is loaded as a column and can buckle if it is too slender.

- If a spring is susceptible to buckling, it can be placed in a hole or over a rod.

The critical length L_{crit} of the spring is a function of material properties (E and G), geometry (E), and end condition (α). The end condition constant α is found in Table 10-2 of the Shigley text.

$$L_{crit} = \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

The spring will be stable (i.e., will not buckle) if $L_0 < L_{crit}$.

For steels, the critical length is $L_{crit} = 2.63\frac{D}{\alpha}$.

12.5.2 Factor of safety at spring closure

The factor of safety for static failure can be found by comparing the yield strength to the shear stress at spring closure.

$$n_s = \frac{S_{ys}}{\tau}$$

12.5.3 Design conditions and approach for static service

A design flow chart for helical compression springs is seen in Figure 10-3 in the Shigley text. Four design conditions are offered.

$$4 \leqslant C \leqslant 12$$

$$3 \leqslant N_a \leqslant 15$$

$$\zeta \geqslant 0.15$$

$$F_s = (1 + \zeta)F_{max}$$

$$n_s \geqslant 1.2$$

12.6 Helical Compression Springs: Fatigue Failure

12.6.1 Factor of safety for infinite life

The factor of safety for infinite life is found using the Goodman criterion, where the criterion has been adapted for the pure shear case.

$$\frac{1}{n_f} = \frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}$$

The shear stresses τ_a and τ_m are:

$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$

$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$

F_a and F_m are:

$$F_a = \frac{F_{max} - F_{min}}{2}$$

$$F_m = \frac{F_{max} + F_{min}}{2}$$

Note that, because compression springs should never be subjected to tensile loads, $F_{min} = 0$ or $F_{min} = F_i$ where F_i is the pre-load.

The torsional modulus of rupture S_{su} is approximately:

$$S_{su} = 0.67 S_{ut}$$

Recall that S_{ut} is a function of wire diameter d.

The approach to S_{se} is unique to springs.

The best data for the torsional endurance limits of spring steels were published by Zimmerli in 1957.

The Zimmerli data show that the size, material, and tensile strength have no effect on S_{se} for wire diameters less than 3/8 inch (10 mm).

$$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}}$$

For <u>unpeened springs</u>: S_{sa} = 35 kpsi = 241 MPa and S_{sm} = 55 kpsi = 379 MPa.

For <u>peened springs</u>: S_{sa} = 57.5 kpsi = 398 MPa and S_{sm} = 77.5 kpsi = 534 MPa.

12.6.2 Spring surge

When compressed, helical compression springs do not deform uniformly. Instead, the deformation resembles a wave moving through the spring.

This phenomenon is called **spring surge**.

To avoid resonance, the natural frequency f of the spring should be 15-20 times higher than the forcing frequency of the application.

$$f = \frac{1}{4}\sqrt{\frac{kg}{W}}$$

Note that f has units of hertz.

The weight of the spring is the product of the wire volume and the wire's specific weight γ.

$$W = AL\gamma = \frac{\pi d^2}{4}(\pi D N_a)\gamma = \frac{\pi^2 d^2 D N_a \gamma}{4}$$

The specific weight γ, also labeled as the unit weight, can be found in Table A-5 of the Shigley text.

For steels, $\gamma = 0.282$ lbf/in^3 = 487 lbf/ft^3 = 76.5 kN/m^3.

12.7 Example Problems: Helical Compression Springs

Example 51 A helical compression spring is wound using 2.5-mm-diameter music wire. The spring has an outside diameter of 31 mm with plain ground ends and 14 total coils.

Determine the following.

(a) Estimate the spring rate.

(b) The force needed to compress the spring to closure.

(c) The free length to ensure that when the spring is compressed solid the torsional stress does not exceed the yield strength.

(d) Is there a possibility that the spring might buckle in service?

Example 52 A helical compression spring is made with oil-tempered wire with wire diameter of 0.2 in, mean coil diameter of 2 in, a total of 12 coils, a free length of 5 in, with squared ends.

Determine the following.

(a) The solid length

(b) The force necessary to deflect the spring to its solid length

(c) The factor of safety guarding against yielding when the spring is compressed to its solid length

(d) The factor of safety for infinite life if the spring is compressed from its free length to its shut height

Example 53 A particular application requires a helical compression spring to handle a dynamic load that varies from 6 lbf to 36 lbf over a working deflection of 2 inches.

Determine the following.

(a) Find a spring on McMaster-Carr that satisfies the force-deflection requirement.

(b) For the spring chosen in part (a), determine the factor of safety for infinite life.

Example 54 Consider part number 9657K246 on McMaster-Carr, a helical compression spring made of music wire steel that is 0.940 inches long and has a 0.240 inch outer diameter and 0.170 inch inner diameter. The shut height is 0.59 inches. The spring has closed and ground ends.

Determine the following.

(a) The spring rate k

(b) The pitch

(c) The force to compress the spring to its shut height, calculated using the spring rate, the free length, and the shut height.

(d) The force to compress the spring to its shut height such that when the spring is compressed solid the torsional stress does not exceed the yield strength.

(e) Compare your answers to parts (a), (b), (c), and (d) to the specifications provided. Explain any differences observed.

(f) Is the spring at risk of buckling? If it is at risk of buckling, how would you mitigate this risk?

(g) The natural frequency of the spring f. If the spring is to be used in a cyclic application, what is your recommendation for the application's forcing frequency?

12.8 Helical Extension Springs

Helical extension springs differ from helical compression springs in several ways.

— Loaded in tension instead of in compression.

— Hooks or loops are provided on the spring ends to allow a pull force to be applied.

— The spring body is wound with an initial tension, F_i.

— Coils are tightly wound.

— Cannot be shot-peened.

No deflection occurs until the applied force exceeds the initial tension (see Figure 46).

$$k = \frac{F - F_i}{y} = \frac{d^4 G}{8 D^3 N_a}$$

The free length of an extension spring is measured inside the end loops or hooks, where N_b is the number of body coils:

$$L_0 = 2(D - d) + (N_b + 1)d = (2C - 1 + N_b)d$$

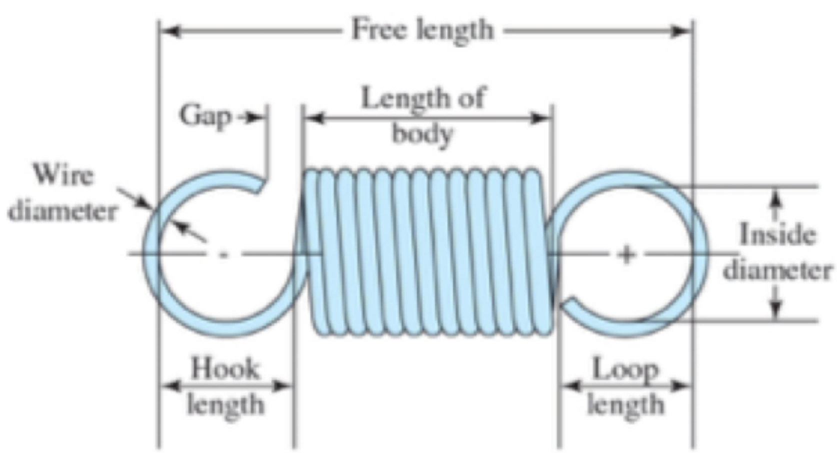


Figure 46: Helical extension spring geometry.

12.9 Helical Extension Springs: Failure Analysis

Three locations must be considered for failure:

1. The spring body.
2. Location A on the hook/loop.
3. Location B on the hook/loop.

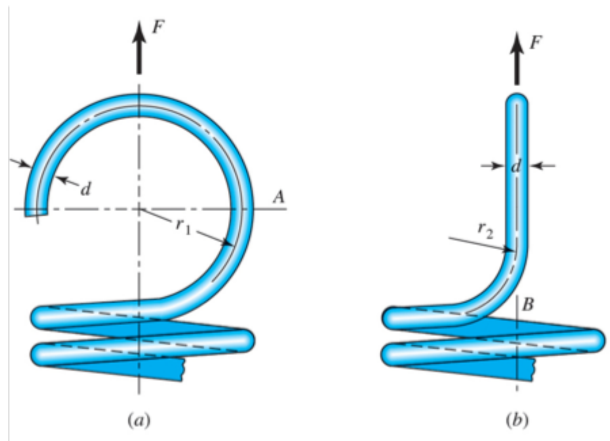


Figure 47: Locations to analyze in the hooks/loops of extension springs.

12.10 Helical Extension Springs: Static Failure

For the spring body, the factor of safety for static loading is found in the same manner as for helical compression springs:

$$n = \frac{S_{ys}}{\tau}$$

The yield strength, S_{ys}, is a fraction of S_{ut}. The fraction is found in Table 10-7.

The shear stress is:

$$\tau = K_B \frac{8FD}{\pi d^3}$$

The coefficient K_B is the Bergsträsser factor:

$$K_B = \frac{4C + 2}{4C - 3}$$

For location A on the hook/loop, the factor of safety for static loading is:

$$n_A = \frac{S_y}{\sigma_A}$$

The yield strength, S_y, is a fraction of S_{ut}. The fraction is found in Table 10-7.

The normal stress at hook/loop location A is:

$$\sigma_A = K_A \frac{16DF}{\pi d^3} + \frac{4F}{\pi d^2}$$

The stress concentration factor for curvature is:

$$K_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

The index for the hook/loop is:

$$C_1 = \frac{2r_1}{d}$$

For location B on the hook/loop, the factor of safety for static loading is:

$$n_B = \frac{S_{ys}}{\tau_B}$$

The yield strength, S_{ys}, is a fraction of S_{ut}. The fraction is found in Table 10-7. Note that S_{ys} is different for the hook/loop than for the spring body.

The shear stress at hook/loop location B is:

$$\tau_B = K_B \frac{8FD}{\pi d^3}$$

Note that K_B is not the Bergsträsser factor; it is the stress concentration factor for curvature at location B:

$$K_B = \frac{4C_2 - 1}{4C_2 - 4}$$

The index for the hook/loop is:

$$C_2 = \frac{2r_2}{d}$$

12.11 Helical Extension Springs: Fatigue Failure

For cyclic loading, the Goodman criterion with Zimmerli data should be used for the three location susceptible to failure (the spring body, location A, and location B). See Example 10-7 in the Shigley text.

12.12 Example Problems: Helical Extension Springs

Example 55 An extension spring has full-twisted loop ends.

The material is AISI 1065 OQ&T wire.

The spring has 84 coils and is close wound with a preload of 16 lbf.

Dimensions are $D = 1.338$ in, $d = 0.162$ in, $r_1 = D/2$, and $r_2 = d/2$.

Determine the following.

(a) The closed length of the spring.

(b) The torsional stress in the spring corresponding to the preload.

