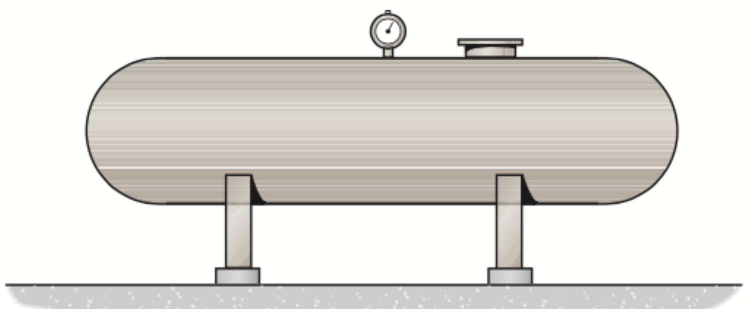
ME 323: Mechanics of Materials — **Homework Set 9**

Spring 2026 — **Due: Friday, Apr. 10, 2026**

Problem 1 (10 points)

A cylindrical vessel has an inner radius $r = 1000\ mm$ and a wall thickness $t = 10\ mm$. The internal pressure is $p = 1.5\ MPa$ and the maximum allowable stress of the vessel is $\sigma_{allow} = 200\ MPa$.

(1) Determine the axial stress σ_a and the hoop stress σ_h in the cylindrical part of the vessel.
(2) Determine the principal stresses, σ_1, σ_2 and σ_3.
(3) Determine the maximum in-plane shear stress τ_{max}.
(4) Determine the maximum allowable pressure p_{allow} so that σ_1 doesn't exceed σ_{allow}.



Solution:

Since $r/t = 100 > 10$, it is a thin wall vessel.

Axial stress:

$$\sigma_a = \frac{pr}{2t} = \frac{1.5 * 1}{2 * 0.01} = 75\ MPa$$

Hoop stress:

$$\sigma_h = \frac{pr}{t} = \frac{1.5 * 1}{0.01} = 150\ MPa$$

The principal stresses:

$$\sigma_1 = \sigma_h = 150\ MPa$$

$$\sigma_2 = \sigma_a = 75\ MPa$$

$$\sigma_3 = 0$$

Maximum in-plane shear stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_h - \sigma_a}{2}\right)^2 + 0^2} = \frac{\sigma_h - \sigma_a}{2} = 37.5\ MPa$$

Maximum allowable pressure:

$$\sigma_{allow} = \frac{p_{allow} r}{t}$$

$$200 = \frac{p_{allow} * 1}{0.01}$$

$$p_{allow} = 2\ MPa$$

Problem 2 (10 points):

The stress element shown below represents the state of stress measured along the x'y' axis in a component loaded under plane stress. No information is known about the stress α except that it is compressive.

(a) Determine the magnitude of the maximum compressive normal stress that can be applied, if the component is made of a material which can withstand a maximum in-plane shear stress of 100 MPa.

(b) Determine the stress components when the element is oriented along the x-y axes.

(c) Draw a stress element oriented along the maximum in-plane shear stress directions. (Show the angle of this rotated element with respect to the axis x')

