15 min .

ME 323: Mechanics of Materials **Homework 5**

Spring 2026 **Due: February 27, 2026**

Problem 1 (10 points):

A beam is supported by a pin at A and a roller at D. The cross-section is triangular as shown below. A concentrated force 8P is applied at B and points downward; a concentrated force 6P is applied at C and points upward; a clockwise moment PL is applied at D. The lengths of AB, BC, and CD are L. The neutral axis of a triangular (**equilateral**) cross-section is 1/3 of its height b to its base side, as shown in the figure. The second area moment of this cross section is I (hint: Use $I = \frac{a^4}{32\sqrt{3}}$, where a is the length of side of the triangle).

a) Construct the shear force and bending moment diagrams for the beam. Mark the critical values on the diagrams.
b) Determine the magnitudes and locations (x,y) of the maximum tensile flexural stress and maximum compressive flexural stress of the beam in terms of P, L, and b.
c) At point M (on the neutral axis) on cross-section n-n, determine the average shear stress.

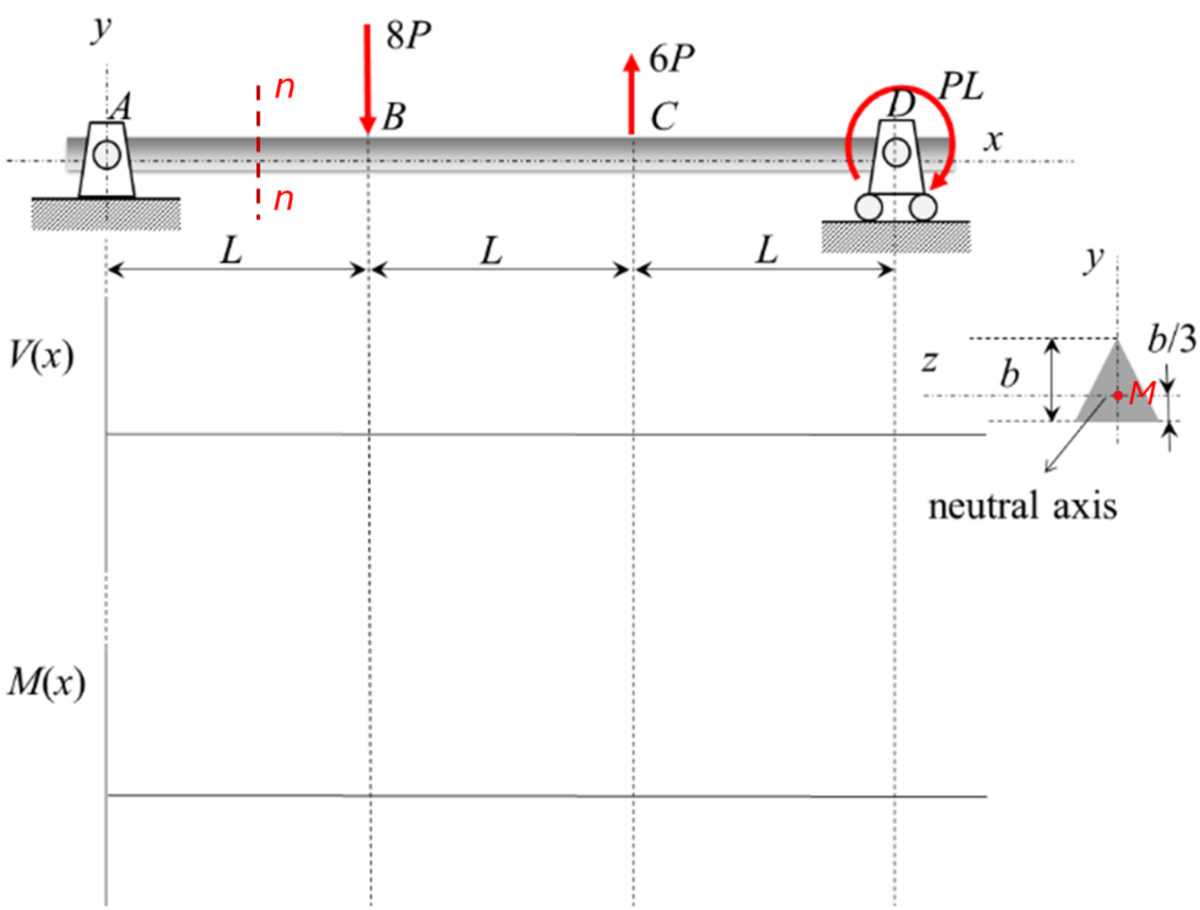


Figure 1: Beam loading for Problem 1

Given: △ w/ side a