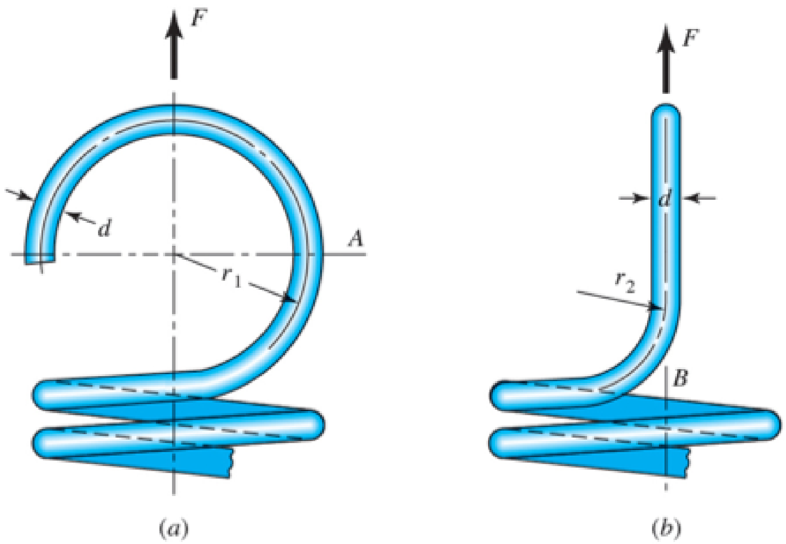
(c) The spring rate.

(d) The load that would cause permanent deflection.

(e) The spring deflection corresponding to the load in part (d).

Example 56 An extension spring is to be designed to support a cyclic load.

The spring will be made of hard-drawn wire with $d = 0.05$ inch and will be unpeened.



Determine the following.

(a) The endurance limit using Zimmerli data and the Goodman criterion for failure at location A.

(b) The endurance limit using Zimmerli data and the Goodman criterion for failure at location B.

Example 57 Consider part number 9654K525 on McMaster-Carr.

This is a helical extension spring that is 2 inches long, has 0.75 inch outside diameter, and 0.115 inch wire diameter.

The spring's maximum load is 73 lbf and the minimum load is 6 lbf.

For the spring subjected to a static load of 73 lbf, determine the following.

(a) The torsional stress in the spring corresponding to the preload.

(b) The factors of safety guarding against yielding for the spring body and for location A in the spring ends.

(c) The radius r_2 characterizing the curvature of location B if the factor of safety guarding against yielding at location B is 1.

12.13 Torsion Springs

A **torsion spring** is a helical coil spring subjected to end torsion.

Figure 48: Torsion springs have many uses, including clothespins.

Torsion springs differ from helical compression springs and from helical extension springs in several ways.

— Usually close wound, like a helical extension spring.
  - Can be wound with a small pitch to avoid intercoil friction.

— Torsion springs should always be loaded to close the coil because the residual stresses from coil-winding are favorable.

— Require reactive support to maintain alignment and to prevent buckling.
  - Contained ends.
  - Used with a support rod.

— Designed to wind tighter in service.
  - Support rod should be no larger than 90% of the smallest ID when ”wound up” to avoid binding.

12.14 Torsion Springs: Geometry

The parameters d, D, C, D_i, D_o, and N_a have the same meanings for torsion springs as for helical compression springs and helical tension springs.

Figure 49 shows parameters describing the unloaded position and the spring deflection.

The initial position of the spring end is angle β.

Applied force F acts at distance l from the spring center and causes deflection θ.

The sum of α and θ is a constant.

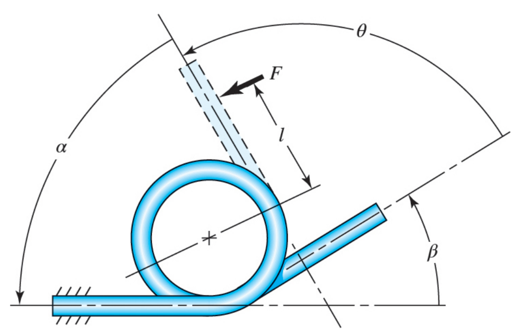


Figure 49: Support rod should be sized to avoid binding.

The partial turn in the coil body is:

$$N_p = \frac{\beta}{360^\circ}$$

The number of body turns, N_b, is always an integer + N_p.

12.15 Torsion Springs: Spring Rate

The deflection, θ, is found using Castigliano's theorem:

$$l\theta = \frac{\partial U}{\partial F}$$

The strain energy for bending is:

$$U = \int \frac{M^2 dx}{2EI} = \int \frac{F^2 l^2 dx}{2EI}$$

Now using Castigliano's theorem, the deflection is:

$$l\theta = \frac{\partial U}{\partial F} = \int_0^{\pi D N_b} \frac{\partial}{\partial F}\left(\frac{F^2 l^2 dx}{2EI}\right) = \frac{64 F l D N_b}{E d^4}$$

The number of body coils, N_b, is replaced with the number of active coils, N_a.

The number of active coils accounts for the angular deflection of the spring ends. The spring ends have lengths l_1 and l_2.

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D}$$

The spring rate in units of torque/radians is;

$$k = \frac{Fl}{\theta} = \frac{d^4 E}{64 D N_a}$$

The spring rate in units of torque/turns is;

$$k' = \frac{2\pi d^4 E}{64 D N_a} \approx \frac{d^4 E}{10.2 D N_a}$$

To account for the effects of friction, constant 10.2 is increased to 10.8:

$$k' = \frac{d^4 E}{10.8 D N_a}$$

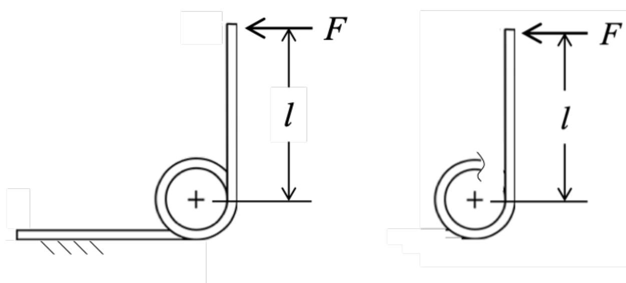
12.16 Torsion Springs: Stress

TRUE · FALSE · IT DEPENDS

A torsion spring, with mean coil diameter D and wire diameter d, is loaded with force F at a distance l.

The internal loads acting in the wire are (select all that apply):

◯ Axial tension
◯ Axial compression
◯ Torsion
◯ Bending
◯ Transverse shear



When a torsion spring is loaded, bending stress is induced in the wire.

$$\sigma = K\frac{Mc}{I}$$

The stress concentration factor K can be calculated at either the inner fiber or the outer fiber of the wire.

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)}$$

$$K_o = \frac{4C^2 + C - 1}{4C(C+1)}$$

Using $M = Fl$, $c = d/2$, and $I = \pi d^4/64$ and recognizing that $K_o < 1$, the bending stress is:

$$\sigma = K_i\frac{32Fl}{\pi d^3}$$

Given the bending stress induced in the wire, torsion springs should be made out of square or rectangular wire. However, round wire is cheaper and more readily available in a variety of sizes.

12.17 Torsion Springs: Static Failure

The factor of safety for static failure is:

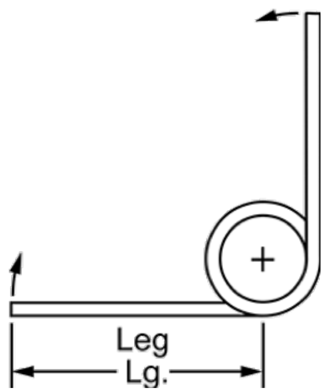
$$n_s = \frac{S_y}{\sigma}$$

The yield strength S_y used for torsion springs can be found by dividing S_{sy} by 0.577. See equation 10-57 in the Shigley text.

12.18 Example Problems: Torsion Springs: Work-out Problems

Example 58 Torsion spring 9271K13 from McMaster-Carr is made of music wire. It has $D_o = 0.16$ in, wire diameter $d = 0.017$ in, leg length 0.5 in, and 3.25 coils.

The spring is subjected to a moment that varies from 0 to 0.1 lbf-in.



Determine the following.

(a) The spring index, C.

(b) the spring rate, k', in units of lbf-in/turn.

(c) The stress concentration factor, K_i.

(d) The factor of safety for infinite life using the Goodman criterion with $S_e = 120$ kpsi.

Example 59 A particular application requires a torsion spring with 120° deflection angle to support a static load of 10 in-lbf.

The spring is to be made of music wire.

The spring index is 8.

Determine the following.

(a) The wire diameter to achieve a static factor of safety of 2.

(b) Find a spring on McMaster-Carr that satisfies the design. For this spring, determine the following.

- The equivalent number of active coils N_a.
- The spring constant k' in units of torque/turns.
- The angular deflection of the coil body θ'.
- The diameter of support pin allowed for the pin diameter to be 90% of spring's inner diameter.

13 Threaded Fasteners and Bolted Joint Design

Threaded fasteners are used in bolted joints to hold things together.

Bolted joints can take tensile loads, shear loads, or both.

— In a tension joint, loads are parallel to the fastener's axis and try to pull the joint apart.

— In a shear joint, loads are perpendicular to the fastener's axis and try to slip the joint members past each other.

A **bolt** is a fastener with a head and a straight, uniformly-threaded shank intended to be used with a **nut** to clamp an assembly together.

A **screw** is the same as a bolt, but is threaded into a tapped hole instead of into a nut. Screws can have any thread form (e.g., tapered, interrupted).

A **stud** is a headless fastener.

Figure 50: A bolted joint.

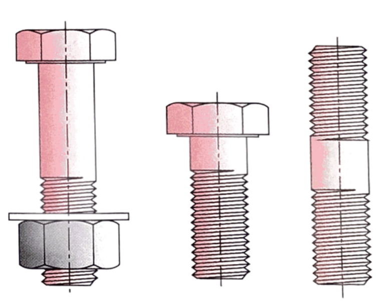


Figure 51: Bolt, screw, and stud.

13.1 Thread Standards

Thread forms were standardized after World War II.

— American National (Unified) thread standard: ASME B1.1

— ISO thread standard: ISO 724

The **thread** is the helix that causes the fastener to advance.

— Right-hand thread will advance away from you when turned clockwise.

— The **lead** (l) is the axial distance the fastener advances with one turn.

— Threads are coarse, fine, or extra-fine.

The **pitch** (p) is the distance between adjacent threads.

The **major diameter** (d) is the largest diameter of a screw thread.

The **minor diameter** (d_r) is the smallest diameter of a screw thread.

The **pitch diameter** (d_p) is a theoretical diameter between d and d_r.

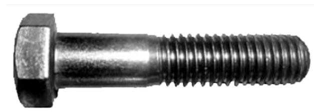
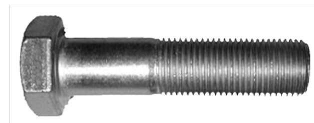
When a fastener fails in pure tension, it typically breaks through the threaded portion. The **tensile stress area** (A_t) is a function of d and p and is tabulated in the Shigley text (Table 8-1 for metric fasteners and Table 8-2 for UNS fasteners).

$$A_t = 0.7854\left[d - \left(\frac{0.9743}{p}\right)\right]^2$$

Figure 52: Screw thread terminology.

