Review for Final Exam

Zhao Section

2026.4.27

- Students must exhibit highest standard of honor. Any misconduct of academic integrity will be addressed.
- The exam is closed-book and closed-notes. There will be three full-length problems and one multiple-choice problem with multiple parts.
- Equation Sheet is posted on course blog and will be handed out in the exam.
- Calculator: please bring the allowed type of calculator as described in syllabus: TI-30X and TI-36X models, fx-115 and fx-991 models.
- **Exam Date & Time: May 6, 2026. Time: 1 – 3 PM**
- Final exam will be comprehensive, with a major focus on the materials after exam 2.
- **Exam Room: WTHR200/WTHR104.**
- Please arrive to exam room at least 15 minutes prior to the start of exam.
- Exam Submission Window (20 Minutes): When you complete your exam, you may use your phone to scan your solution and upload to Gradescope. Specifically, your solutions will be scanned and submitted to Gradescope session "ME 323 – S26 – Final Exam". You are responsible for scanning your exam into a single PDF and uploading your exam into Gradescope immediately after completion of your exam. To accommodate the time needed to do this, the deadline to have your exam scanned and uploaded to Gradescope will be 3:20PM (EST), giving 20 minutes to complete this process. The time limit will be strictly enforced.
- Assigning Pages for Your Exam: As part of the submission process, you will need to identify the page numbers for Problem 1, 2, ... separately. If you need extra papers, please use your own but make sure to arrange the pages in the correct order in your submission. Do not submit the equation sheet.

Coverage: **Comprehensive**

27 F	27-Mar	Energy methods – introduction to finite element methods	Chap. 17	
28 M	30-Mar	Energy methods – introduction to finite element methods	Chap. 17	
29 W	1-Apr	Energy methods – introduction to finite element methods	Chap. 17	
30 F	3-Apr	Thin-walled pressure vessels – axial and hoop stresses	Chap. 12	HW 8
31 M	6-Apr	Stress transformation – principal /maximum shear stresses	Chap. 13	
32 W	8-Apr	Stress transformation – Mohr's circle	Chap. 13	
33 F	10-Apr	Stress transformation – absolute maximum shear stress	Chap. 13	HW 9
34 M	13-Apr	Stresses – combined loading	Chap. 14	
35 W	15-Apr	Stresses – combined loading	Chap. 14	
36 F	17-Apr	Stresses – combined loading	Chap. 14	HW 10
37 M	20-Apr	Failure analysis – stress theories	Chap. 15	
38 W	22-Apr	Failure analysis – stress theories	Chap. 15	
39 F	24-Apr	Failure analysis – buckling of columns	Chap. 18	HW 11
40 M	27-Apr	Practice with combined loadings and failure analysis		
41 W	29-Apr	Practice with combined loadings and failure analysis		
42 F	1-May	Review		

Finite element method

- *Defining the nodes and elements for the problem*. Choose a set of N+1 *nodes* along the length of the rod at locations $x_1(=0), x_2, x_3, \ldots, x_N, x_{N+1}(=L)$. The subdomain of $x_i < x < x_{i+1}$ is known as the ith *element* of length $L_i = x_{i+1} - x_i$, for $i = 1, 2, \ldots, N$. The value of $k_i = (EA)_i / L_i$ is determined through the average value of EA over the ith element and the element length L_i.

- *Constructing the global stiffness matrix*. Construct the stiffness matrix [K]. The resulting matrix will be tri-diagonal and of size $(N+1)\times(N+1)$.

- *Constructing the force vector*
Construct the force vector $\{F\}$ as being made up on the resultant *external* force acting on each node. The resulting vector will be of length *N+1*.

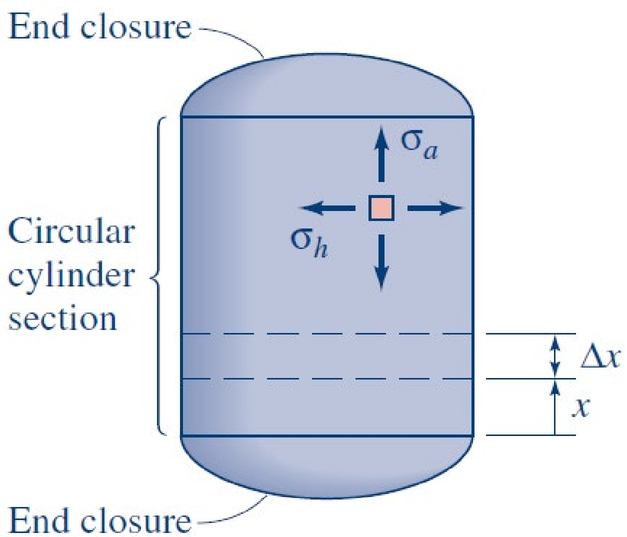
- *Enforcing fixed-displacement boundary conditions*. The fixed-displacement boundary conditions are enforced through the elimination of appropriate terms in the resulting stiffness matrix $[K]$ and forcing vector $\{F\}$. For example, if the ith node has a fixed (zero) displacement, we eliminate the ith row and ith column of $[K]$ and the ith row of $\{F\}$. If the problem has "n" fixed nodal displacements, then the stiffness matrix and force vector will be of sizes $(N-n+1)\times(N-n+1)$ and $N-n+1$, respectively[1].

