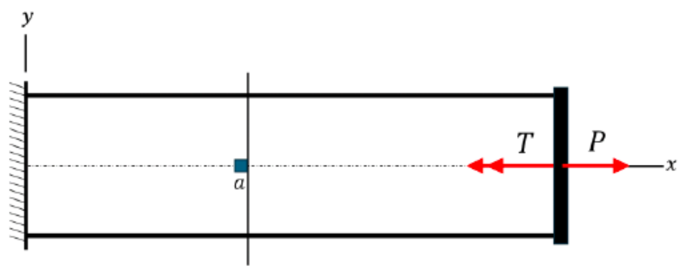
ME 323: Mechanics of Materials **Homework 10**

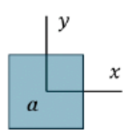
Spring 2026 **Due: April 17, 2026**

Problem 1 (10 points):

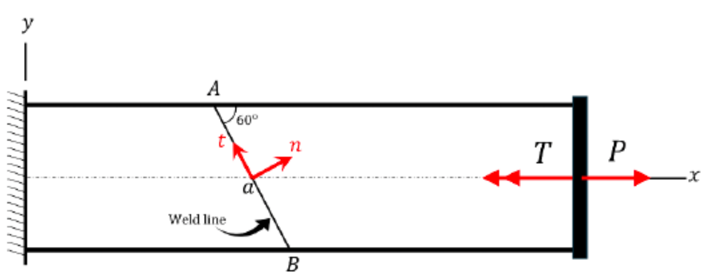
A solid circular shaft is subjected to a tensile axial load P and an applied torque T, as illustrated below. The axial load produces a uniform normal stress of σ =160 ksi, and the torque T generates a shear stress component of τ = 60 ksi.

a) At the surface point labeled a, sketch the stress element showing all x- and y-components of stress under this combined loading condition.



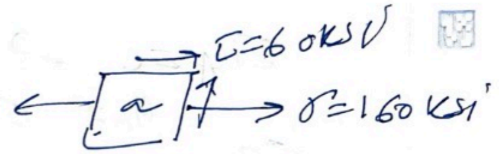


b) Now, assume the cylinder was made by welding two parts together along an inclined plane AB. Using the stress transformation equations, determine the normal stress σ_n and shear stress τ_{nt} acting on the weld if the cylinder is still under the same loading conditions as above. Be careful when finding the correct angle to use in the stress transformation equations.

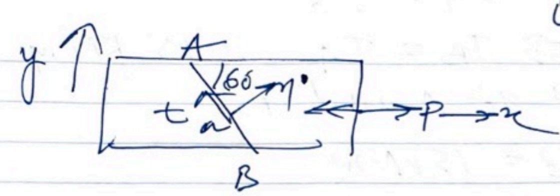


c) Find the principal stresses σ_{p1}, σ_{p2} and CCW rotation angle (θ_{p1}) corresponding to the principal stress σ_{p1}. Plot these principal stresses in a stress element oriented at a CCW angle of θ_{p1} with respect to x-axis. Also calculate absolute max. shear stress.

① a)



b)



$\sigma_x = 160$ ksi, $\sigma_y = 0$, $\tau_{xy} = 60$ ksi

$2\theta = 60$

Normal stress

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \left(\frac{160+0}{2}\right) + \left(\frac{160-0}{2}\right)\cos 60 + 60 \sin 60$$

$$= 171.96 \text{ ksi}$$

Shear stress

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -80 \sin 60 + 60 \cos 60$$

$$= -39.28 \text{ ksi}$$

c) Principal stresses

$$\sigma_{p1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 80 + \sqrt{80^2 + 60^2}$$

$$= 80 + 100 = 180 \text{ ksi}$$

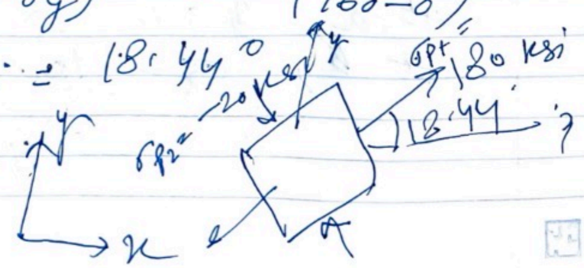
$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 80 - \sqrt{60^2 + 80^2}$$

$$= -20 \text{ ksi}$$

Rotation angle

$$\theta_{p1} = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2 \cdot 60}{160 - 0}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right) = 18.44^\circ$$