TRUE · FALSE · IT DEPENDS

Compare the performance of a coarse-thread fastener and a fine-thread fastener.

	Coarse thread 	**Fine thread** 
Strength in tension		
Adjustment accuracy		
Stripping strength		
Fatigue life		
Assembly speed		
Use in brittle materials		

TRUE · FALSE · IT DEPENDS

Decipher the naming convention for a UNS bolt:

1/4-20 UNC-2A

Decipher the naming convention for a metric bolt:

M8 x 1.25

13.2 Thread Production

Threads can be produced by either cutting or rolling operations.

— Cut threads: material is removed from a bolt blank with a cutting die or lathe.

— Rolled threads: the bolt blank is rolled between two reciprocating serrated dies.

Advantages of cut threads include the following.

— Lower tooling costs.

— Better for large diameter or non-standard externally threaded fasteners.

— Most common for internal threads.

Advantages of rolled threads include the following.

— More accurate and uniform thread dimension.

— Smoother thread surface

— Greater thread strength.

13.3 Fastener critical points: Tension Joints

When fasteners fail, they fail at one of the following locations.

- The bolt underhead
- The root of the first engaged thread
- Thread stripping
  - In the bolt
  - In the nut

Breaking the fastener is often the desired failure mode.

Figure 53: Fastener critical points.

Figure 54: A bolt with stripped threads.

13.4 Thread Engagement

The **length of thread engagement** is the axial length that the fully formed threads of the nut and the bolt are in contact.

— Too much thread engagement? The torque to tighten the bolt may be too high.

— Not enough thread engagement? The threads may strip.

Theoretically, all threads in engagement should share the load.

In actuality, virtually all the load is taken by the first engaged thread.

Nuts are available in standard sizes.

— See ASME B18.2.2 or Tables A-29, A-30, and A-31 in the Shigley text.

— If the nut is long enough, the fastener will fail in tension before the threads will strip.

— For UNS or ISO threads with $d \leqslant 1$ in, the nut length should be at least $0.5d$.

Guidelines for tapped hole depth.

— For same-material combinations, thread engagement should be at least d.

— For a steel screw in cast iron, brass, or bronze, use $1.5d$.

— For a steel screw in aluminum, use $2d$.

Figure 55: All engaged threads do not share the load due to inaccuracies in thread spacing.

13.5 Stresses in Threaded Fasteners: Tension Joints

13.5.1 Axial stress

A threaded fastener typically sees only axial tension, not compression.

— Due to the initial preload.

— Due to the applied load.

The tensile stress is the force in the fastener (F_b) divided by the fastener's tensile stress area.

$$\sigma = \frac{F_b}{A_t}$$

13.5.2 Torsional shear stress

Torsional shear stress develops in the fastener when the fastener is tightened.

Torsional shear stress is due to friction.

— Between the fastener head and/or the nut and the clamped material.

— Between the fastener and the nut.

  – Worst case: when the nut is rusted to the bolt.

The worst case for torsional shear stress is high thread friction. For high thread friction, the total applied torque is transmitted to the fastener.

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d_r^3}$$

Torsional shear stress dissipates after installation and can be neglected.

TRUE · FALSE · IT DEPENDS

Determine the critical cross-section(s) for a threaded fastener while the fastener is being tightened.

Identify the critical element(s) on the critical cross-section(s).

For the critical cross-section(s) identified, draw the state of stress.

13.5.3 Transverse shear stress

Threads strip due to shear stress. The shear stress for thread stripping is:

$$\tau_s = \frac{F_b}{A_s N_{th}}$$

The number of threads is:

— $N_{th} = 1$ if one thread takes all of the load.

— $N_{th} = \frac{H}{p}$ if the load is shared by all engaged threads.

For the bolt threads to strip, the stripping area A_s is the area of the cylinder at the bolt's minor diameter:

$$A_s = \pi d_r w_i p$$

For the nut threads to strip, the stripping area A_s is the area of the cylinder at the bolt's major diameter:

$$A_s = \pi d w_o p$$

Area factors for thread-stripping shear area of UNS/ISO fasteners are $w_i = 0.80$ and $w_o = 0.88$.

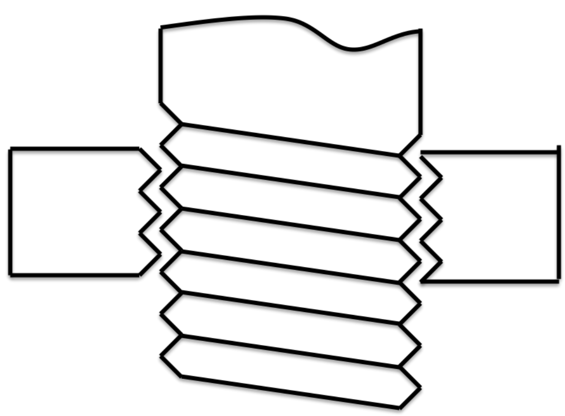


Figure 56: Shear planes for thread stripping.

TRUE · FALSE · IT DEPENDS

For the nut and the bolt made of the same material, will the nut threads strip first or the bolt threads?

Based on this, should one material be stronger than the other?

13.6 Tensile forces in bolts

The total external tensile load applied to a bolted joint is P_{total}.

The external tensile load per bolt is P_{total} divided by the number of fasteners in the bolted joint. P is shared between the bolt (P_b) and the member (P_m).

$$P = \frac{P_{total}}{N} = P_b + P_m$$

The tensile force in a bolt ($F_b = F_i + P_b$) comes from two sources.

— The preload (F_i) is generated during the assembly process.

- Recommended preload for reused fasteners: $F_i = 0.75F_p = 0.75S_pA_t$
- Recommended preload for permanent fasteners: $F_i = 0.9F_p = 0.9S_pA_t$

— The external applied load taken by the bolt (P_b).

Bolted joints are often over-designed because it is difficult and/or expensive to accurately generate and/or measure preload.

— Over-designed bolted joints have more and/or larger fasteners.

— It is cheaper to over-design a bolted joint than to control the assembly process.

Indirect methods to control preload include torque control, the torque-turn method, tension control bolts, and stretch control.

Direct methods to control preload include washer control, hydraulic tensioners, and ultrasonics.

— Torque control

- Only about 10% of the input energy is converted to bolt stretch; the majority of the input energy is used to overcome friction.
- The relationship between applied torque T and pre-load F_i is usually linear.

$$T = KF_i d$$

- The nut factor K represents anything that increases or decreases friction in the threaded fastener.
- There are published tables for K, including Table 8-15 in the Shigley text.
  - If the bolt condition is unknown, use $K = 0.2$.

— Torque-turn method

- A fastener is tightened to "snug tight" and then a prescribed amount of turn develops the required preload.

— Tension control (TC) bolts

— Stretch control

- Requires access to both ends of the bolt.
- A micrometer is the easiest way to measure bolt length (see Figure 57).

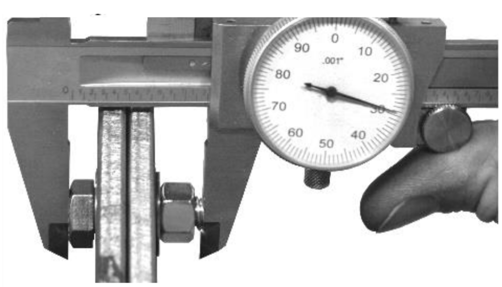


Figure 57: Stretch control is a very accurate method of determining bolt tension.

TRUE · FALSE · IT DEPENDS

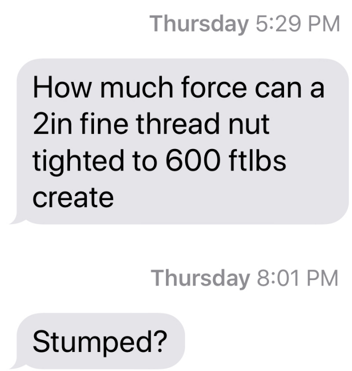
No two bolts respond exactly the same to a given torque.

From the list below, select the variable(s) that could impact the nut factor K.

◯ Dirt in a tapped hole
◯ Damaged threads
◯ Hole misalignment
◯ Bolt hardness
◯ Nut hardness
◯ Bolt material
◯ Nut material
◯ Class of fit
◯ Plating
◯ The number of times the fastener was used
◯ Surface finish
◯ If washers are present
◯ Thread type (cut or rolled)
◯ Type of tool used for tightening
◯ Tightening speed
◯ Element being torqued (bolt or nut)
◯ Lubricant (type, amount, condition, method of application, contamination, temperature)
◯ It depends
◯ All of the above

TRUE · FALSE · IT DEPENDS

Dr. Hess received the following text messages from her dad, a farmer in central Kansas.



Please answer Dr. Hess's dad's question.

13.7 Joint constant

The bolt and the member can both be modeled as linear springs, where force is linearly related to displacement.

The bolt has stiffness k_b and the member has stiffness k_m.

When a bolt is tightened to the preload F_i, the bolt is elongated by:

$$\delta_b = \frac{F_i}{k_b}$$

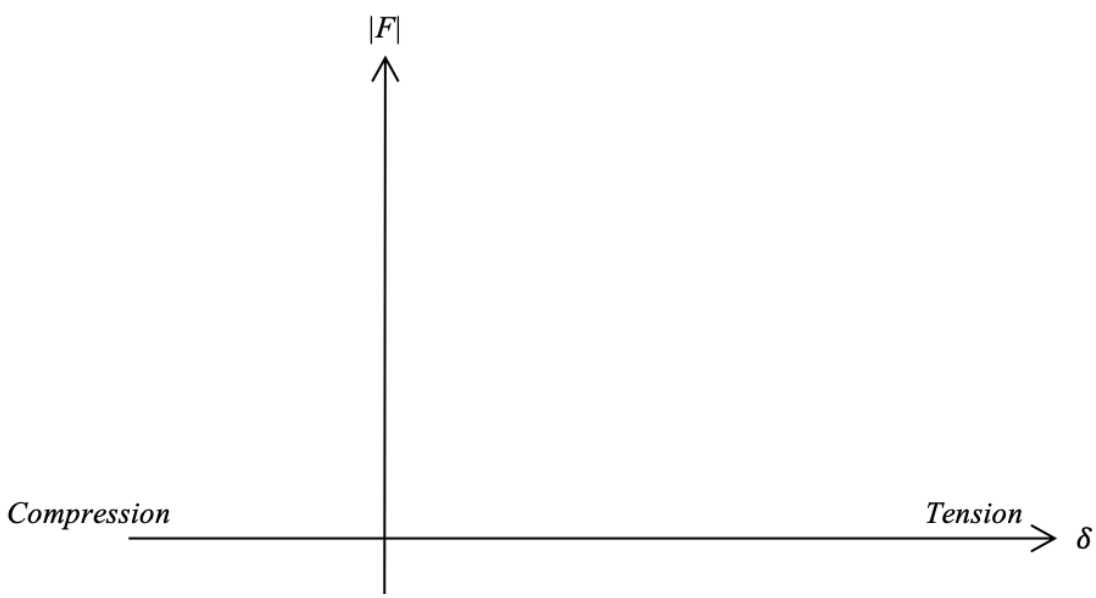
As the bolt is tightened, the member is compressed by:

$$\delta_m = \frac{F_i}{k_m}$$

The member is stiffer than the bolt ($k_m > k_b$), so $|\delta_m| < |\delta_b|$.