- *Solving*. The nodal displacements u_k ; $k = 1, 2, \ldots, N-n+1$ are found from the solution of the algebraic equilibrium equations:
$$[K]\{u\} = \{F\}$$
through a linear equation solver in an application such as Matlab or Mathematica.

- *Stress calculations*
The average stress across the ith element is found from:
$$\sigma_i = \frac{E_i}{L_i}(u_{i+1} - u_i)$$

Thin wall pressure vessels

Cylindrical pressure vessel

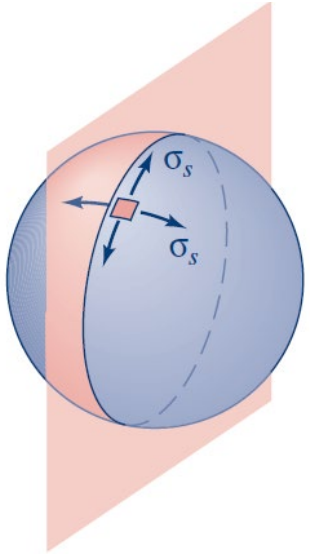


Axial stress $\sigma_a = \frac{pr}{2t}$

Hoop stress $\sigma_h = \frac{pr}{t}$

Mohr's circle?

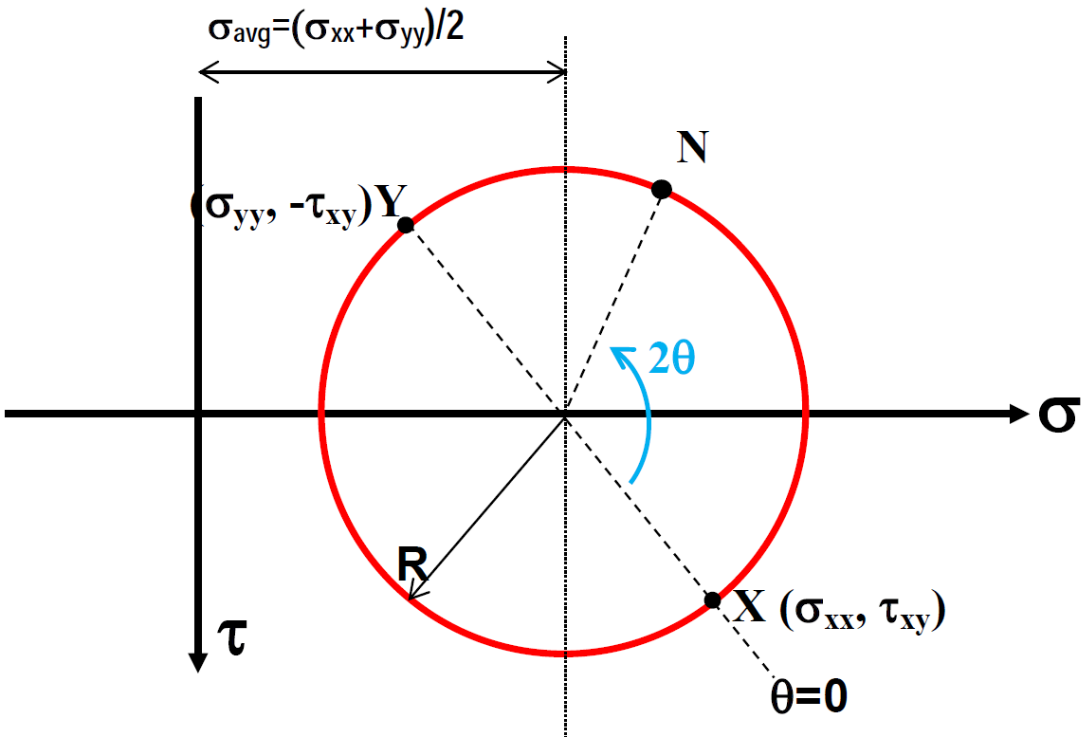
Spherical pressure vessel



$$\sigma_s = \frac{pr}{2t}$$

Mohr's circle?

