Useful Equations

Bending deformation:

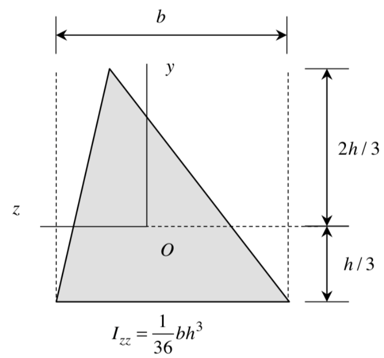
$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x) \qquad M = EIv'' \qquad \Delta V = P \qquad \Delta M = -M_0$$

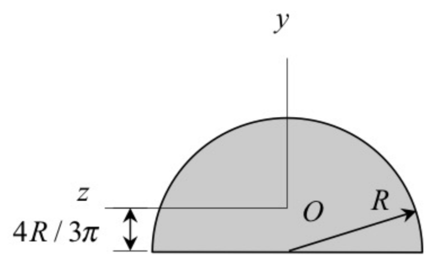
$$\sigma(x,y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}} \qquad I_{zz} = \frac{bh^3}{12} \text{ (rectangle)}, \quad I_{zz} = \frac{\pi r^4}{4} \text{ (circle)}$$

$$\tau(x,y) = \frac{VQ}{I_{zz}t} = \frac{VA^* y^*}{I_{zz}t},$$

$$\tau_{\max} = \frac{3V}{2A} \text{ (rectangle)},$$

$$\tau_{\max} = \frac{4V}{3A} \text{ (circle)}$$





Strain energy density:

$$\bar{u} = \frac{1}{2}[\sigma_x(\varepsilon_x - \alpha\Delta T) + \sigma_y(\varepsilon_y - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods:

$$U = \frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx \qquad U = \frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx \qquad U = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}dx$$

Work-energy principle: $U = W$

Castigliano's 2nd theorem:

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \qquad \theta_{M_i} = \frac{\partial U}{\partial M_i} \qquad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

$$\delta_{P_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial P_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial P_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial P_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial P_i}dx$$

$$\theta_{M_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial M_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial M_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial M_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial M_i}dx$$

$$\phi_{T_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial T_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial T_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial T_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial T_i}dx$$

f_s=6/5 (rectangular cross section), f_s=10/9 (circular cross section)

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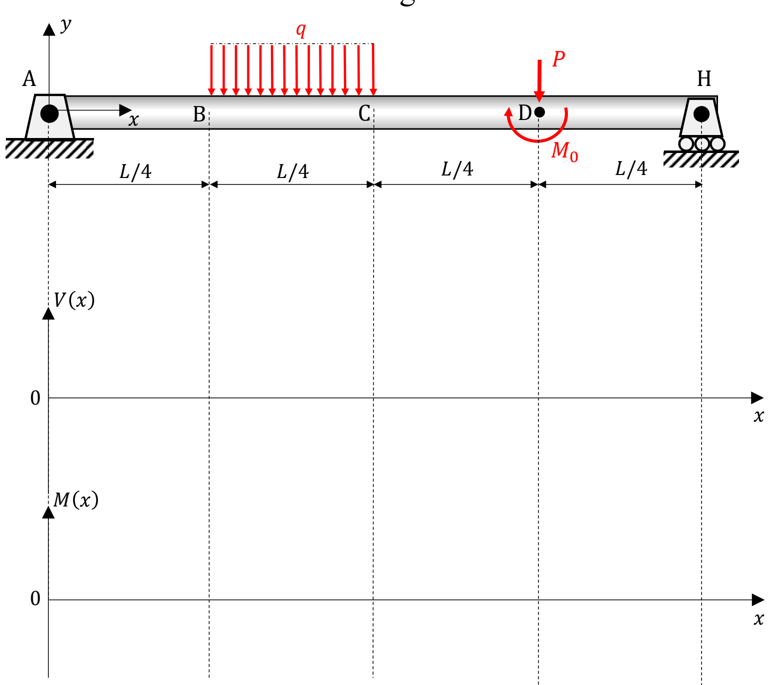
PROBLEM # 1 (25 points).