TRUE · FALSE · IT DEPENDS

On the figure below, sketch the deflection of the bolt and the member for a preload of F_i.



After the bolt is tightened to preload F_i, an external load P is applied to the bolted joint.

Load P is shared by the bolt (P_b) and by the member (P_m).

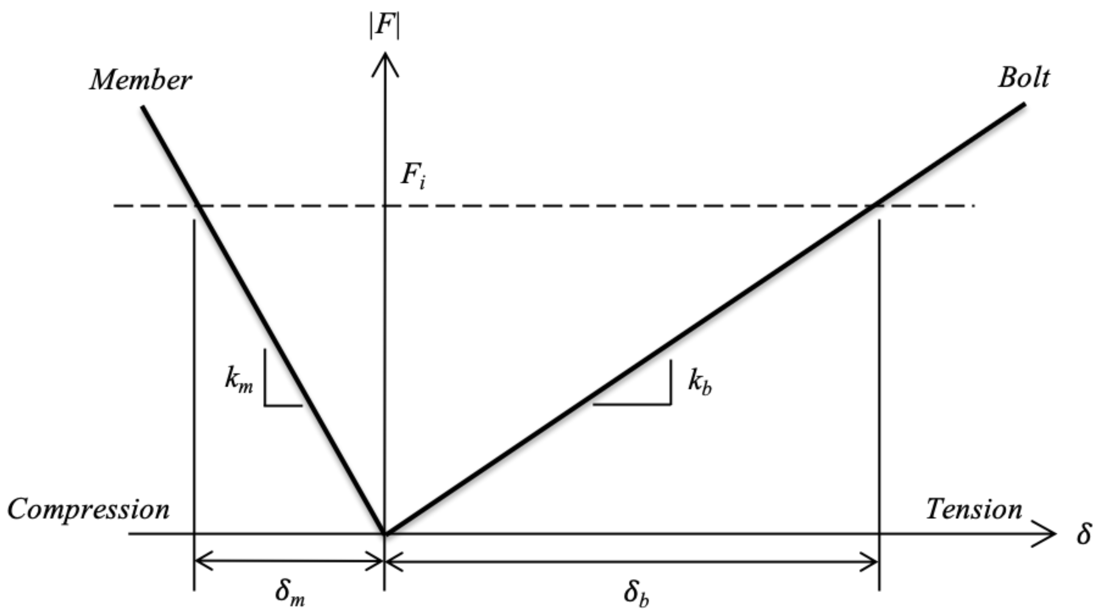
— The bolt elongates by $\Delta\delta_b$.

— The member elongates by the same amount ($\Delta\delta_b = \Delta\delta_m$).

— $P = P_b + P_m$

TRUE · FALSE · IT DEPENDS

Sketch the deflection of the bolt and the member for an external load P.



Because $\Delta\delta_b = \Delta\delta_m$, P_b and P_m can be related.

$$P_m = \frac{k_m}{k_b} P_b$$

Since $P = P_m + P_b$, then $P_m = P - P_b$. Substituting and rearranging gives:

$$P_b = \frac{k_b}{k_m + k_b} P = CP$$

$$P_m = P - P_b = (1 - C)P$$

C is the **joint stiffness constant**.

The tensile load in the bolt is:

$$F_b = P_b + F_i = CP + F_i$$

The compressive load in the member is:

$$F_m = P_m - F_i = (1 - C)P - F_i$$

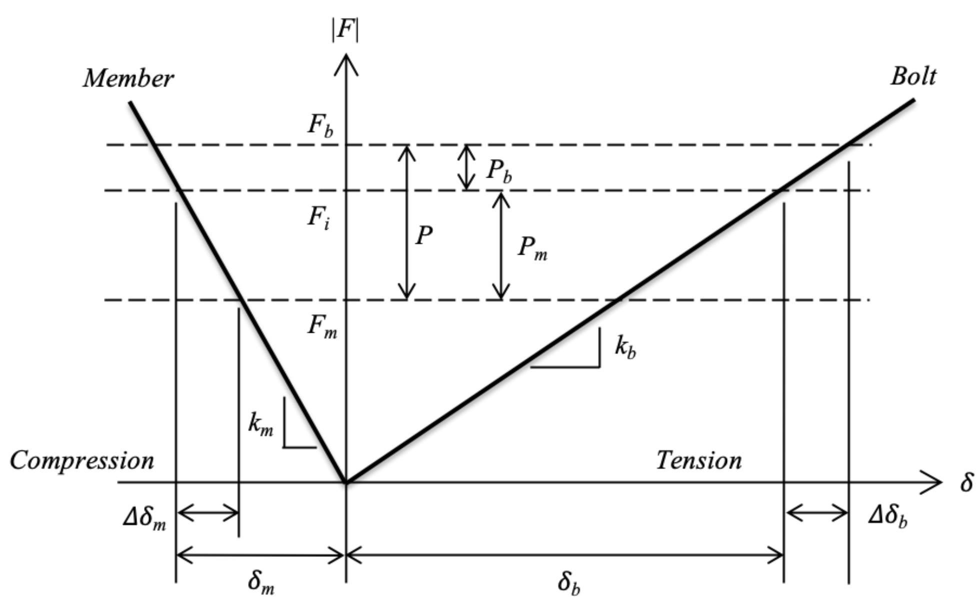


Figure 58: Force-deflection response of a bolted joint.

13.8 Fastener stiffness

Consider two springs in series, one with stiffness k_1 and one with stiffness k_2. The stiffness of the two springs in series is:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

For a threaded fastener, the "two springs in series" are the length of the threaded portion of the grip (l_t) and the length of the unthreaded portion in the grip (l_d).

— The **grip** (l) is the total thickness of the clamped material.

Recall from Mechanics of Materials that the stiffness of a linearly elastic material loaded in tension is a function of cross-sectional area (A), modulus of elasticity (E), and length (L):

$$k = \frac{AE}{L}$$

The recipe for finding the fastener stiffness, k_b, is found in Table 8-7 in the Shigley text.

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

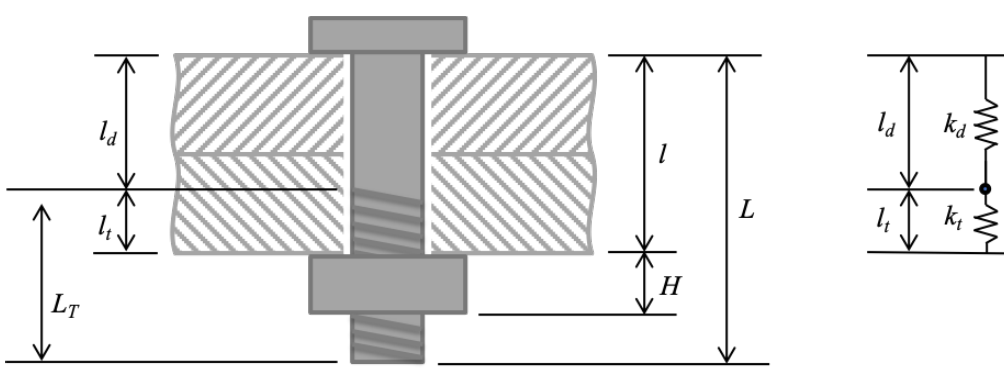


Figure 59: The portion of a threaded fastener in the grip of a bolted joint can be modeled as two springs in series.

13.9 Member stiffness

Compression in the bolted joint not uniform. In theory, the compression is represented by the frustrum of a cone (see Figure 60).

A **frustum** is the portion of a solid that lies between one or two parallel planes cutting it.

Each segment of the member acts like a spring in compression. The member stiffness, k_m, is found by the sum of these n "springs" acting in series.

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ... + \frac{1}{k_n}$$

The stiffness of the ith segment of the frustrum is found using equation 8-20 in the Shigley text. The stiffness k_i is a function of the segment's modulus of elasticity (E), the bolt diameter (d), and lengths D and t (see Figure 61).

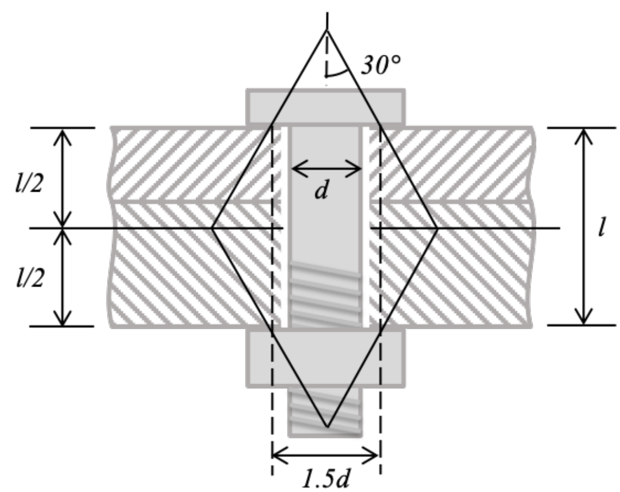


Figure 60: Compression in a bolted joint is represented by the frustrum of a cone.

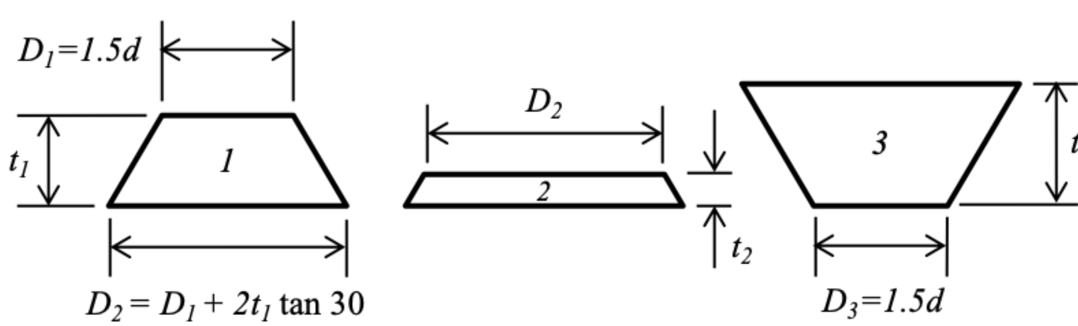


Figure 61: Finding D to calculate member stiffness.

13.10 Bolt strength

Tensile strength is the material property most widely associated with threaded fasteners.

— The **proof strength** is the tensile load that can be applied without permanent deformation.