Stress transformation & Mohr's circle



$$\sigma_1 = \sigma_{avg} + R,$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = R$$

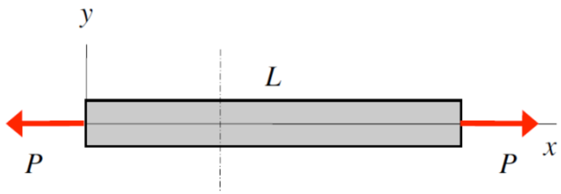
3-dimensional:

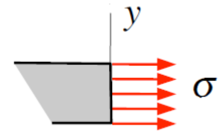
- σ_1 is the *largest* of the three
- σ_3 is the *smallest* of the three
- σ_2 is the *intermediate* of the three

$$(\tau_{max})_{abs} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

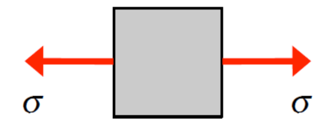
Combined loads

Axial load

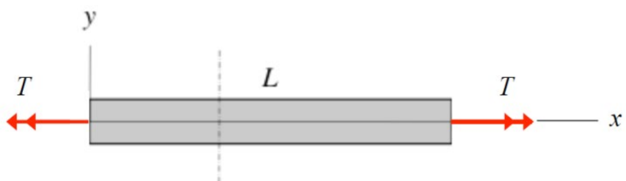


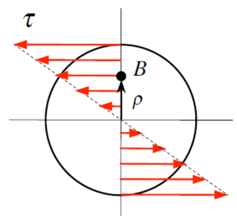


$$\sigma = \frac{P}{A}$$

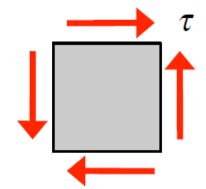


Torsion

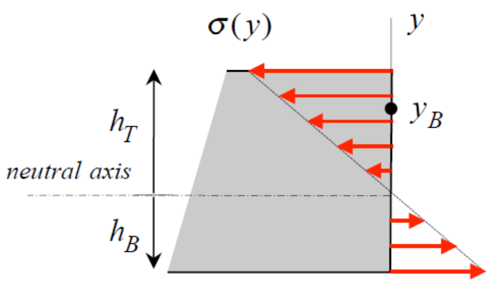




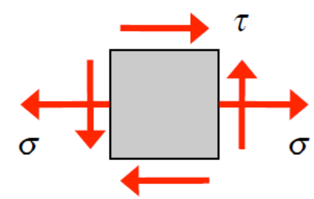