Absolute max shear stress = 180- (-20) /2=
100 ksi

Problem 2 (10 points):

A cantilevered beam AC of length $L = 1.5$ m is subjected to a point load $P = 20$ kN and torque T = 15kN.m at C, as shown in the figure below. The beam has a circular cross section with radius $R = 0.3$ m.

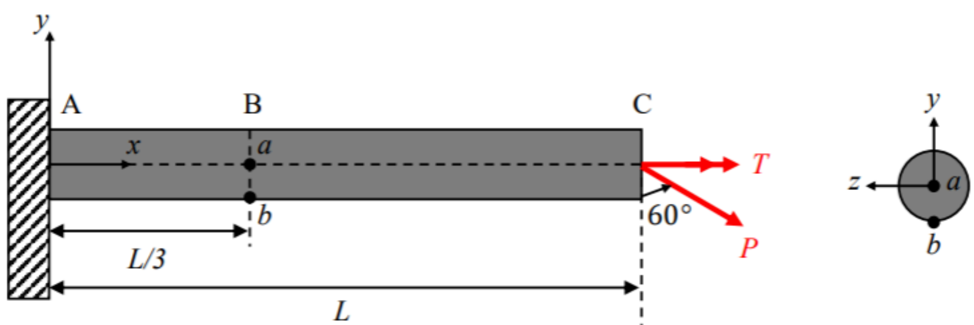
a) Find the reactions at A.

~~b) Construct the shear force and bending moment diagrams for the beam.~~

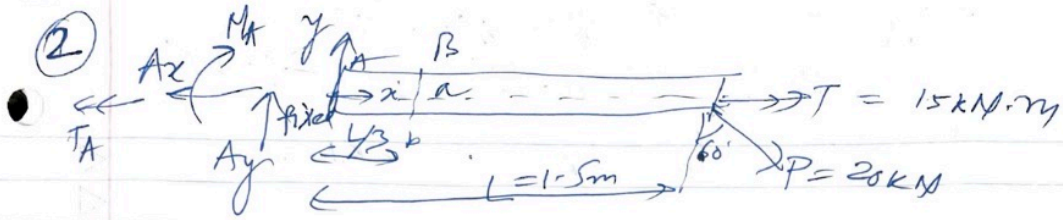
c) Determine the stresses induced at points "a" and "b" at section B.

d) Draw a three-dimensional stress element for each point. On your stress elements, show the directions of the components of stress and their magnitudes.

Write your answers in decimal form and box in your answers at each step.



(2)



a) Equilibrium

$\Sigma F_x = 0 \Rightarrow A_x = P\sin 60° = 20 \times 10^3 \times \frac{\sqrt{3}}{2}$
$= 17.32\ kN$

$\Sigma F_y = 0 \Rightarrow A_y = P\cos 60° \Rightarrow A_y = 20 \times \frac{1}{2} = 10\,kN$

$\Sigma M_A = 0 \Rightarrow -M_A - P\cos(60)L \Rightarrow M_A = -20 \times \frac{1}{2} \times 1.5$

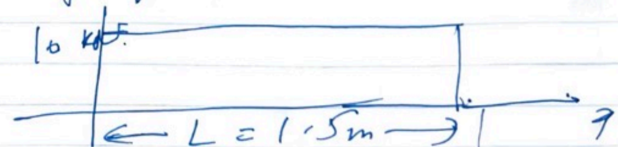
$M_A = -15\,kNm$

$\Sigma T = 0 \Rightarrow +T - T_A = 0 \Rightarrow T_A = T \Rightarrow T_A = 15\,kNm$

b)