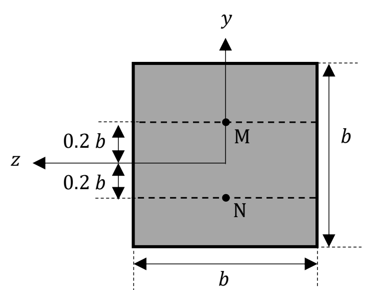
A simply supported beam AH is subject to a constant distributed load q over the section BC, a moment M_0 and a concentrated force P at D. The cross section of the beam is shown below.

The parameters are following: L=8 ft., q=10 lb/ft, M_0=40 lb×ft, P=10 lb, b= 2 in.

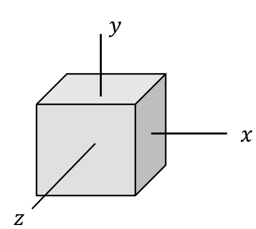
a) Draw the shear force and bending moment diagrams. Mark the values at the cross sections A, B, C, D, and H, and the maximum and minimum values along the beam.

b) Determine the stress state at the points M and N which are located at the cross section C. Sketch their stress state on the given stress elements.

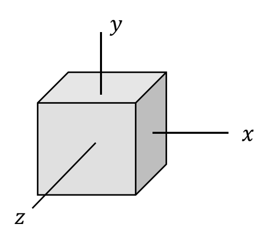




Cross section



Stress element M



Stress element N

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PROBLEM # 1 (cont.)

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PROBLEM # 1 (cont.)

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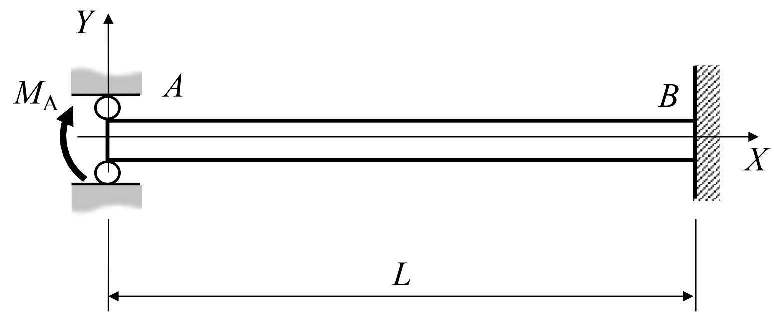
PROBLEM # 2 (25 points).



The linearly elastic beam shown in the figure supports a couple M_A at end A. The beam is homogeneous, with Young's modulus E, and has constant cross-section, with moment of inertia I.

(a) Using the following free body diagram, write the equations of equilibrium and identify whether the structure is statically determinate or indeterminate.

Using the second-order integration method:

(b) Determine the bending moment $M(x)$ of the beam (as a function of the reactions at A, the external loads and the geometric parameters).
(c) Determine the slope $v'(x)$ and deflection $v(x)$ of the beam.
(d) Indicate the boundary conditions at supports A and B.
(e) Solve for the reaction at A, i.e., R_A.

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PROBLEM # 2 (cont.)

Mechanical Engineering

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PROBLEM # 2 (cont.)

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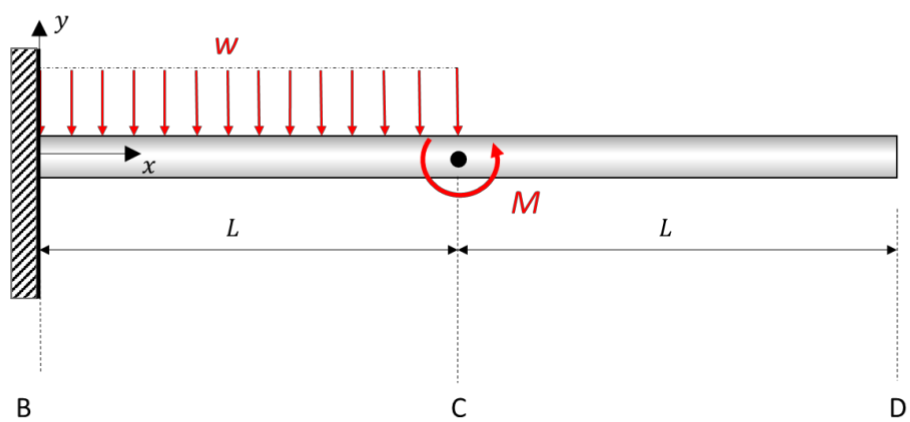
PROBLEM # 3 (25 points).