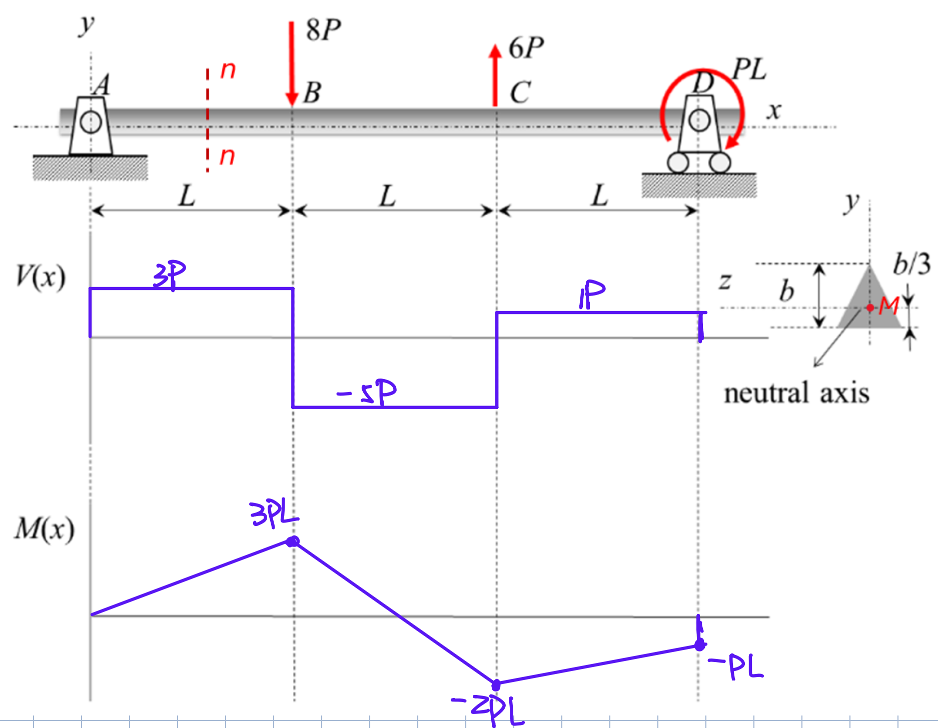
$I = \frac{a^4}{32\sqrt{3}}$

b, L, P

Find: VM Diagram.

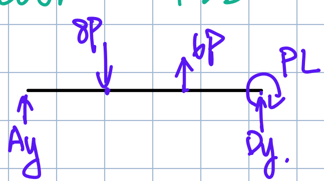
σ_{max} and location. (+ and −)

@ n, τ_M.



Solution.

1. coor & FBD



2. Reaction & VM diagram.

$\Sigma M_D = -(3L)A_y + (2L)(8P) - (L)(6P) - PL = 0$ ①

$\Sigma F_y = A_y - 8P + 6P + D_y = 0$ ②

① → $A_y = 3P$

② → $D_y = -P$

3. σ_{max}.

$|\sigma_{max}|$ is always @ $y = \frac{2}{3}b$.

σ_{max} comp @ B, $x = L$, $M = 3P$.

σ_{max} Tens. @ either B, $x = L$, $y = -\frac{1}{3}b$

or C, $x = 2L$, $y = \frac{2}{3}b$

@ B: $\sigma_{Top} = -\cancel{3}PL\cdot\frac{2}{\cancel{3}}b\cdot\frac{18\sqrt{3}}{b^{\cancel{4}3}} = -\frac{36\sqrt{3}PL}{b^3}$

$\sigma_{Bot} = -3PL\cdot\frac{-1}{3}b\cdot\frac{18\sqrt{3}}{b^4} = \frac{18\sqrt{3}PL}{b^3}$

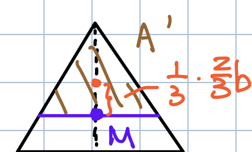
@ C: $\sigma_{Top} = -(-2PL)\frac{2}{3}b\cdot\frac{18\sqrt{3}}{b^4} = \frac{24\sqrt{3}PL}{b^3}$

max comp. $\sigma(L, \frac{2}{3}b) = -\frac{36\sqrt{3}PL}{b^3}$

max tens. $\sigma(2L, \frac{2}{3}b) = \frac{24\sqrt{3}PL}{b^3}$

$a = \frac{2}{\sqrt{3}}b \Rightarrow I = \frac{16}{9\cdot 32\sqrt{3}}b^4 = \frac{b^4}{18\sqrt{3}}$

4. τ_M.