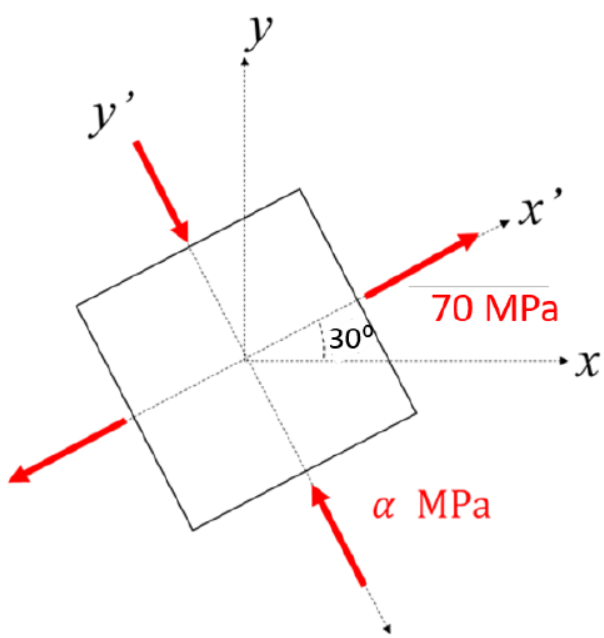


Figure 2: Stress element for Problem 2

(a) Let the stress along the y'direction be $\sigma_{y'}$. In the $x'y'$ orientation as seen in the problem figure, the element is in oriented along the principal stress direction (as shear stress $\tau_{x'y'}$ is 0). Hence $\sigma_{x'}(= 70\ MPa)$ and $\sigma_{y'}$ are principal stresses.

Given max shear stress that the material can withstand is 100 MPa, we have

$$|\tau_{max}| = 100 \text{ MPa}$$

$$=> \frac{|\sigma_{y'} - \sigma_{x'}|}{2} = 100 \text{ MPa}$$

$$=> \sigma_{y'} - 70 = \pm 200$$

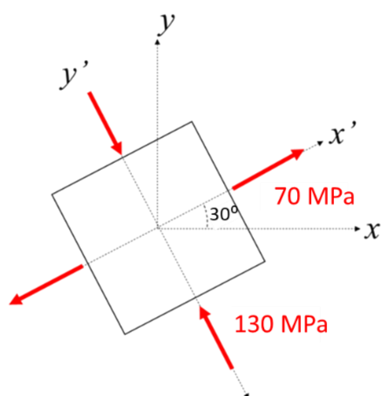
$$=> \sigma_{y'} = +270 \text{ MPa } or - 130 \text{ MPa}$$

Since $\sigma_{y'}$ is a compressive state of stress

$$\sigma_{y'} = -130 \text{ MPa}$$

Hence magnitude of $\sigma_{y'}$, i.e.,

$$\alpha = 130 \text{ MPa}$$



(b) The stress components along the xy axes can be obtained by rotating $\theta = -30°$

$$\sigma_x = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\theta) + \tau_{x'y'}\sin(2\theta)$$

$$\sigma_x = 20 \text{ MPa}$$

$$\sigma_y = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\theta) - \tau_{x'y'}\sin(2\theta)$$

$$\sigma_y = -80 \text{ MPa}$$

$$\tau_{xy} = -\left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\sin(2\theta) + \tau_{x'y'}\cos(2\theta)$$

$$\tau_{xy} = 86.603 \text{ MPa}$$

(c) Max shear stress is oriented at an angle of $\pm 45°$ to the principal axis. In this problem, the principal axis is oriented along $x'y'$ axis. For the solution we will consider a rotation of $\beta = -45°$ to get to max shear orientation

$$\tau_{x''y''} = \tau_{max} = -\left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\sin(2\beta)) + \tau_{x'y'}\cos(2\beta)$$

$$\tau_{max} = 100 \text{ MPa}$$

The correspnding normal stress along x'' is

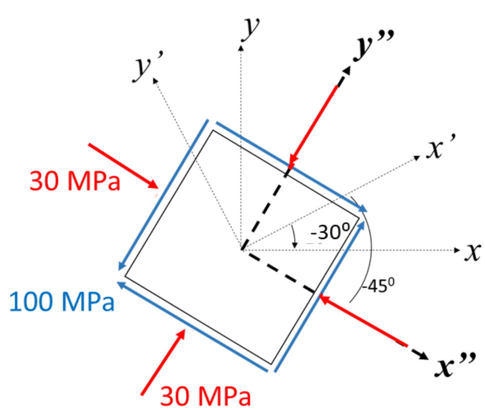
$$\sigma_{x''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\beta) + \tau_{x'y'}\cos(2\beta)$$

$$\sigma_{x''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = -30 \text{ MPa} = \sigma_{ave}$$

And along y'' axis is

$$\sigma_{y''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\beta) - \tau_{x'y'}\cos(2\beta)$$