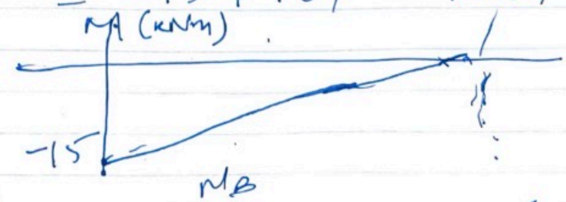
$\Sigma F_y = 0 \Rightarrow A_y - V = 0 \Rightarrow V = 10\,kN$

Shear force diagram

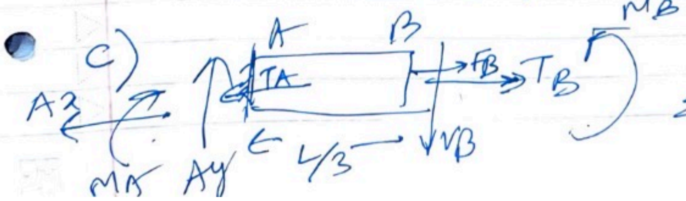


Ignore part b

$\Sigma M = 0 \Rightarrow M = M_A + Vx$
$= -15 + 10x$ kNm



c)



$\Sigma F_x = 0 \Rightarrow F_B = A_x = 17.32\,kN$

$\Sigma F_y = 0 \Rightarrow V_B = A_y = 10\,kN$

$\Sigma M_A = 0 \Rightarrow M_B = M_A + V_B L/3$

$\cdot\ M_B = -15 + 10 \cdot \frac{1.5}{3} = -10\ kNm$

$\Sigma T = 0 \Rightarrow T_B = T_A = 15\ kNm$

Now, stresses at A and B

Torque $T_B = 15\ kNm$

$$\tau = \frac{Tr}{I_p}$$

At point A, $r_A = 0 \Rightarrow \tau_A = 0$

At b, $\tau_{xz} = -\frac{15 \times 0.3}{\frac{\pi}{2}(0.3)^4} = -353.68\ kPa$

Axial force $F_B = 17.32\ kN$, $\sigma = \frac{F}{A}$

At point a & b $\sigma_x = \frac{17.32}{\pi(0.3)^2} = 61.26\ kPa$

Shear force $V_B = 10\ kN$

$$\tau = \frac{V_B A^* y^*}{It} = -\frac{10 \cdot \frac{\pi(0.3)^2}{2} \cdot \frac{4(0.3)}{3\pi}}{\frac{\pi}{4}(0.3)^4\ (2 \times 0.3)}$$

Point a $\tau_{xy} = -47.16\ KPa$

Point b, $\tau_{xy} = 0$

Bending Moment $M_B = -10\ kN$

$\sigma = My/I$

At a, $y = 0 \Rightarrow \sigma = 0$

At b, $\sigma_x = \frac{-10 \cdot (-3)}{\frac{\pi(0.3)^4}{4}} = -471.57\ kPa$

Summary

Loads	Stress	Point "a"	Point "b"
$T_B = 15\,kNm$	$\tau = \frac{Tr}{I_p}$	0	$\tau_{xz} = -353.68\,kPa$
$F_B = 17.32\,kN$	$\sigma = F/A$	$\sigma_x = 61.26\,kPa$	$\sigma_x = 61.26\,kP$
$V_B = 10\,kN$	$\tau = \frac{VA^* y^*}{It}$	$\tau_{xy} = -47.16\,kPa$	0
$M_B = -10\,kN$	$\sigma = My/I$	0	$\sigma_x = -471.57\,kPa$

At point "a"