— The **yield strength** is the tensile load at which a specified amount of permanent deformation occurs.

Bolted joints should be designed such that the bolt functions in the elastic range.

— The threaded fastener should always return to its original shape.

See Tables 8-9, 8-10, and 8-11 in the Shigley text for material properties and bolt head markings.

TRUE · FALSE · IT DEPENDS

Sketch a typical stress-strain diagram for steel. Label the typical pre-load, the proof strength, the yield strength, the ultimate tensile strength, the point of failure, and the elastic and plastic ranges.

13.11 Static failure

The factor of safety guarding against the static stress exceeding the proof strength is:

$$n_P = \frac{S_P}{\sigma_b} = \frac{S_p A_t}{CP + F_i}$$

n_P is often not much greater than 1.

Another indicator of yielding is the **load factor**.

$$n_L = \frac{S_p A_t - F_i}{CP}$$

When the joint separates, the fastener carries the entire applied load. The factor of safety guarding against the joint separating is:

$$n_O = \frac{F_i}{P(1 - C)}$$

13.12 Fatigue failure

For a bolt with constant pre-load that experiences an external load that varies from P_{min} to P_{max}, the maximum and minimum forces in the bolt are:

$$F_{min} = CP_{min} + F_i$$

$$F_{max} = CP_{max} + F_i$$

The alternating and mean stress components are:

$$\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t}$$

$$\sigma_m = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t} = \frac{C(P_{max} + P_{min})}{2A_t} + \sigma_i$$

The fatigue factor of safety is:

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)}$$

The endurance limit for bolts (S_e) is tabulated in Table 8-17 in the Shigley text.

The ultimate tensile strength for bolts (S_{ut}) is tabulated in Tables 8-9, 8-10, and 8-11 in the Shigley text.

TRUE · FALSE · IT DEPENDS

You are working as a structures engineer.

A peer has asked you to review their bolted joint design. You are not impressed with their design. Explain your concern with each of the following.

Use of Grade 12.9 M16 x 1.5 screws in an Imperial design.

Use of Grade 12.9 M16 x 1.5 screws to grip a combination of low carbon, brass (gasket), and cast iron members.

Use of Grade 12.9 M16 x 1.5 screws to be screwed into a cast aluminum housing that is 0.357 inches thick.

The Grade 12.9 M16 x 1.5 screws are called out as having a length of 83.5 mm.

Use of 12.9W washers.

Calculated factors of safety $n_p = 4.3$ and $n_0 = 8.9$.

13.13 Fasteners in Shear

The previous discussion of threaded fasteners and bolted joints has focused on tension joints. Threaded fasteners can also be used to support shear loads.

— More common in structural design than machine design.

— Fasteners are still loaded in tension, with high tensile preloads.

  - Preload creates large frictional forces between the elements of the bolted joint.
  - If the friction in the joint is not sufficient to support the shear loads, the fasteners will be placed in direct shear.

A combination of threaded fasteners and dowel pins should be used to support shear loads.

— The threaded fasteners clamp the joint in compression.

— The dowel pins provide accurate transverse location and shear resistance.

Analysis considerations for fasteners in shear include the following.

— Bearing in the bolt (all bolts participate) and in members (all holes participate)

— Bolt shear (all members participate eventually)

— Thread shear vs. shank shear

— Edge shearing and tearing of member

— Tensile yielding of member across bolt holes

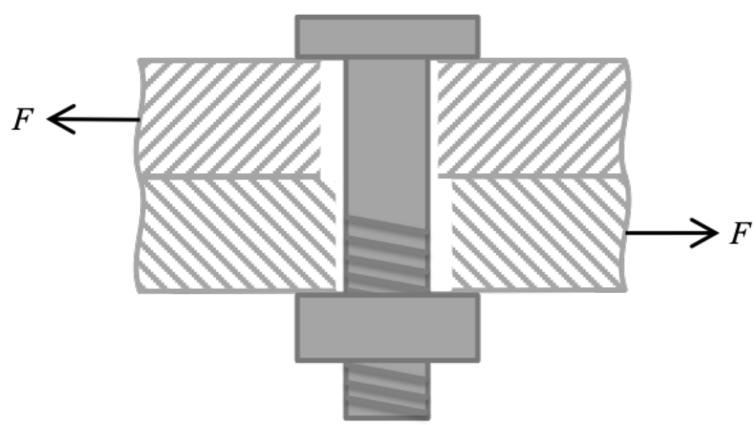


Figure 62: Holes for threaded fasteners are made with clearance for assembly. The clearance leads to eccentricity in the joint.

13.14 Example Problems: Bolted Joints

Example 60 An M14 x 2 steel hex-head bolt is used to clamp together two 15-mm steel plates.

Determine the following.

(a) A suitable bolt length.

(b) The bolt stiffness.

Example 61 A 30-mm steel plate is sandwiched by two 10-mm steel plates. The bolt is a M10 x 1.5.

Determine the following.

(a) The member stiffness.

(b) The member stiffness if the 10-mm plates are aluminum.

(c) The member stiffness if the bottom aluminum plate is 20-mm thick.

(d) The member stiffness if the bottom plate is threaded.

Example 62 A bolted assembly has 8 bolts, each having stiffness $k_b = 1.0$ MN/mm and $k_m = 2.6$ MN/mm per bolt.

The bolts are M6x1 class 5.8 and the joint is occasionally dissembled.

Determine the maximum static load that can be applied.

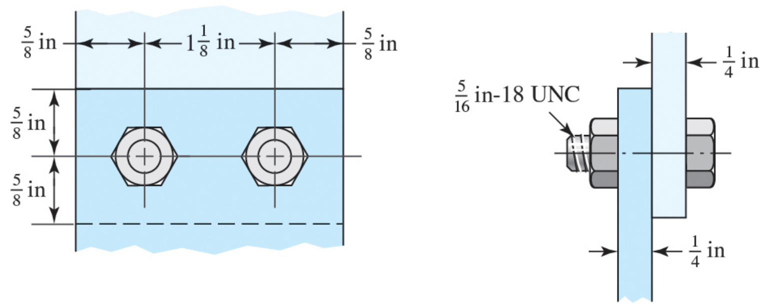
Example 63 A bolted assembly has 8 bolts, each having stiffness $k_b = 1.0$ MN/mm and $k_m = 2.6$ MN/mm per bolt.

The pre-load is 75% of the proof strength. $P_{max} = 60$ kN and $P_{min} = 20$ kN.

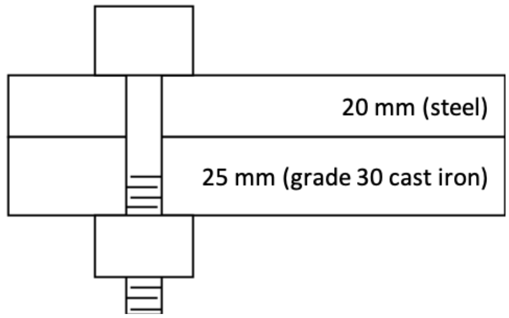
The bolts are M6x1 class 5.8 with rolled threads.

Determine n_f.

Example 64 The bolted connection shown in the figure uses SAE grade 8 bolts. The members are hot-rolled AISI 1040 steel. A tensile shear load $F = 5000$ lbf is applied to the connection. Assume the bolt threads do not extend into the joint. Find the factor of safety for all possible modes of failure.



Example 65 A metric hex-head bolt with a nut is used to clamp together a 20-mm thick piece of steel and a 25-mm thick piece of grade 30 cast iron.



Determine the following.

(a) Specify a coarse-pitch bolt to provide a joint constant C of approximately 0.2.

You may use the following table to compare bolts of various diameters.

d	A_t	A_d	H	L	L_T	l	l_d	l_t	k_b	k_1	k_2	k_3	k_m	C

(b) For the bolt selected in part (a), determine the static load P that can be applied in order for factors of safety n_L, n_0, and n_p to be at least 1.2 for a Class 9.8 bolt. Use a preload of $F_i = 0.75F_p$.

(c) For the bolt selected in part (a), the bolted joint is now subjected to a repeated load with $P_{max} = P$ and $P_{min} = 0$. Determine P_{max} using the Goodman criterion and a factor of safety of 1.2 for a Class 9.8 bolt.

Example 66 Compare the tensile load capacity of a 5/16-18 UNC thread and a 5/16-24 UNF thread made of the same of the same material. Which is stronger?

Compare the tensile load capacity of a M8x1.25 thread and a M8x1 thread made of the same of the same material. Which is stronger?

Example 67 An M14x2, class 8.8 bolt is used to clamp together a 3-cm-thick sandwich of solid aluminum.

(a) Determine a suitable length of the bolt.

(b) Determine the bolt stiffness.

(c) Determine the member stiffness.

(d) Determine the joint constant.

Example 68 Each bolt in a bolted assembly has stiffness $k_b = 4.5 \times 10^6$ M lbf/in.

The member stiffness per bolt is $k_m = 5.5 \times 10^6$ lbf/in.

It has been determined to use 1/2-13 UNC SAE Grade 8 bolts with rolled threads.

The assembly is subjected to a tensile load that fluctuates between 0 and 8000 lbf. Assume the load is distributed equally among all bolts.

The pre-load is 90% of the proof load.

Determine the following.

(a) The joint constant, C.

(b) The minimum number of bolts required to achieve a fatigue factor of safety of $n_f = 1.5$ using the Goodman criterion.

(c) The yielding factor of safety.

(d) The overload factor of safety.

(e) The factor of safety based on joint separation.

(f) The torque necessary to develop the pre-load.

14 Geometric Dimensioning and Tolerancing

Geometric Dimensioning and Tolerancing (GD&T) is a system for defining and communicating engineering tolerances.

14.1 Why do we need GD&T?

For the simple figure seen in Figure 63, the designer's intent is clear and most machine shops could manufacture such a part.