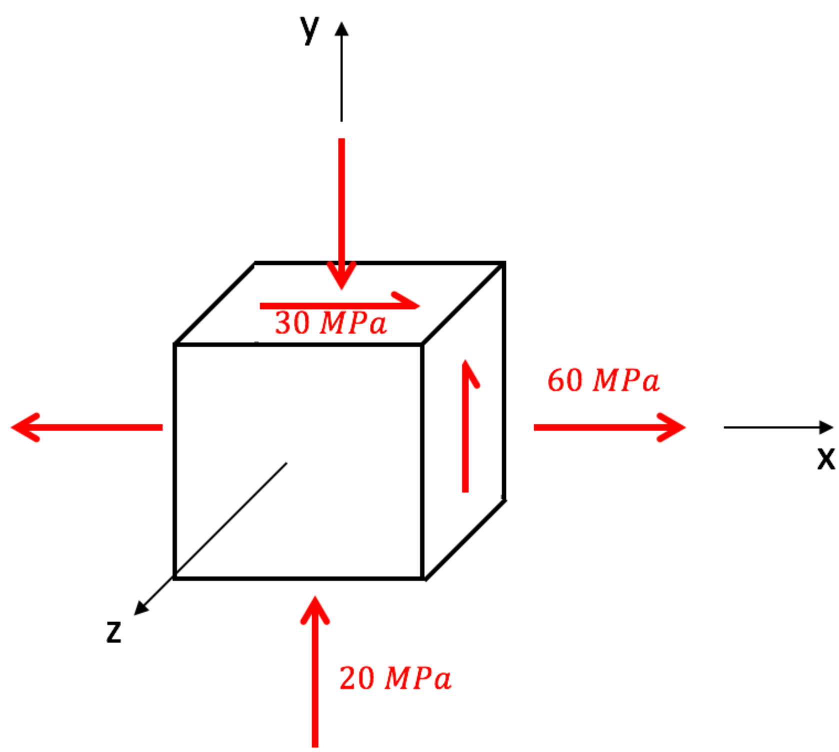
$$\sigma_{y''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = -30 \text{ MPa} = \sigma_{ave}$$



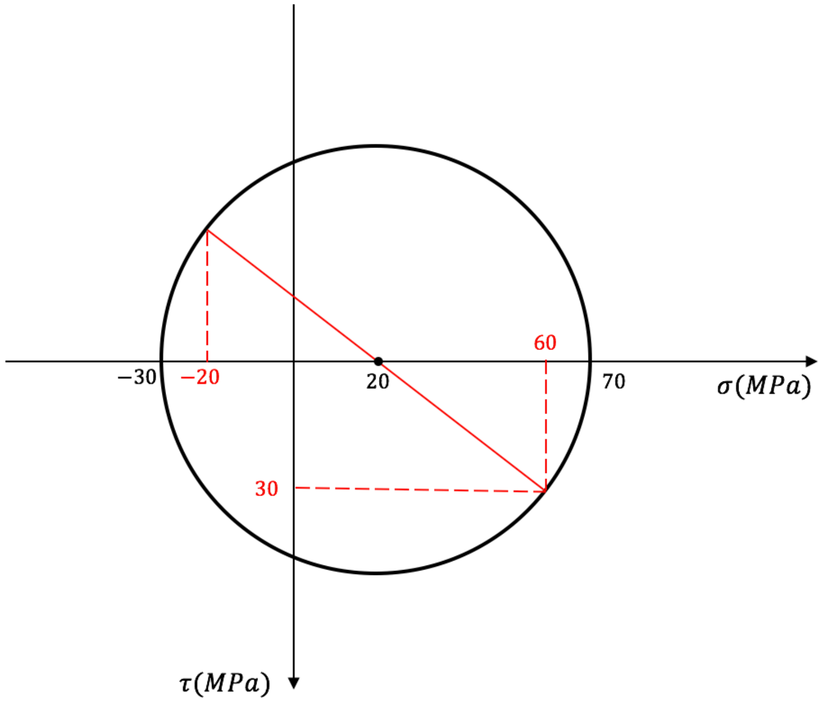
Problem 3 (10 points)

Use the given state of stress

(1) Draw the Mohr's circle.
(2) Using the Mohr's circle, determine the principal stresses σ_1, σ_2 and σ_3.
(3) Sketch a 2D element under the principal stresses with proper orientation.
(4) Using the Mohr's circle, determine the maximum shear stress.
(5) Sketch a 2D element under the stress state found in (4) with proper orientation.