A cantilevered beam BCD is subjected to a distributed load w (in the unit of load/length) between BC and a concentrated moment M at point C. The structure is made of a material with elastic modulus E, second moment of area I and cross-sectional area A.

1) Assuming **the shear strain energy due to bending is negligible**, **use Castigliano's theorem** to determine the vertical (y-direction) deflection of point D.
2) Use the attached superposition tables to calculate the deflection of point D using **the superposition method.**



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PROBLEM # 3 (cont.)

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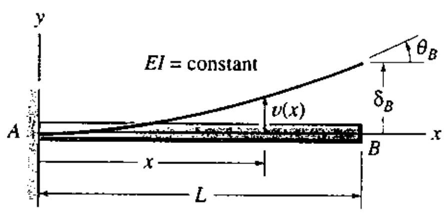
PROBLEM # 3 (cont.)

APPENDIX

DEFLECTIONS AND SLOPES OF BEAMS; FIXED-END ACTIONS

E

E.1. Deflections and Slopes of Uniform Cantilever Beams*



Notation

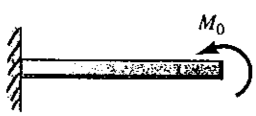
$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\delta_B \equiv v(L)$ = deflection at end B

$\theta_B \equiv v'(L)$ = slope at end B

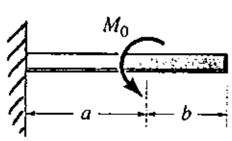
1



$$v = \frac{M_0x^2}{2EI} \qquad v' = \frac{M_0x}{EI}$$

$$\delta_B = \frac{M_0L^2}{2EI} \qquad \theta_B = \frac{M_0L}{EI}$$

2

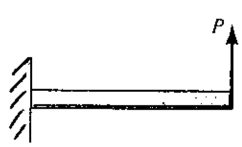


$$v = \frac{M_0x^2}{2EI} \qquad v' = \frac{M_0x}{EI} \qquad 0 \le x \le a$$

$$v = \frac{M_0a}{2EI}(2x - a) \qquad v' = \frac{M_0a}{EI} \qquad a \le x \le L$$

$$\delta_B = \frac{M_0a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0a}{EI}$$

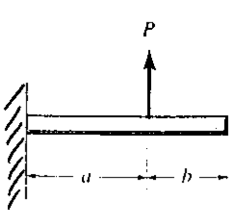
3



$$v = \frac{Px^2}{6EI}(3L - x) \qquad v' = \frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

4

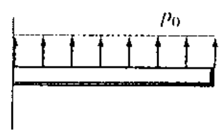


$$v = \frac{Px^2}{6EI}(3a - x) \qquad v' = \frac{Px}{2EI}(2a - x) \qquad 0 \le x \le a$$

$$v = \frac{Pa^2}{6EI}(3x - a) \qquad v' = \frac{Pa^2}{2EI} \qquad a \le x \le L$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

5

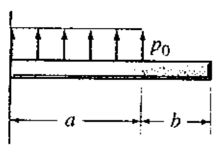


$$v = \frac{p_0 x^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$v' = \frac{p_0 x}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{p_0 L^4}{8EI} \qquad \theta_B = \frac{p_0 L^3}{6EI}$$

6



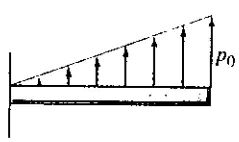
$$v = \frac{p_0 x^2}{24EI}(6a^2 - 4ax + x^2) \qquad 0 \le x \le a$$

$$v' = \frac{p_0 x}{6EI}(3a^2 - 3ax + x^2) \qquad 0 \le x \le a$$

$$v = \frac{p_0 a^3}{24EI}(4x - a) \qquad v' = \frac{p_0 a^3}{6EI} \qquad a \le x \le L$$

$$\delta_B = \frac{p_0 a^3}{24EI}(4L - a) \qquad \theta_B = \frac{p_0 a^3}{6EI}$$