$$\tau = \frac{T\rho}{J}$$



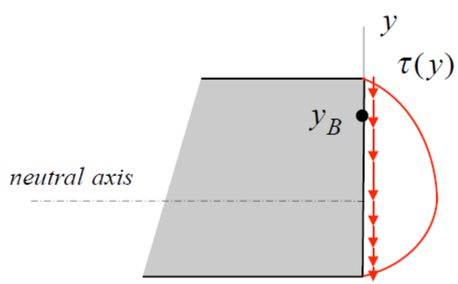
Flexural stress



$$\sigma = -\frac{My_B}{I}$$



Shear stress



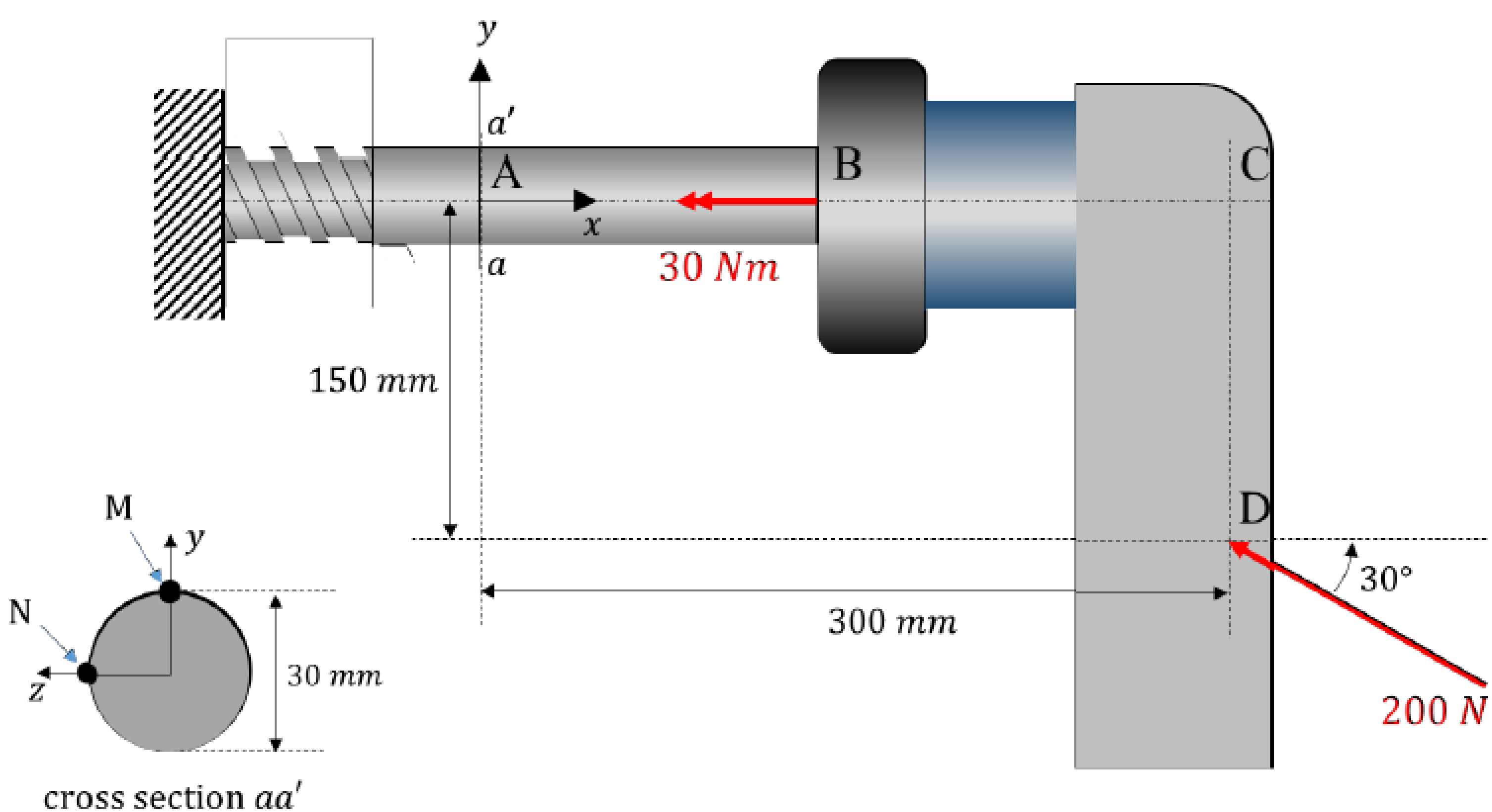
$$\tau = \frac{VA^{*}\overline{y}^{*}}{It}$$

Problem X (XX points): A drill jammed in the wall is acted upon by a point load at D, and a torque at B, as shown in the figure below. For the given state of loading,

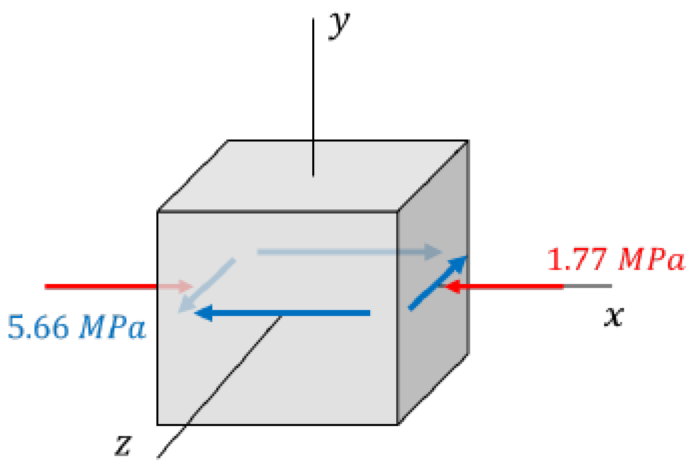
a) Determine the stress state at the point M on the cross section aa', and represent the stress state on an appropriate stress element.

b) Determine the stress state at the point N on the cross section aa', and represent the stress state on an appropriate stress element.

