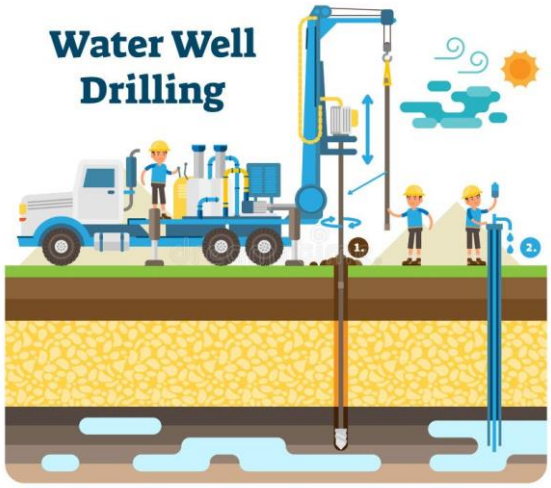
Quiz 12 – Failure in Drills

A drill rig has a shaft with a length of L and a diameter of d. The shaft experiences a compressive force of 1.5P and an applied torque of $T = Pd/4$. The Young's modulus is 80 GPa and the shear modulus is 30 GPa.



a) If the material is ductile with a yield stress of σ_y, use the **maximum shear stress theory** (MSS) to find the maximum value of P that can be applied. (in terms of σ_y and d)

b) If the material is brittle with failure stresses of $\sigma_{UC} = 4\sigma_{UT}$, find the maximum value of P that can be applied. (in terms of σ_{UT} and d)

Solution:

$$\sigma_y = -\frac{1.5P}{A} = -\frac{6P}{\pi d^2}$$

$$\tau_{xy} = \frac{TR}{I_p} = \left(\frac{Pd}{4}\right)\left(\frac{\frac{d}{2}}{\frac{\pi}{32}d^4}\right) = \frac{4P}{\pi d^2}$$

$$\sigma_{avg} = -\frac{3P}{\pi d^2}$$

$$R = \sqrt{\left(\frac{3P}{\pi d^2}\right)^2 + \left(\frac{4P}{\pi d^2}\right)^2} = \frac{5P}{\pi d^2}$$

(a) One positive and one negative principal stress

$$\rightarrow \tau_{max,abs} = R = \frac{5P}{\pi d^2} = \frac{\sigma_Y}{2}$$

$$P = \left(\frac{1}{10}\right)\pi d^2 \sigma_Y$$

(b) One positive and one negative principal stress
→ use Mohr's criterion

$$\frac{\sigma_{P1}}{\sigma_{UT}} = \frac{\sigma_{P2}}{\sigma_{UC}} + 1$$

$$\frac{\frac{2P}{\pi d^2}}{\sigma_{UT}} + \frac{\frac{8P}{\pi d^2}}{4\sigma_{UT}} = 1$$

$$\frac{2P}{\pi d^2 \sigma_{UT}} + \frac{2P}{\pi d^2 \sigma_{UT}} = 1$$

$$P = \frac{\pi d^2 \sigma_{UT}}{4}$$