7

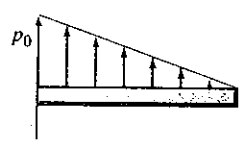


$$v = \frac{p_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$v' = \frac{p_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

$$\delta_B = \frac{11 p_0 L^4}{120EI} \qquad \theta_B = \frac{p_0 L^3}{8EI}$$

8

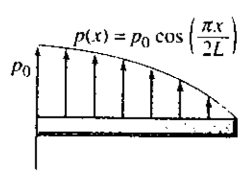


$$v = \frac{p_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$v' = \frac{p_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

$$\delta_B = \frac{p_0 L^4}{30EI} \qquad \theta_B = \frac{p_0 L^3}{24EI}$$

9



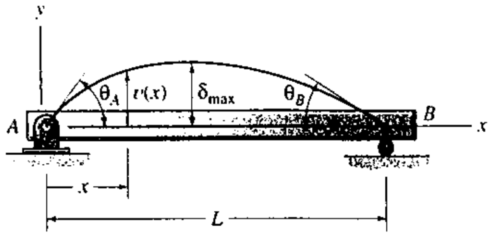
$$v = \frac{p_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 Lx^2 - \pi^3 x^3\right)$$

$$v' = \frac{p_0 L}{\pi^3 EI}\left(2\pi^2 Lx - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$

$$\delta_B = \frac{2p_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{p_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

*Beam-deflection theory is covered in Chapter 7. The sign convention used here is the same as in Chapter 7.

E.2. Deflections and Slopes of Uniform Simply-Supported Beams*



EI = constant

Notation

$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\theta_A \equiv v'(0)$ = slope (angle) at end A

$\theta_B \equiv -v'(L)$ = angle of rotation at end B

x_m = distance from end A to the point of maximum deflection

$\delta_C \equiv |v(L/2)|$ = deflection at the center of the beam

$\delta_{max} \equiv \max |v(x)|$ = maximum deflection

1

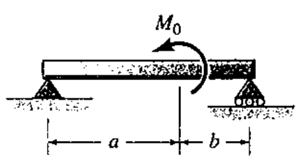
$$v = \frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$$

$$v' = \frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_m = L\left(1 - \frac{\sqrt{3}}{3}\right) \text{ and } \delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$$

2

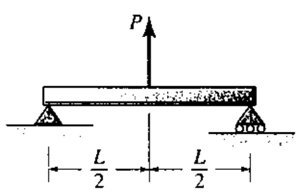


$$v = \frac{-M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad 0 \le x \le a$$

$$v' = \frac{-M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad 0 \le x \le a$$

$$\theta_A = \frac{-M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{-M_0}{6LEI}(3a^2 - L^2)$$

3

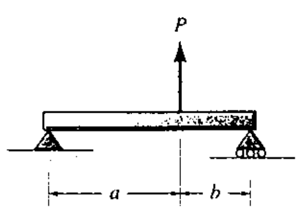


$$v = \frac{Px}{48EI}(3L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$v' = \frac{P}{16EI}(L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

4



$$v = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad 0 \le x \le a$$

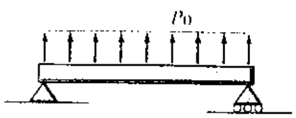
$$v' = \frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad 0 \le x \le a$$

$$\theta_A = \frac{Pab(L + b)}{6LEI}$$

$$\theta_B = \frac{Pab(L + a)}{6LEI}$$

$$\text{If } a \ge b,\ x_m = \sqrt{\frac{L^2 - b^2}{3}} \text{ and } \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$$