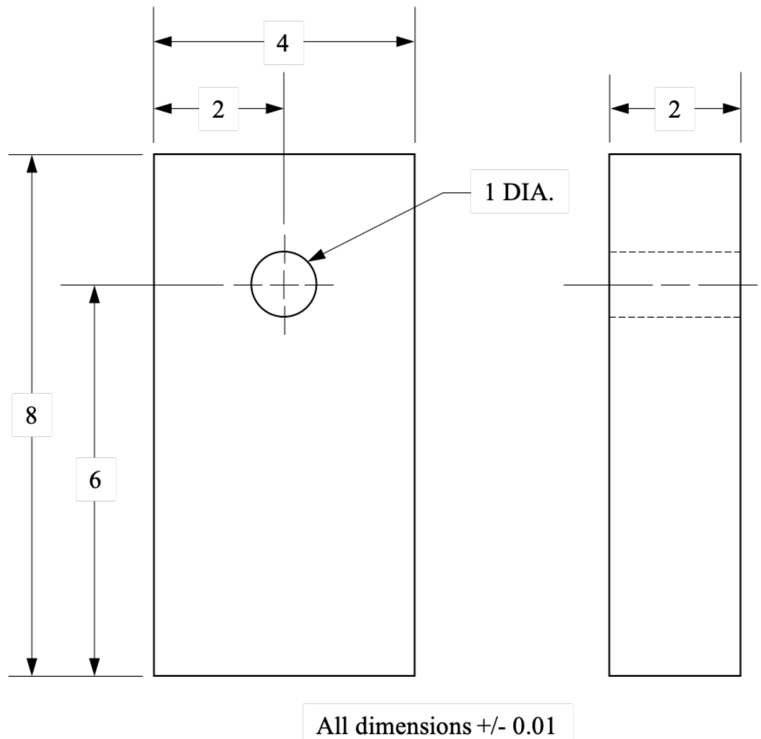
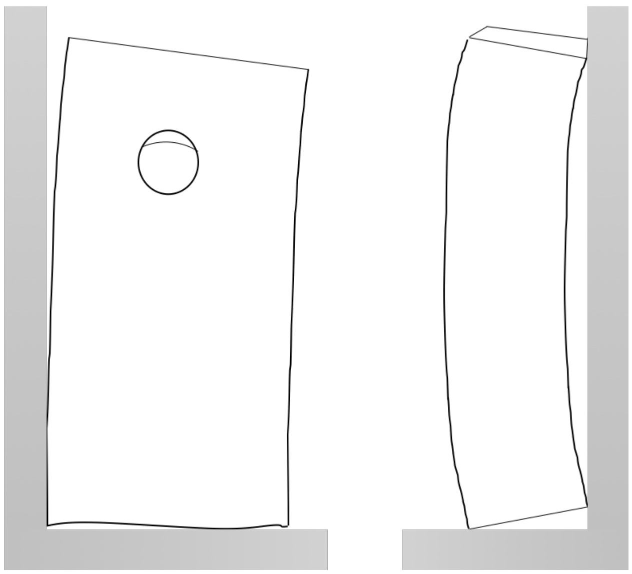


Figure 63: A simple part dimensioned with TD&T.

TRUE · FALSE · IT DEPENDS

You are asked to inspect a manufactured part.

Does it meet the print in Figure 63? Or should you reject it?



Note the following.

— The bar stock is not perfectly flat.

— The corners are not perfectly square.

— The hole is not perfectly perpendicular to the part face.

— The hole is not perfectly round.

How will you measure the dimensions?

— If the hole is not perfectly round, how is the center defined?

— If the corner is not square, from where should the 2 inches be measured? From the bottom corner? The top corner? The closest edge?

A GD&T version of the part from Figure 63 is seen in Figure 64.

— The **ASME Y14.5 standard** establishes symbols, rules, definitions, requirements, defaults, and recommended practices for stating and interpreting dimensioning, tolerancing, and related requirements for use on engineering drawings, models defined in digital data files, and related documents.

— Engineering drawings created in accordance with the principles put forth in ASME Y14.5 should have no ambiguity. A GD&T drawing will yield a single, exact part.

A GD&T drawing will specify the four geometric attributes of every feature.

— A **feature** is a clearly identifiable physical portion of a part.

— The four geometric attributes are size, location, orientation, and form.

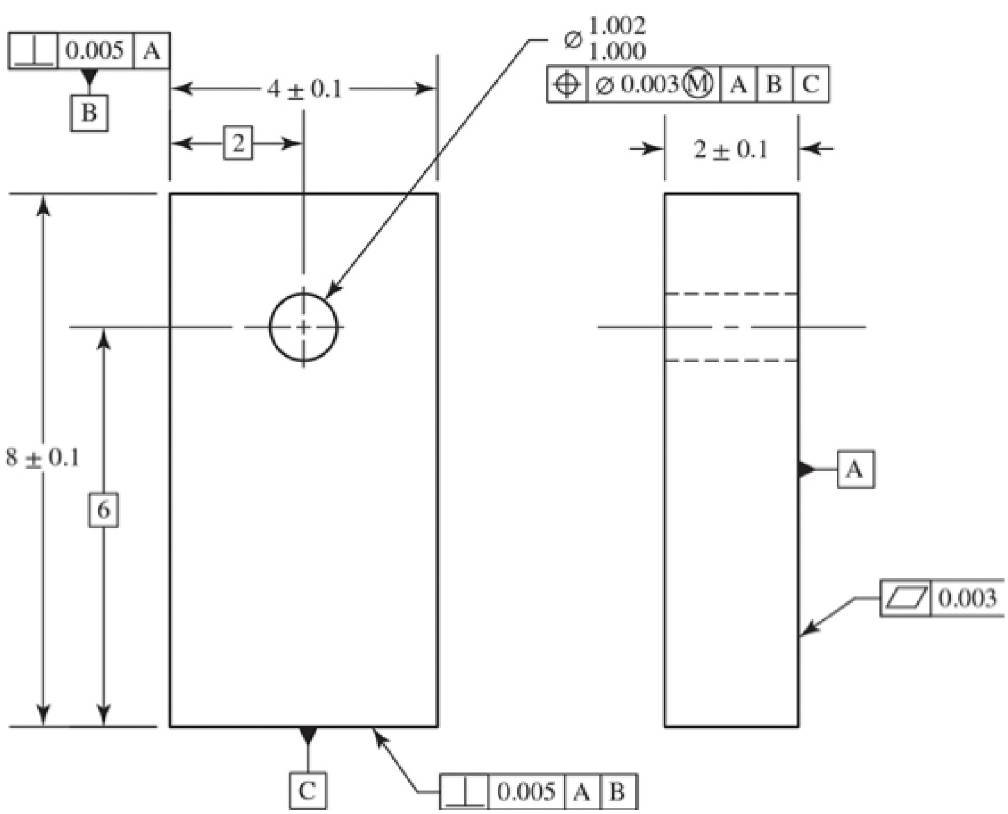


Figure 64: A simple part dimensioned with GD&T.

14.2 GD&T vs. TD&T

A GD&T drawing requires much more effort to create compared to a traditionally dimensioned and toleranced drawing (TD&T).

TD&T is sufficient for the following.

— Inter-office documents.

— When the drawing's purpose is to communicate ideas, not exact part specifications.

— For parts that will be manufactured in-house.

— For prototype or low quantity parts.

GD&T is a better choice for the following.

— For parts that will be mass produced.

— For parts that have important tolerance requirements.

— For parts that will be manufactured by another facility.

— For parts that have features that cannot be clearly communicated via TD&T.

Keep in mind when creating a GD&T drawing...

— How good is good enough?

— The manufacturer needs to be able to cut corners as much as possible.

— Balancing functional requirements and tolerancing will achieve cost-effective and functional parts.

14.3 Terminology

Nominal size is the size used when referring to a part or element.

The maximum and minimum dimensions are the **limits**.

Tolerance is the difference between the two limits.

— **Bilateral tolerance** permits variation in both directions.

— **Unilateral tolerance** permits variation in only one direction.

Clearance refers to the mating of cylindrical parts.

— **Diametral clearance** is the difference between two diameters.

— **Radial clearance** is the difference between two radii.

Interference is the opposite of clearance.

14.4 Geometric Attributes of Features

A **size** attribute is measured across two opposing points. Size dimensions are controlled with plus/minus tolerancing.

Location, orientation, and form attributes are controlled with the 14 geometric characteristic controls shown in Figure 65.

- A **location** attribute gives the distance between a feature and some origin of measurement.
- An **orientation** attribute gives the angle of a feature with respect to some origin of measurement.
- A **form** attribute refers to imperfections in the shape of a feature.

TYPE OF FEATURE	TYPE OF TOLERANCE	CHARACTERISTIC	SYMBOL
INDIVIDUAL (No Datum Reference)	FORM	FLATNESS	⏥
		STRAIGHTNESS	—
		CIRCULARITY	○
		CYLINDRICITY	⌭
INDIVIDUAL or RELATED FEATURES	PROFILE	LINE PROFILE	⌒
		SURFACE PROFILE	⌓
RELATED FEATURES (Datum Reference Required)	ORIENTATION	PERPENDICULARITY	⊥
		ANGULARITY	∠
		PARALLELISM	//
	RUNOUT	CIRCULAR RUNOUT	↗
		TOTAL RUNOUT	⌰
	LOCATION	CONCENTRICITY	◎
		POSITION	⌖
		SYMMETRY	⌯

Figure 65: Geometric characteristic controls and symbols.

14.5 Datums

A **datum** is a theoretically exact point, axis, line, or plane that is used as an origin for repeatable measurements.

— A **datum feature** is a non-ideal surface of the physical part that is used to establish a datum.

— A **datum feature simulator** is a precision embodiment, such as a surface plate, gauge pin, or machine tool bed, of the datum described by an imperfect datum feature.

— Datums are sequentially selected to immobilize a part in a precise, repeatable location.

  – Immobilization is useful for manufacturing the part and for inspecting the part.

Datums are shown on GD&T drawings as capital letters enclosed in a square frame.

— The square frame is attached to a leader line that terminates at the datum feature with a triangle; the triangle can be filled or empty.

— Almost any letter can be used for a datum. Do not use I, O, or Q.

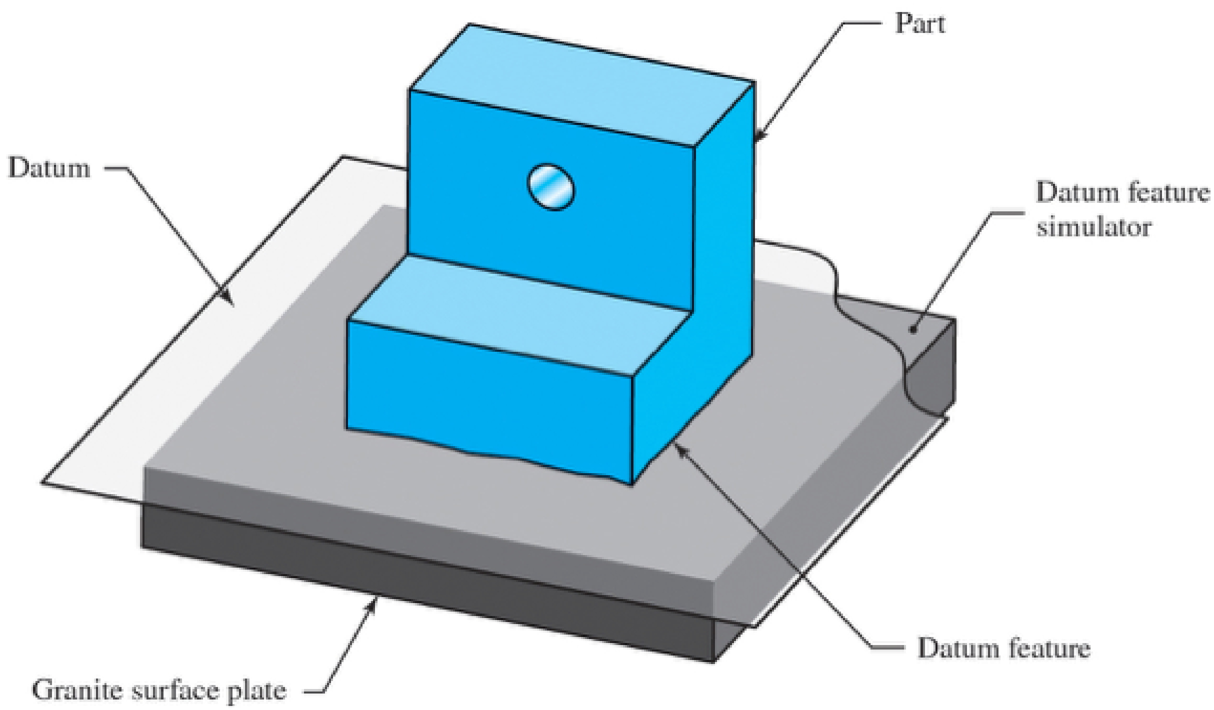


Figure 66: Datum terminology.