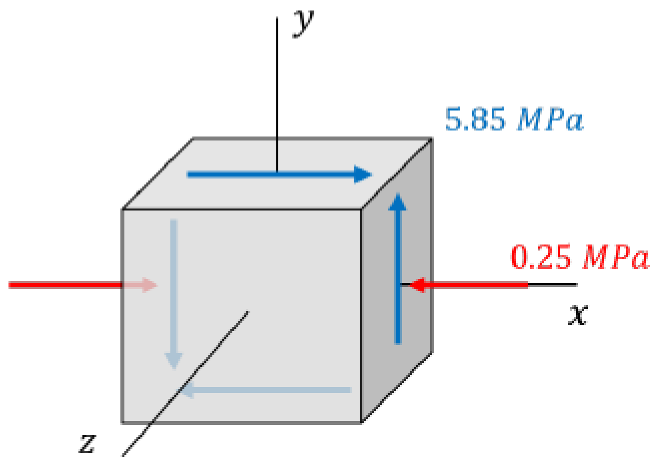
c) Using a Mohr's circle, determine the absolute maximum shear stress $\tau_{max,abs}$ for the points M and N.



	M	N
$F_x = -200\cos 30°$	$\sigma_x = \frac{F_A}{A} = -0.25\ MPa$	$\sigma_x = \frac{F_x}{A} = -0.25\ MPa$
$F_y = 200\sin 30°$	$\tau = 0 \#(\text{free surface})$	$\tau_{xy} = \frac{4F_y}{3A} = 0.19\ MPa$
$F_z = 0$	0	0
$M_{x,A} = -30000\ Nmm$	$\tau_{xz} = \frac{16M_{x,A}}{\pi d^3} = -5.66\ MPa$	$\tau_{xy} = -\frac{16M_{x,A}}{\pi d^3} = 5.66\ MPa$
$M_{y,A} = 0$	0	0
$M_{z,A} = 4019.24\ Nmm$	$\sigma_x = -\frac{32M_{z,A}}{\pi d^3} = -1.52\ MPa$	$\sigma_x = 0 \#(\text{neutral plane})$



Element M



Element N

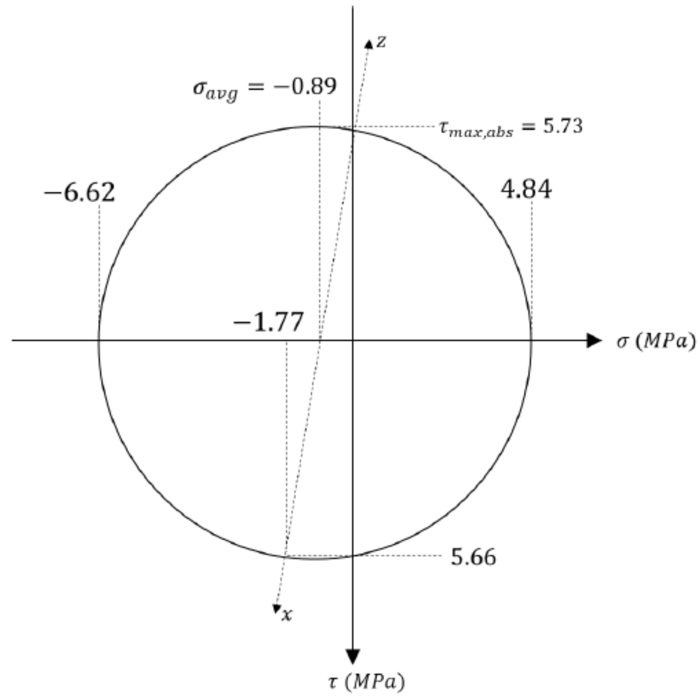
c) For element M:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = -0.89\ MPa; \quad R = \sqrt{\left(\sigma_x - \sigma_{avg}\right)^2 + \tau_{xz}^2} = 5.73\ MPa$$