5

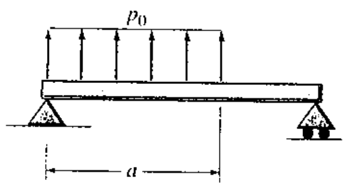


$$v = \frac{p_0 x}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = \frac{p_0}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{max} = \frac{5p_0 L^4}{384EI} \qquad \theta_A = \theta_B = \frac{p_0 L^3}{24EI}$$

6



$$v = \frac{p_0 x}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad 0 \le x \le a$$

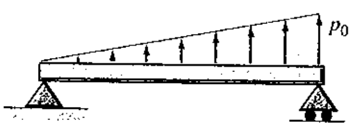
$$v' = \frac{p_0}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad 0 \le x \le a$$

$$v = \frac{p_0 a^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad a \le x \le L$$

$$v' = \frac{p_0 a^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad a \le x \le L$$

$$\theta_A = \frac{p_0 a^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{p_0 a^2}{24LEI}(2L^2 - a^2)$$

7



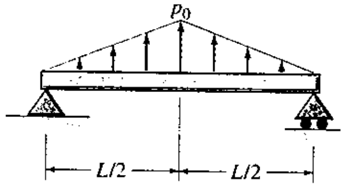
$$v = \frac{p_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = \frac{p_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

$$\theta_A = \frac{7p_0 L^3}{360EI} \qquad \theta_B = \frac{p_0 L^3}{45EI}$$

$$x_m = 0.5193\,L \qquad \delta_{max} = 0.00652\frac{p_0 L^4}{EI}$$

8

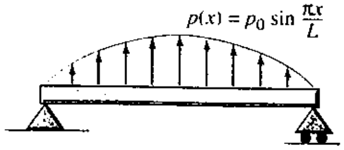


$$v = \frac{p_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad 0 \le x \le \frac{L}{2}$$

$$v' = \frac{p_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$\delta_C = \delta_{max} = \frac{p_0 L^4}{120LEI} \qquad \theta_A = \theta_B = \frac{5p_0 L^3}{192EI}$$

9



$$v = \frac{p_0 L^4}{\pi^4 EI}\sin\left(\frac{\pi x}{L}\right)$$

$$v' = \frac{p_0 L^3}{\pi^3 EI}\cos\left(\frac{\pi x}{L}\right)$$

$$\delta_C = \delta_{max} = \frac{p_0 L^4}{\pi^4 EI} \qquad \theta_A = \theta_B = \frac{p_0 L^3}{\pi^3 EI}$$

*Beam-deflection theory is covered in Chapter 7. The sign convention used here is the same as in Chapter 7.

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November 5th, 2020

Name (Print) ______________________________
(Last) (First)

PROBLEM #4 (25 Points)

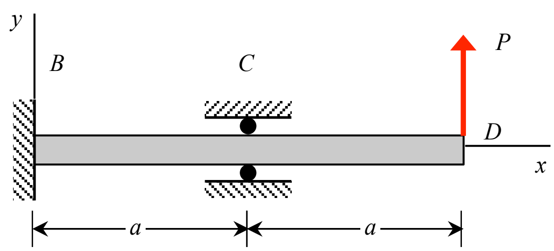
Part A – 5 points

The beam shown below has a second area moment *I* for its cross-section, and is made of a material with a Young's modulus of *E*. Using the method of superposition:

a) Determine the reaction on the beam at location C. Leave your answer in terms of the load *P*.

b) Determine the deflection of the beam at end D. Leave your answer in terms of *P*, *a* and *EI*.

Superposition tables are provided.