14.6 Feature Control Frames

A **feature control frame** contains the necessary information to define the tolerance zone of a specified feature.

The feature control frame always contains a geometric control symbol (see Figure 65), tolerance information, and datum references.

The feature control frame is read from left to right, and from top to bottom (see Figure 67).

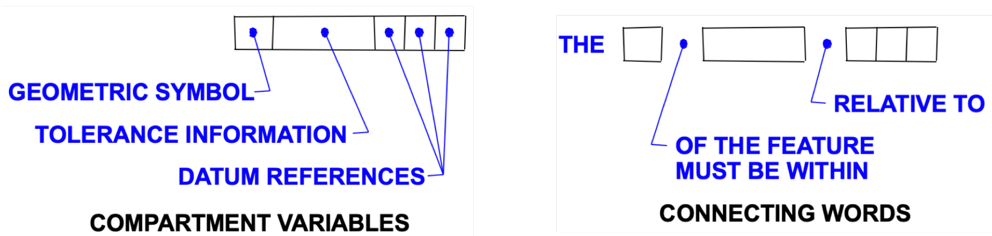


Figure 67: Interpretation of a feature control frame.

TRUE · FALSE · IT DEPENDS

What is the difference between a Datum and a Datum Feature?

(a) A Datum and Datum Feature are synonymous.

(b) A Datum is theoretical, a Datum Feature is real.

(c) A Datum is a feature of size, and a Datum Feature is a surface feature.

The advantage(s) of using GD&T as compared to TD&T is/are:

(a) Tighter tolerances can be achieved in manufacturing.

(b) The true design requirements for a part can be communicated to the manufacturer.

(c) More tolerance is made available to manufacturing than is possible with only plus/minus tolerancing.

(d) Both (b) and (c).

How can properly implemented GD&T save money in the manufacturing process?

(a) Better definition of the design requirements, increased availability of tolerances, better communication between design, manufacturing and inspection, fewer engineering changes.

(b) It can't. GD&T is just a way to illustrate how to inspect parts.

(c) By creating extra tolerance that did not physically exist without using GD&T.

(d) By defining the datum references that must be used by manufacturing for fixturing.

What is a Basic Dimension?

(a) An exact dimension with no tolerance associated with it.

(b) A dimension in a box that must be strictly adhered to by manufacturing.

(c) The nominal size of a feature.

(d) A numerical value that describes a theoretically exact size, profile, orientation or location of a feature or a datum target. It is the basis from which permissible variations (tolerances) are established.

14.7 Tolerance Zones

Tolerance zones define the limiting boundaries for the physical surfaces of the parts.

— Tolerance zones are defined relative to a theoretically exact location or shape.

— The actual shape and location of the part surfaces may vary from the theoretically exact shape and location, so long as they stay within the limiting boundaries of the tolerance zones.

In GD&T, the only direct application of plus/minus tolerances is for size attributes.

All other attributes must be controlled to stay within **tolerance zones**.

14.8 More Terminology

A **basic dimension** is a theoretically exact dimension that ideally locates and/or orients the tolerance zone of a feature.

The **maximum material condition** (MMC) is when the part will weigh the most.

— For a shaft, MMC specifies the largest allowable size.

— For a hole, MMC specifies the smallest allowable size.

The **least material condition** (LMC) is when the part will weigh the least.

— For a shaft, LMC specifies the smallest allowable size.

— For a hole, LMC specifies the largest allowable size.

Regardless of feature size (RFS) is the default condition for a tolerance (i.e., the condition when neither LMC nor MMC are stated). RFS indicates the stated tolerance applies, regardless of the feature's actual size.

14.9 Tolerance Zones for Attributes of Size

In ASME Y14.5-2018, **Rule #1**, also known as the **envelope principle**, specifies a default tolerance zone for features of size.

— When only a tolerance of size is specified for a feature of size, the limits of size prescribe the extent of which variation in its geometric form, as well as size, are allowed.

Rule #1 is illustrated in Figure 68 for a simple example of a dowel pin.

— If the pin is manufactured at MMC, it must have perfect form.

— If the pin diameter is reduced from MMC, the pin may deviate from perfect form.

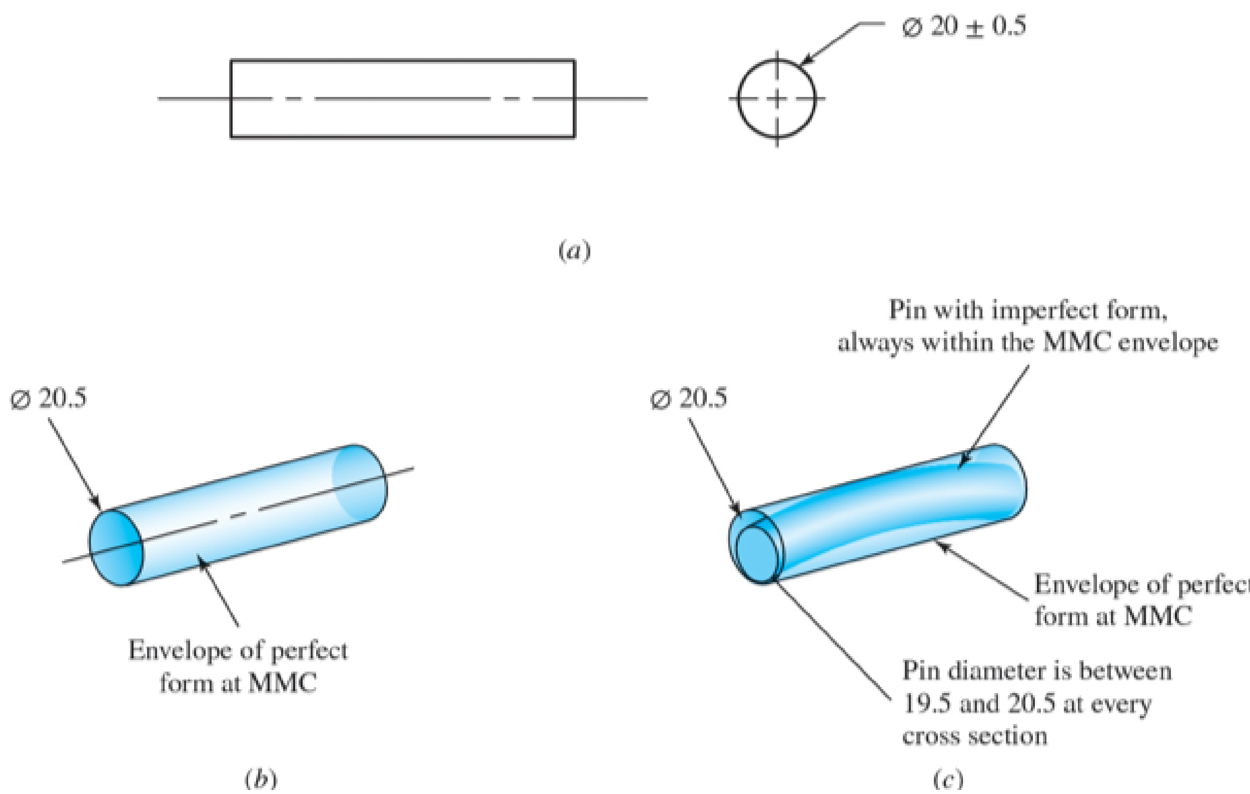


Figure 68: Rule #1 applied to a dowel pin.

The **position control** defines the allowed location and orientation of the axis, center-line, or center plane of a feature of size.

The position control does not control the size or form of the feature.

The position control is applied as follows. See Figure 69 for an example.

1. Use basic dimensions to locate the true position of a feature of size with respect to datums.
2. Directly dimension the size of the feature, using a plus/minus tolerance.
3. Apply the position control with a feature control frame.

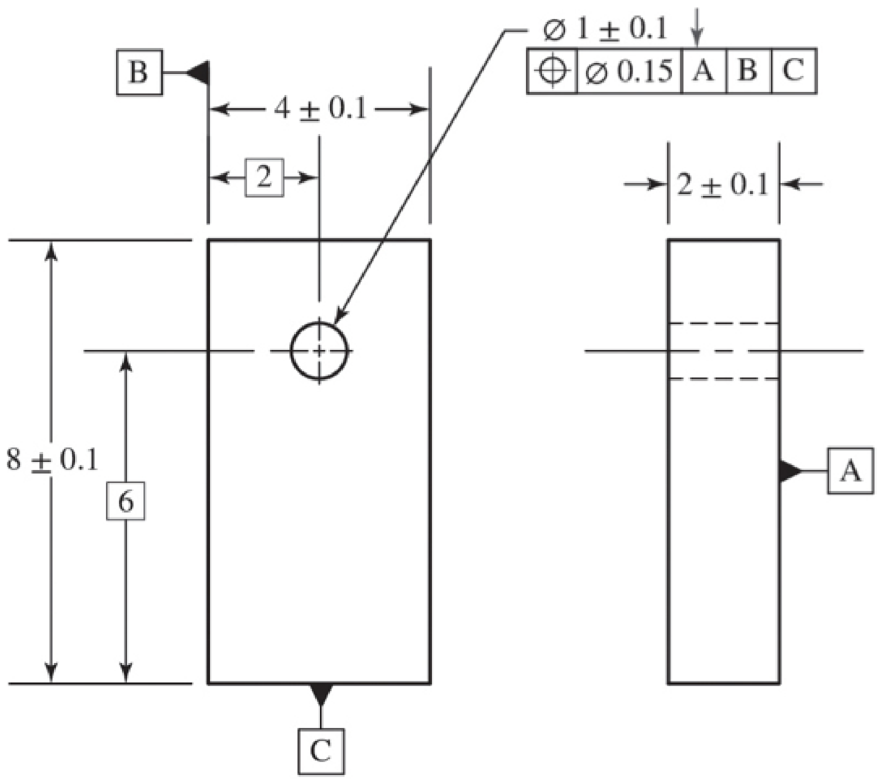


Figure 69: Position control applied to a hole.