Solution:

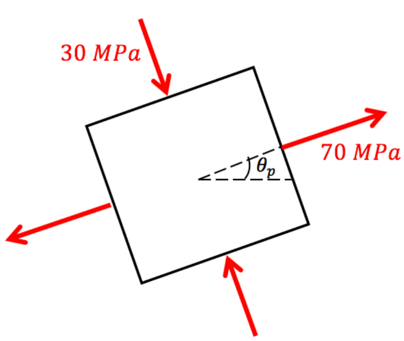


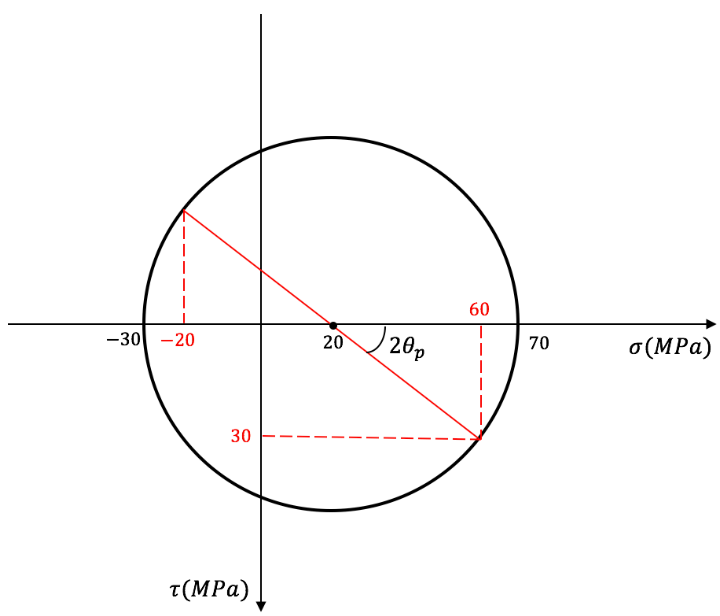
Radius of the Mohr's circle:

$$R = \sqrt{\left(\frac{60-(-20)}{2}\right)^2 + 30^2} = 50\ MPa$$

(2) Pricinpal stresses:

$$\sigma_1 = \sigma_{avg} + R = 20 + 50 = 70\ MPa$$
$$\sigma_2 = 0$$
$$\sigma_3 = \sigma_{avg} - R = 20 - 50 = -30\ MPa$$