$\tau = \frac{VQ}{It}$.

$V = 3P$

$I = \frac{b^4}{18\sqrt{3}}$

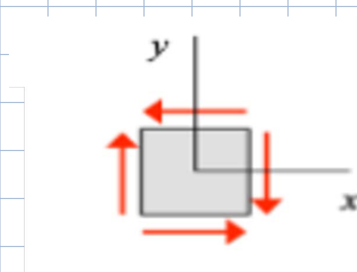
$t = \frac{2}{3}\cdot\frac{2}{\sqrt{3}}b = \frac{4}{3\sqrt{3}}b$.

$A' = (\frac{2}{3})^2\cdot(\frac{\cancel{2}}{\sqrt{3}}b)\cdot b\cdot\frac{1}{\cancel{2}} = \frac{4b^2}{9\sqrt{3}}$

$\bar{y}' = \frac{1}{3}\cdot\frac{2}{3}b = \frac{2b}{9}$

$\tau = \frac{\cancel{3}P\cdot\frac{4b^2}{\cancel{9}\sqrt{3}}\cdot\frac{2b}{9}}{\frac{b^{\cancel{4}2}}{2\cancel{18}\sqrt{3}}\cdot\frac{4\cancel{b}}{\cancel{3}\sqrt{3}}} = \frac{4\sqrt{3}P}{b^2}$

$\tau_{ave} = \frac{4\sqrt{3}P}{b^2}$



20 min. mostly calculation.

Problem 2 (10 points):

Shown in figure 2 is a beam supported by pin joints at A and D. It is acted upon by a line load that increases uniformly from zero at A to B and maintains a constant value of 600 N/m between B and C. It is also acted upon by a concentrated couple at E.

a) Using the integral approach, determine expressions for shear force V(x) and bending moments M(x) between x = 0 m to x = 1 m.
b) Construct shear force and bending moment diagrams using expressions determined in (a).