14.10 Material Condition Modifiers and Bonus Tolerances

First consider a hole dimensioned with TD&T (see Figure 70).

The center of the hole can be ± 0.005 in both directions. This creates a square tolerance zone, as seen as Figure 71.

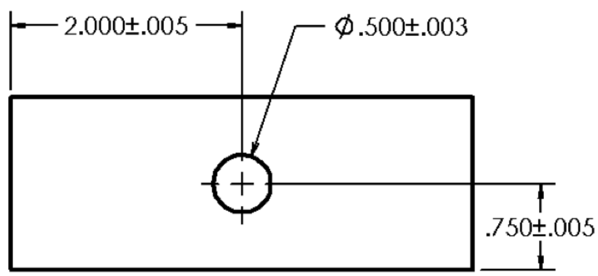


Figure 70: Hole dimensioned with TD&T.

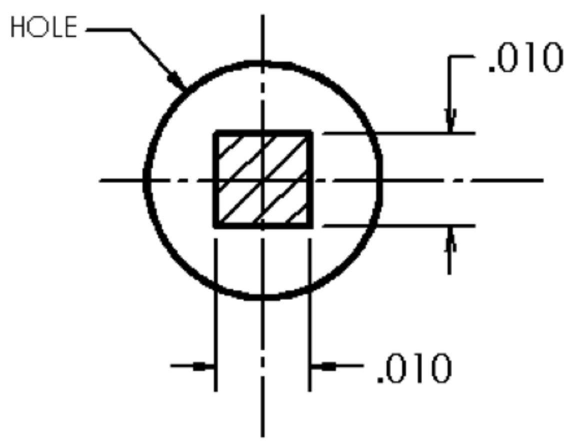


Figure 71: Tolerance zone for a hole dimensioned with TD&T.

Now consider the same hole dimensioned with GD&T (see Figure 72).

The tolerance zone is seen in Figure 73.

The actual center of the hole (axis) must lie in the round tolerance zone. The same tolerance is applied, regardless of the direction.

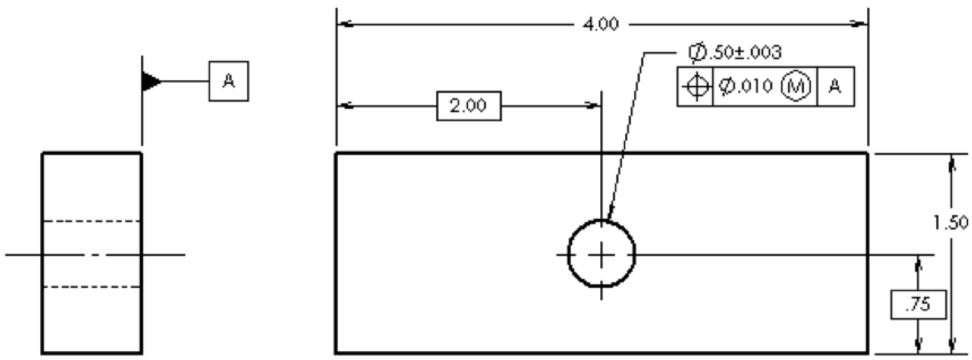


Figure 72: Hole dimensioned with GD&T.

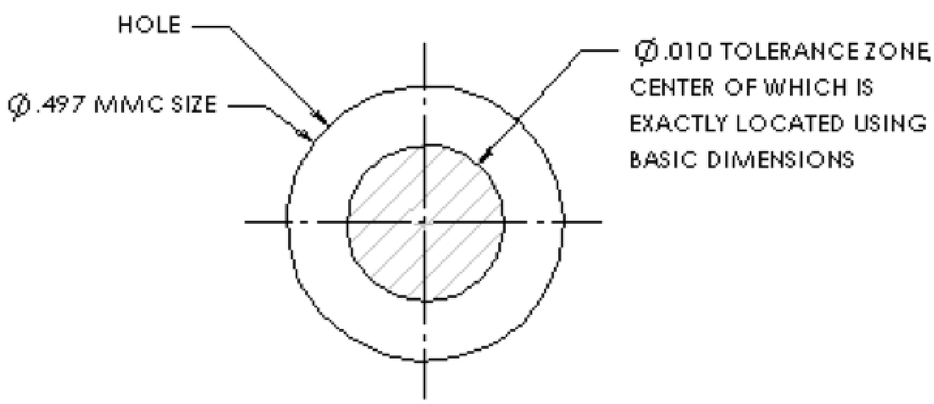


Figure 73: Tolerance zone for a hole dimensioned with GD&T.

Because the feature control frame includes the MMC modifier for tolerance, a **bonus tolerance** is allowed when the hole is larger than the MMC size.

The bonus tolerance is the difference between the MMC size and the actual size.

Using the bonus tolerance system, it is observed that the larger a hole is, the more it can deviate from true position and still fit in the mating condition.

14.11 Example Problems: GD&T

Example 69 Determine the following.

(a) Identify all size attributes.

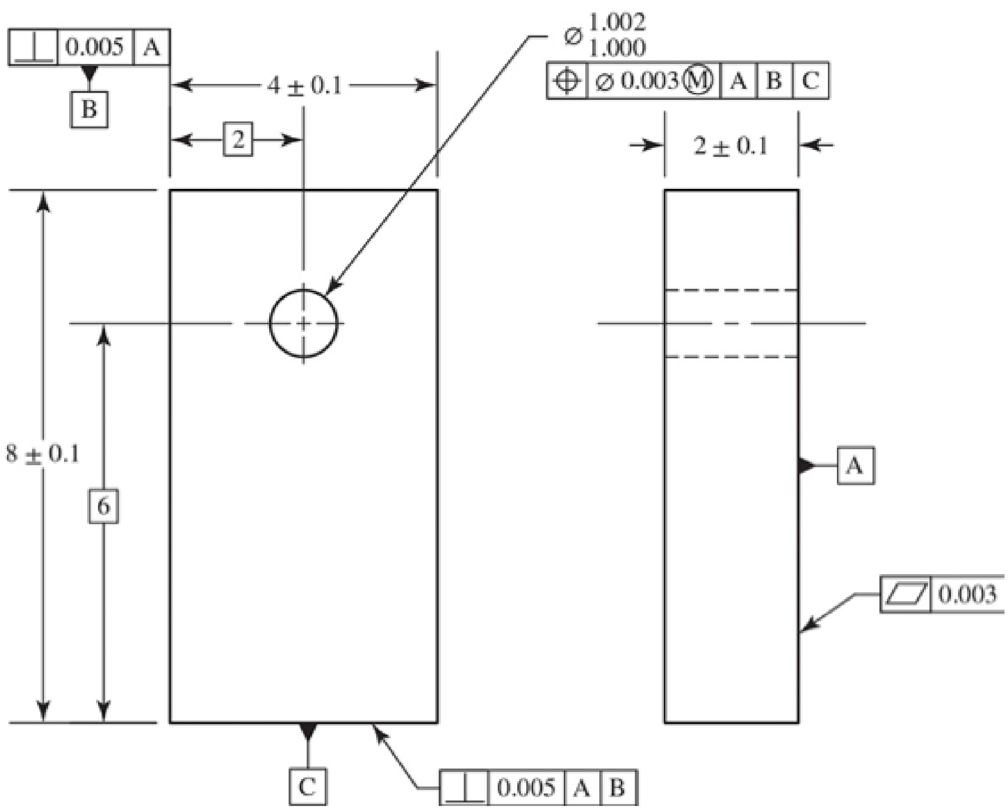
(b) Identify all location attributes.

(c) Identify all orientation attributes.

(d) Identify all form attributes.

(e) Express each control reference frame in a sentence.

(f) Identify all basic dimensions.



Example 70 Determine the following.

(a) Identify all size attributes.

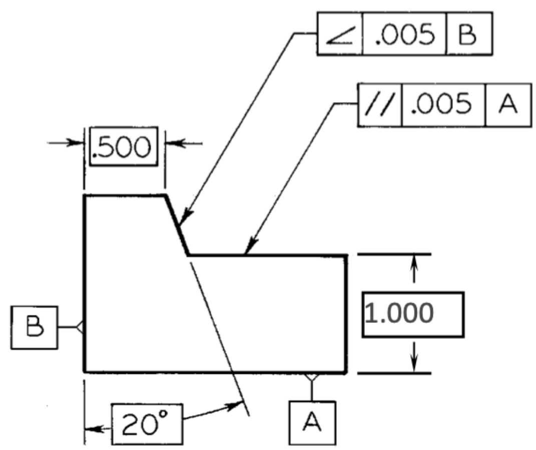
(b) Identify all location attributes.

(c) Identify all orientation attributes.

(d) Identify all form attributes.

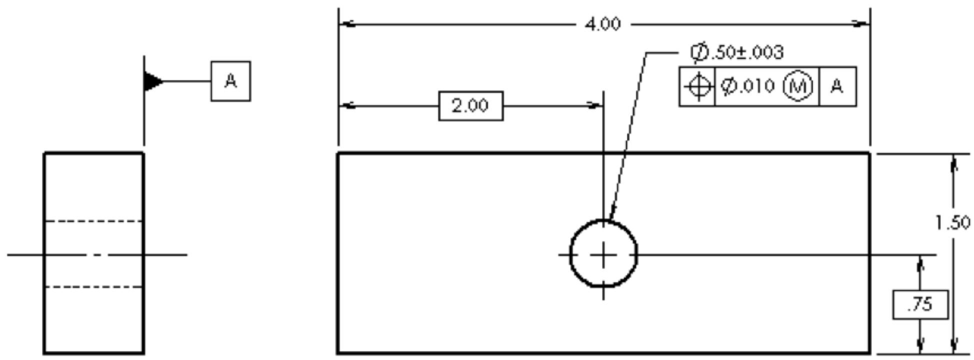
(e) Express each control reference frame in a sentence.

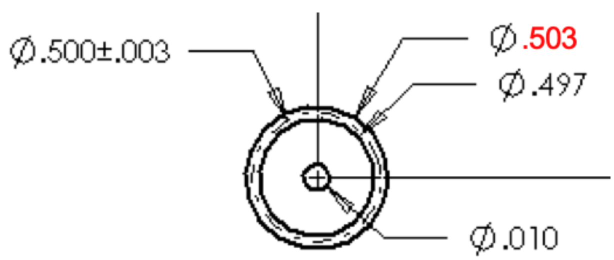
(f) Identify all basic dimensions.



Example 71 Consider the hole shown in the drawing shown below.

Complete the table below.





Actual Hole Size	Bonus Tolerance	Diameter of Tol. Zone
Ø .497 (MMC)		
Ø .499		
Ø .500		
Ø .502		
Ø .503 (LMC)		

Example 72 For the GD&T drawing provided, answer the following.

(a) Identify all basic dimensions.

(b) Identify all features of size.

(c) What is the MMC size of one of the holes?

(d) What is the LMC size of one of the holes?

(e) How much total positional tolerance would be available for one hole if that hole was produced at a size of 0.255?