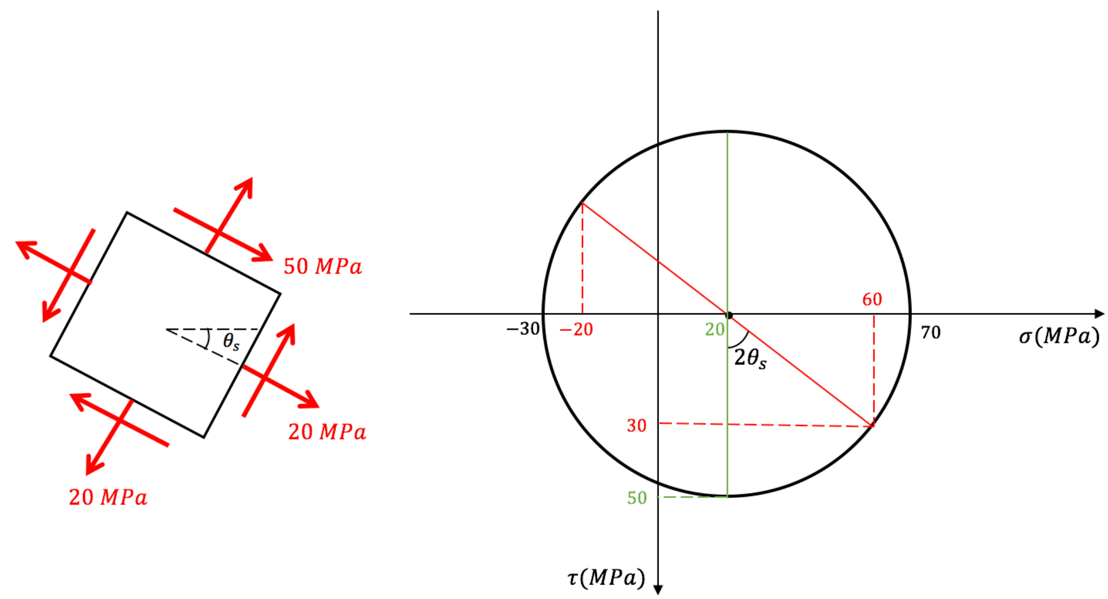


$$\theta_p = \frac{1}{2}\text{atan}\left(\frac{30}{40}\right) = 18.43°$$

(counter-clockwise)

(4) Maximum shear stress:

$$\tau_{max} = R = 50\ MPa$$



$$\theta_s = \frac{1}{2}\text{atan}\left(\frac{40}{30}\right) = 26.57°$$

(clockwise)

Problem 4 (2.5 + 2.5 points):

I. A moment M (<u>about positive z</u>) and torque T (<u>about positive x</u>) are applied to a circular rod as shown in *Figure 4.1*. Choose the correct in plane Mohr's circle, from the given options, for the stress states at Point a and Point b. Justify. (Note that **<u>location of Point a is at (L/2, 0, -R)</u>** where R is the radius of the cross section.)

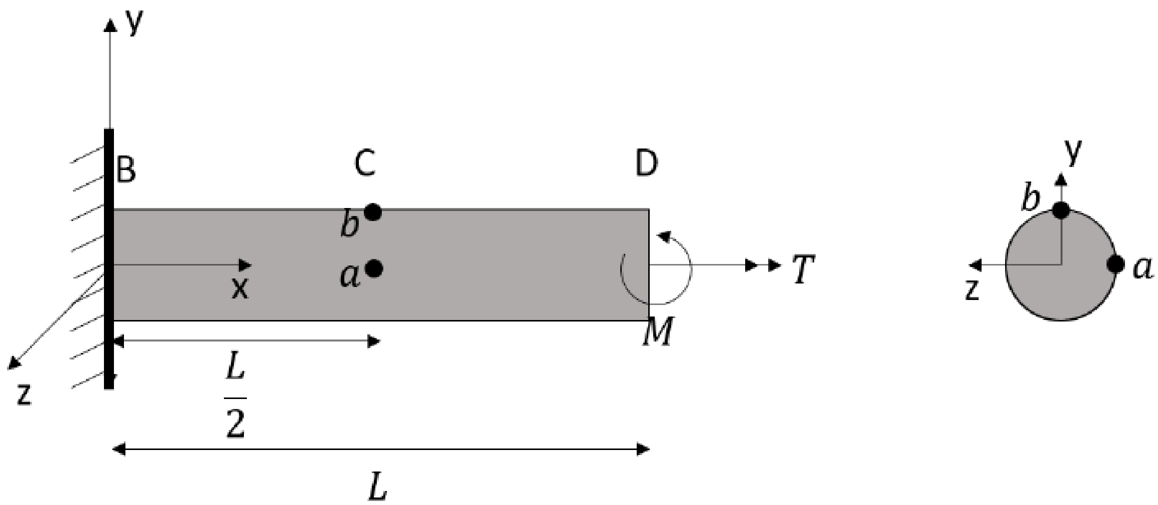
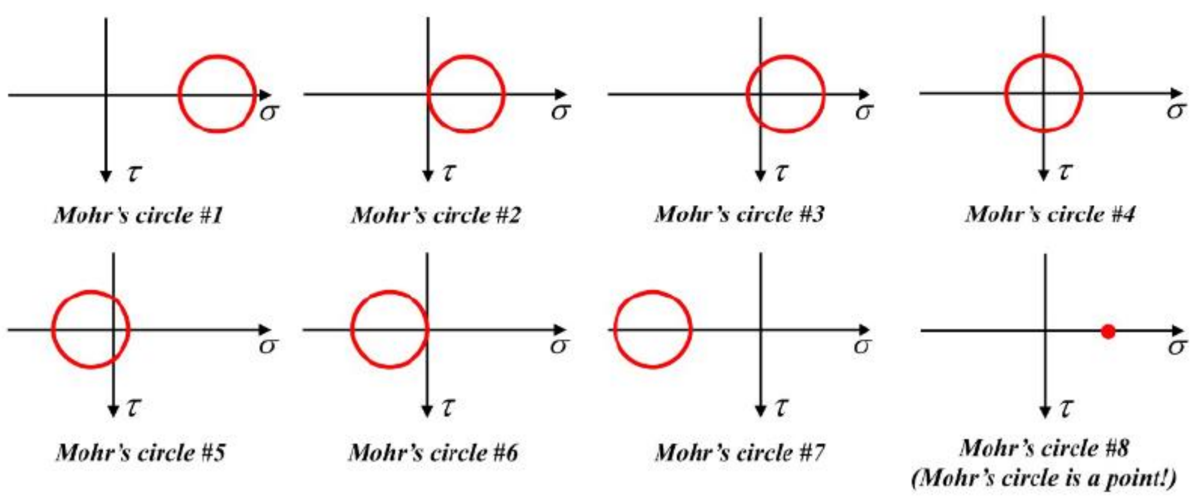