$$\sigma_{p1} = \sigma_{avg} + R = 4.84\ MPa\ (= \sigma_{max})$$

$$\sigma_{p2} = \sigma_{avg} - R = -6.62\ MPa\ (= \sigma_{min})$$

$$\therefore \tau_{max,abs} = R = 5.73\ MPa$$



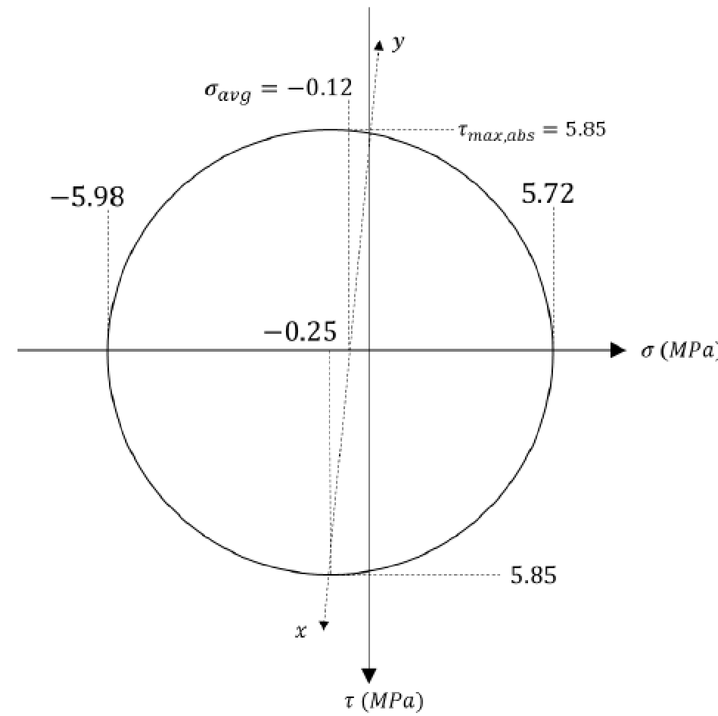
For element N:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -0.13\ MPa; \quad R = \sqrt{\left(\sigma_x - \sigma_{avg}\right)^2 + \tau_{xy}^2} \approx 5.85\ MPa$$

$$\sigma_{p1} = \sigma_{avg} + R = 5.72\ MPa\ (= \sigma_{max})$$

$$\sigma_{p2} = \sigma_{avg} - R = -5.98\ MPa\ (= \sigma_{min})$$

$$\therefore \tau_{max,abs} = R = 5.85\ MPa$$



Failure theories

Ductile materials

Maximum shear stress failure theory

$$\tau_{max,abs} = \frac{\sigma_Y}{2}$$

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2}$$

Maximum distortional energy failure theory

$$\sigma_Y = \sigma_M$$

$$\sigma_M = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

Brittle materials

Maximum normal stress failure theory

$$|\sigma_1| = \sigma_U \quad \text{OR} \quad |\sigma_3| = \sigma_U$$

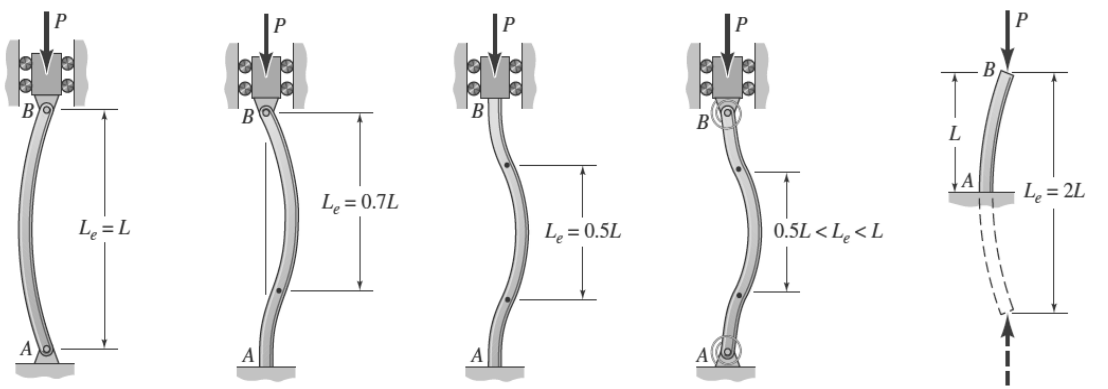
Mohr's failure theory

$$\sigma_{\max} = \sigma_{TU} \quad OR \quad \sigma_{\min} = -\sigma_{CU}\,.$$

$$\frac{\sigma_{P1}}{\sigma_{UT}} = \frac{\sigma_{P2}}{\sigma_{UC}} + 1\,.$$

Buckling of columns

Euler buckling $$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2}$$



(*a*) Pinned-pinned column, $K = 1$.

(*b*) Fixed-pinned column, $K = 0.7$.

(*c*) Fixed-fixed column, $K = 0.5$.

(*d*) Partially-restrained column. $0.5 < K < 1$.

(*e*) Fixed-free column, $K = 2$.