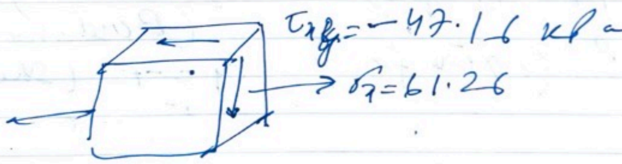
$\sigma_x = 61.26\,kPa \quad \tau_{xy} = -47.16\,kPa$

At point "b"

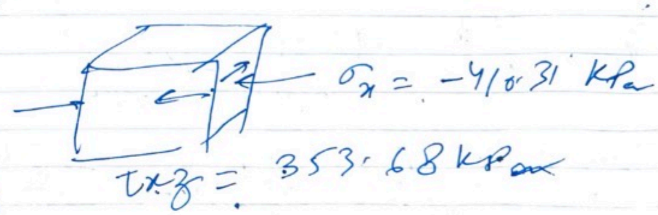
$\sigma_x = 61.26 - 471.57 = -410.31\,kPa$

$\tau_{xz} = -353.68\,kPa$

d)



a)



b)

Problem 3 (10 points):

A structure is attached to a fixed surface at its bottom C, as shown in the figure below. The weight of the structure is negligible. The length of sections BO and OH is L. Both BO and OH have radii of r, with $L = 10r$. The end H is acted upon by a force $\boldsymbol{F}_{\mathbf{H}} = 3P\,\hat{\imath} - P\,\hat{\jmath} + 0\,\hat{k}$. It is desired to understand the states of stress at points "a" and "b" at B.

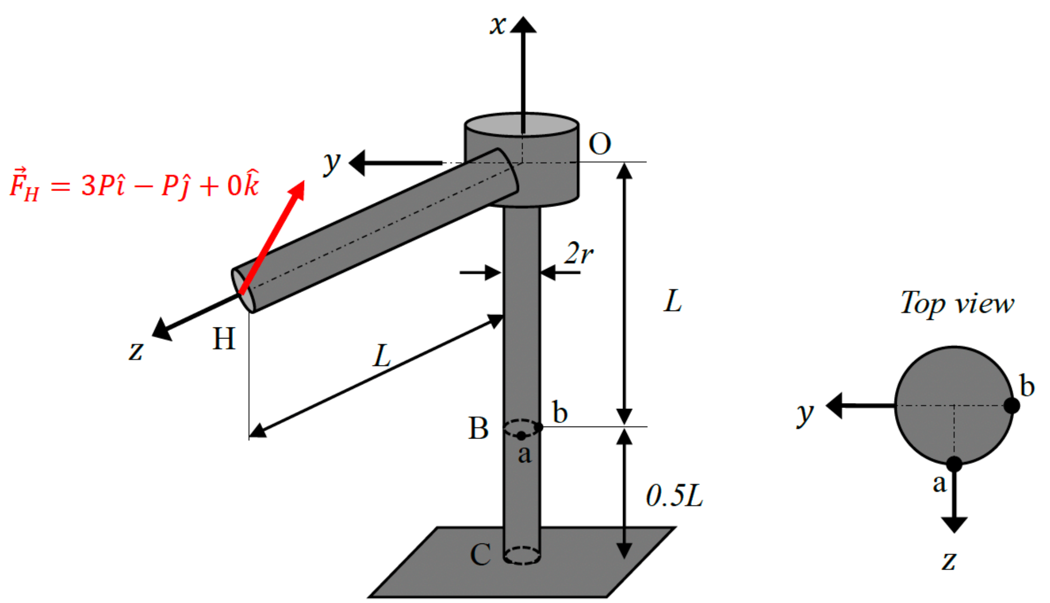
~~a) Determine the internal resultants at cross section B. (i.e., axial force, two shear forces, torque, and two bending moments).~~

b) Identify the components of stress associated with each of the internal resultants at locations "a" and "b" on the cross section of the cut, in the table on the next page.

c) Draw the stress distributions at location B on the cross-sections on the next page.