Figure 4.1: Loading of circular rod for Problem 4.I



Point a - The correct Mohr's circle is #4

$$\tau_{xy} > 0$$

$$\sigma_x = 0, \qquad \sigma_y = 0$$

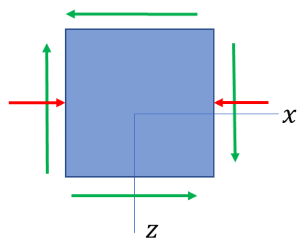
$$\sigma_{avg} = \frac{(\sigma_x + \sigma_y)}{2} = 0$$

Point b – The correct Mohr's circle is #5

$$\tau_{xz} > 0$$

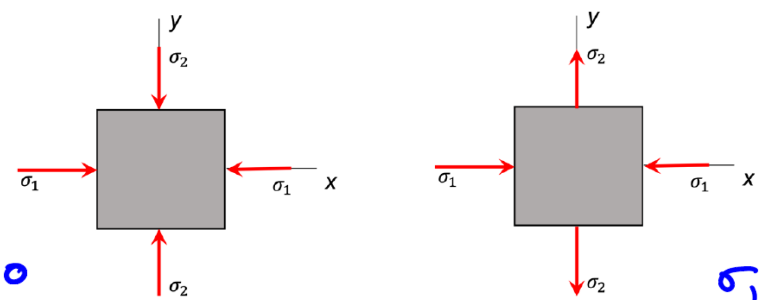
$$\sigma_x < 0, \qquad \sigma_z = 0$$

$$\sigma_{avg} = \frac{(\sigma_x + \sigma_y)}{2} < 0$$



II.

Consider stress states (a) and (b) shown above, with $|\sigma_1| > |\sigma_2|$. Let $\left(|\tau|_{max,abs}\right)_a$ and $\left(|\tau|_{max,abs}\right)_b$ represent the absolute maximum shear stress corresponding to stress states (a) and (b), respectively. Choose the response below that describes the relative sizes of these stresses.



$\sigma_2 < \sigma_1 < 0$

Stress state (a)

Stress state (b)

$\sigma_1 < 0 < \sigma_2$

i. $\left(|\tau|_{max,abs}\right)_a > \left(|\tau|_{max,abs}\right)_b$

ii. $\left(|\tau|_{max,abs}\right)_a = \left(|\tau|_{max,abs}\right)_b$

iii. $\left(|\tau|_{max,abs}\right)_a < \left(|\tau|_{max,abs}\right)_b$

Once, all 3 mohr's circles are drawn, (b) produces a larger $\tau_{max,abs}$