PROBLEM #4 ***(continued)***

Part B **– 8 points**

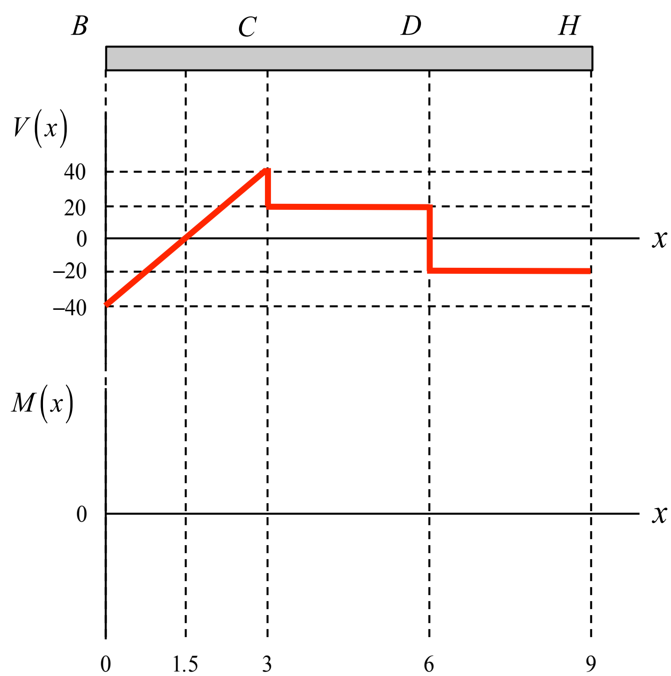
The loading on the beam shown below is not provided in the figure. The shear force $V(x)$ for the beam is given below the beam, with the shear force being provided in terms of *kips* and the position variable x in *ft*. In addition, it is known that the bending moment at the left end is $M(0) = 0$, and there are no concentrated couples applied to the beam at any locations except, possibly, at H.

For the shear force diagram provided:

a) Draw the bending diagram $M(x)$ on the axes provided.

b) Show the loading on the beam in the figure below.

No justification is needed for your answers.



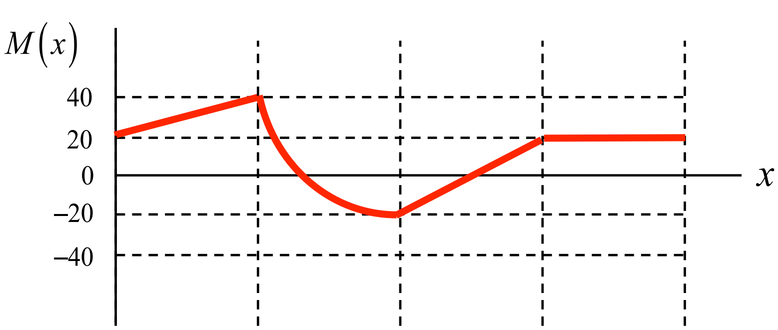
PROBLEM #4 ***(continued)***

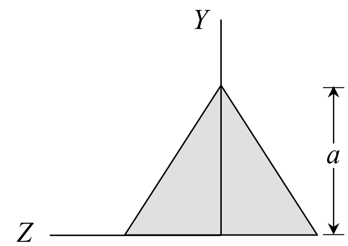
***Part C* – 6 points**

The bending moment diagram for a loaded beam is shown below. The beam is known to have the triangular cross section shown below. Provide a justification for each answer.

a) At what location(s) on the beam does the maximum *tensile* normal stress exist? Provide both x and Y components of the location of this point(s). You are *not* asked to solve for this value of stress.

b) At what location(s) on the beam does the maximum *compressive* normal stress exist? Provide both x and Y components of the location of this point(s). You are *not* asked to solve for this value of stress.





beam cross-section

PROBLEM #4 *(continued)*

Part D **– 6 points**

The cross-sections for Beams 1 and 2 are shown below. Let I_1 and I_2 represent the centroidal second area moment (about the z-axis) for beams 1 and 2, respectively. Each beam is experiencing the same shear force of V at the cross section. Let τ_{1B} and τ_{2B} be the shear stress at points B on Beams 1 and 2, respectively.

a) Circle the correct answer below in regard to the relative sizes of I_1 and I_2. You are _not_ asked to provide numerical values for these second area moments, or justification for your answers.

- $I_1 > I_2$
- $I_1 = I_2$
- $I_1 < I_2$

b) Circle the correct answer below in regard to the relative sizes of $|\tau_{1B}|$ and $|\tau_{2B}|$. You are _not_ asked to provide numerical values for these stresses, or justification for your answers.

- $|\tau_{1B}| > |\tau_{2B}|$
- $|\tau_{1B}| = |\tau_{2B}|$
- $|\tau_{1B}| < |\tau_{2B}|$