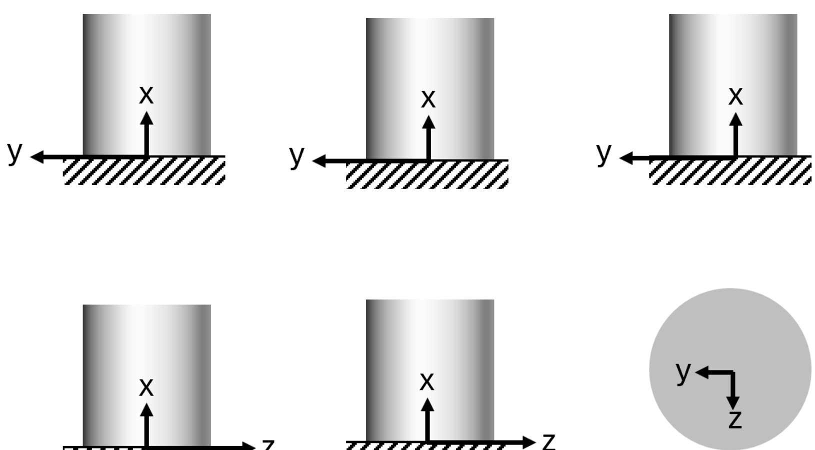
d) Draw the stress elements at locations "a" and "b".

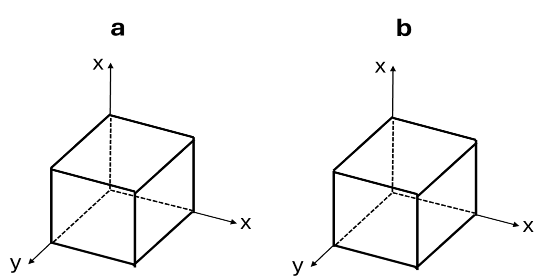
Write your answers in terms of P and r.



Reaction	**Stress at a**	**Stress at b**

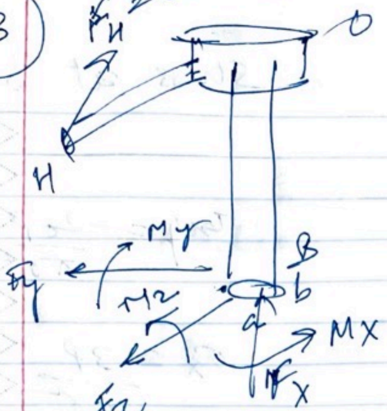
Stress Distribution at location B

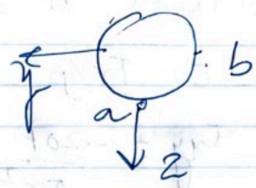




(3) $\vec{F}_H = 3P\hat{i} - P\hat{j} + 0\hat{k}$

a)





$$\vec{F}_B = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{M}_B = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

$$\Sigma F = 0 \Rightarrow 3P\hat{i} - P\hat{j} + 0\hat{k} + F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = 0$$

$F_x = -3P$, $F_y = P$, $F_z = 0$

$$\Sigma M_B = 0 \Rightarrow M_B + \vec{r}_{BH} \times \vec{F}_H = 0$$

$$M_x\hat{i} + M_y\hat{j} + M_z\hat{k} + (L\hat{i} + 0\hat{j} + L\hat{k}) \times (3P\hat{i} - P\hat{j} + 0\hat{k}) = 0$$

$M_x = -LP = -10Pr$

$M_y = -3LP = -30Pr$

$M_z = LP = 10Pr$

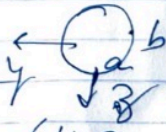
Reactions $M_x = 10Pr$ (Torque) $M_y = 30Pr$ $M_z = -10Pr$ (Bending moment)

$F_x = 3P$ (Axial force) , $F_y = -P$ (Shear force)

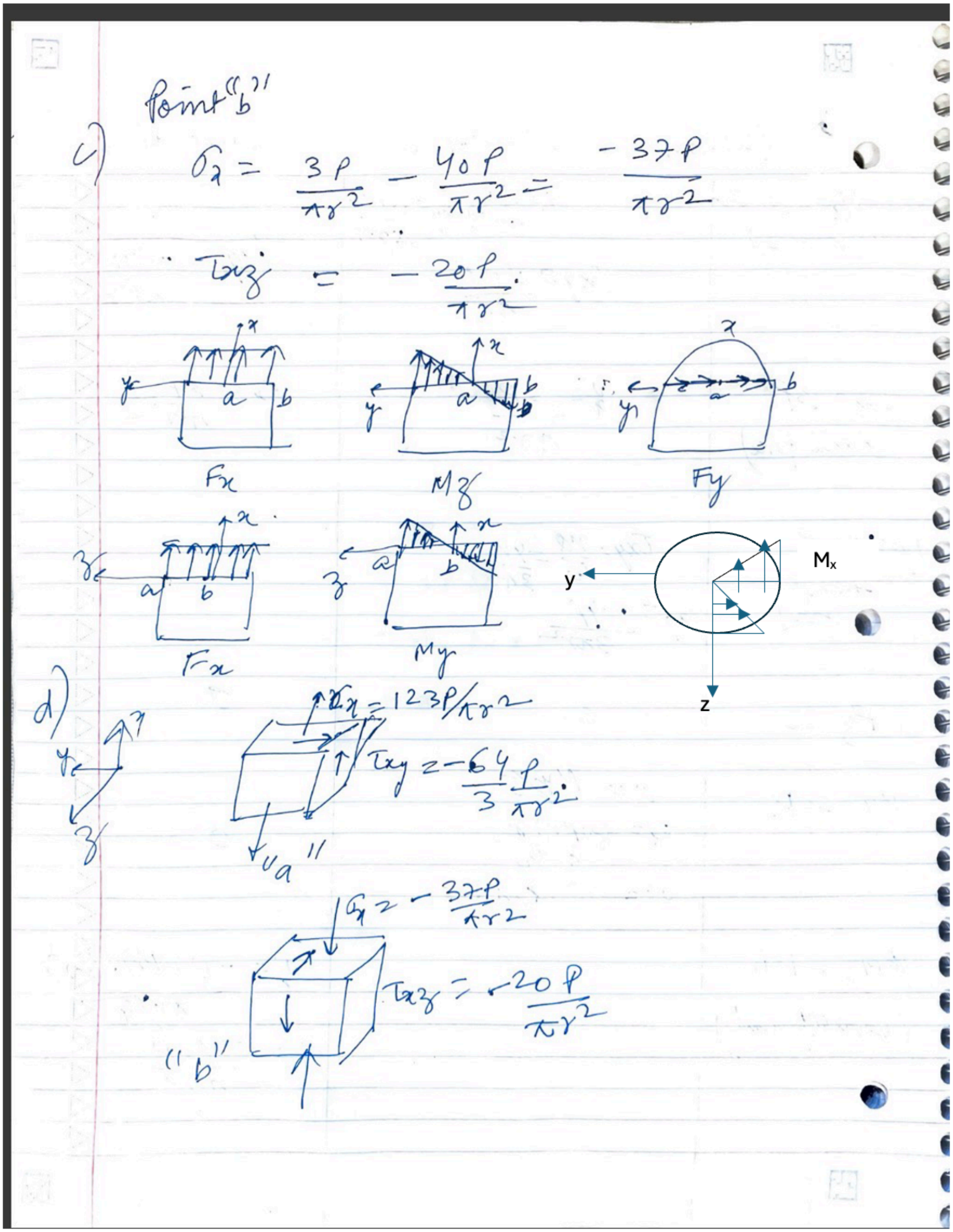
$F_z = 0$

Reaction	Stress at a	Stress at b
$M_x = Pr$ (Torque)	$\tau = Tr/J$ $\tau_{xy} = -\frac{20P}{\pi r^2}$	$\tau_{xy} = -\frac{20P}{\pi r^2}$
$F_x = 3P$ (Axial)	$\sigma_x = \frac{3P}{\pi r^2}$	$\sigma_x = \frac{3P}{\pi r^2}$
$F_y = V = -P$ (Shear force)	$\tau = \frac{VQ}{It} = \frac{4V}{3A}$ $\tau_{xy} = -\frac{4P}{3\pi r^2}$	0
$F_z = 0$	0	0
$M_y = 30Pr$	$\sigma = My/I$ $\sigma_x = \frac{120P}{\pi r^2}$	0
$M_z = -10Pr$ (Bending moment)	0	$\sigma_x = -\frac{40P}{\pi r^2}$

At point "a", $\sigma_x = \frac{3P}{\pi r^2} + \frac{120P}{\pi r^2} = \frac{123P}{\pi r^2}$



$\tau_{xy} = -\frac{20P}{\pi r^2} - \frac{4P}{3\pi r^2} = -\frac{64P}{3\pi r^2}$



Problem 4.1 (2 points)

For a state of plane stress, $\sigma_x = 5$ MPa, $\tau_{xy} = 0$ and σ_y is negative. What is the possible range of the magnitude of the maximum in-plane shear stress, $|\tau|_{max}$?

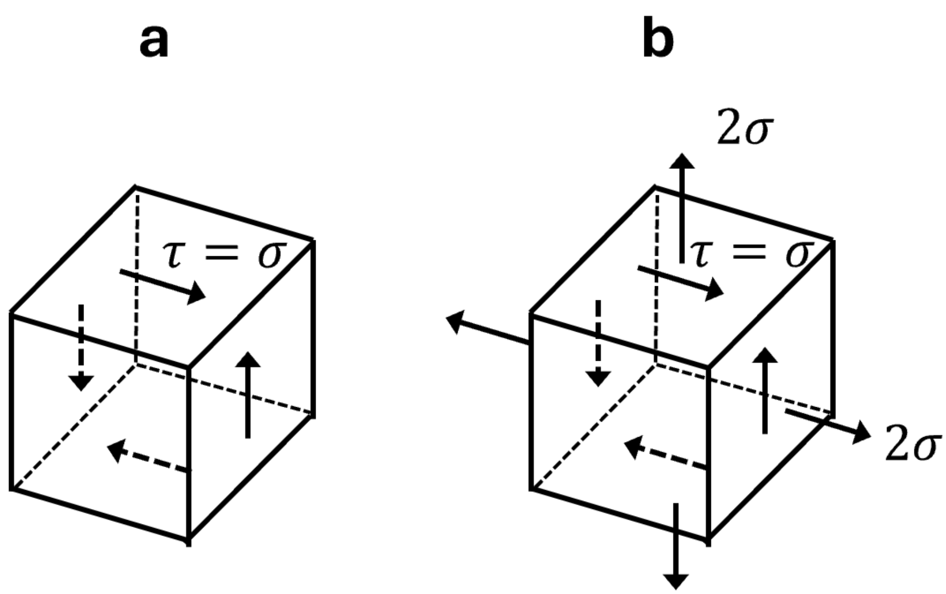
a) $|\tau|_{max}$ = 2.5 MPa
b) $|\tau|_{max}$ < 2.5 MPa
c) $|\tau|_{max}$ > 2.5 MPa
d) More information needed

Problem 4.2 (3 points)

a) Considering stress transformation, the smallest angle of rotation for which the stress components on a stress element repeat their original values is 360 degrees. Indicate whether it is TRUE or FALSE?

b) Stress elements a and b are shown below, "a" is under pure shear while "b" is pure shear along with normal stresses, write the max in-plane shear stress and absolute maximum shear stress in terms of σ.

	a	**b**
$\tau_{max,in-plane}$		
$\tau_{max,abs}$		



solutions
4.1 c

4.2
a) False

	a	**b**
$\tau_{max,in-plane}$	σ	σ
$\tau_{max,abs}$	σ	1.5 σ