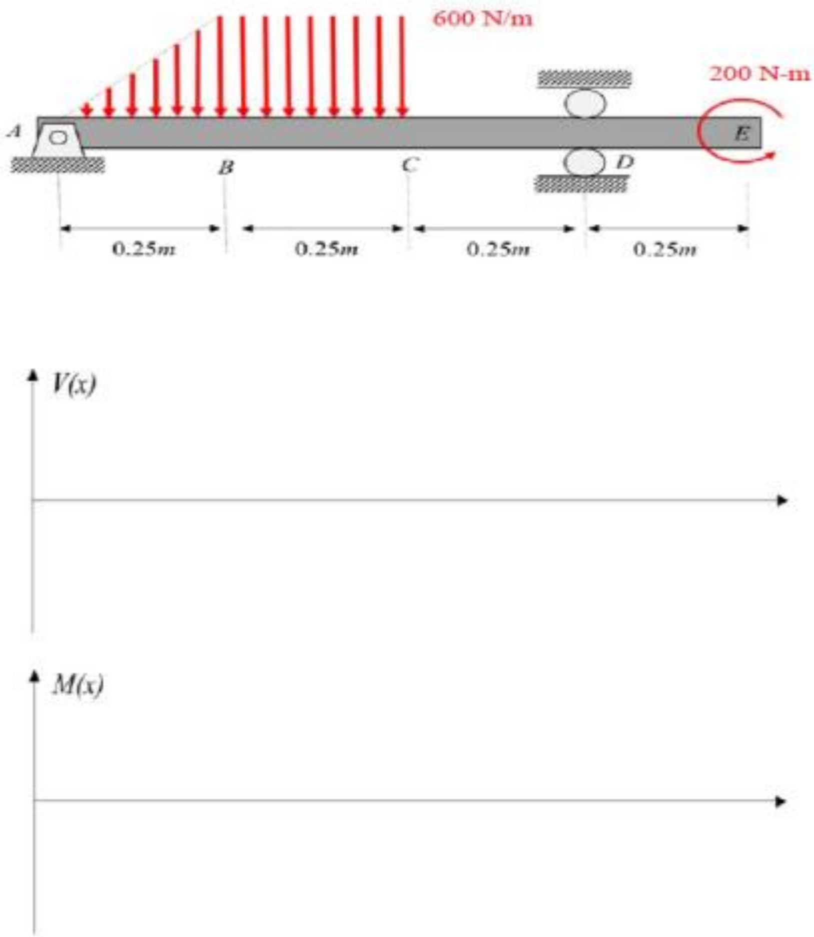
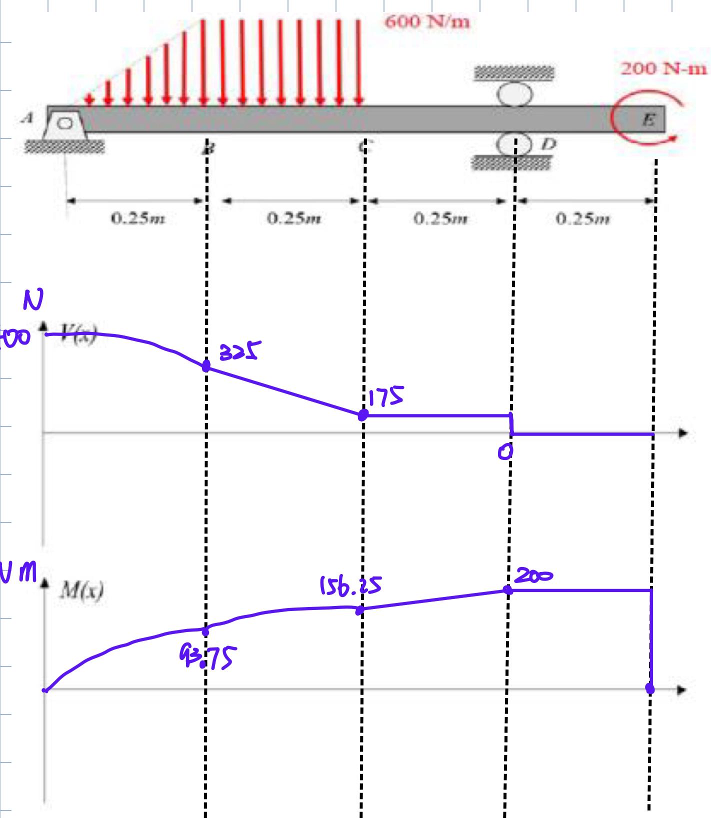


Figure 2: Beam structure for Problem 2

Given: shown.

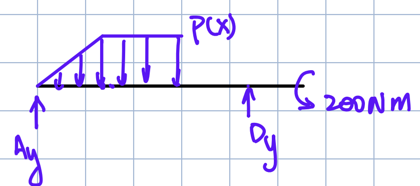
Find: $V(x)$ $M(x)$ w/ integral approach

VM Diagram.



Solution:

1. coor & FBD.



2. Equilibrium Eq.

$\Sigma M_A: -\frac{1}{2}(0.25m)(600N/m)\cdot\frac{2}{3}(0.25m)$

$-(0.25m)(600N/m)\cdot(0.375m)$

$+D_y(0.75m) - 200NM = 0$ ①

$\Sigma F_y: A_y + D_y - \frac{1}{2}(0.25m)(600N/m) - (0.25m)(600N/m) = 0$ ②

3. Solve for Reactions.

① → $D_y = -175N$.

② → $A_y = 400N$

4. V/M Diagram w/ integration.

$x \in [0, 0.25]$. $p(x) = -2400x$ N/m

$x \in (0, 0.5]$ $p(x) = -600$ N/m.

$x \in (0.5, 0.75]$ $p(x) = 0$

$x \in (0.75, 1]$ $p(x) = 0$

$x \in [0, 0.25]$.

$V(x) = \int_0^x A_y + p(x)\,dx$

$= 400N + \int_0^x -2400x\ N/m\,dx$

$= -1200x^2 + 400$

$M(x) = \int_0^x V(x)\,dx$

$= -400x^3 + 400x$

Continue

$x \in (0.25, 0.5]$.

$V(0.25) = 325N$. $M(0.25) = 93.75NM$

$V(x) = V(0.25) + \int_{0.25}^x -600\,dx$

$= 325 - 600x + 0.25\cdot 600 = -600x + 475$

$M(x) = M(0.25) + \int_{0.25}^x -600x + 475\,dx$

$= 93.75 + (-300x^2 + 475x) - (-300\cdot 0.25^2 + 475\times 0.25)$

$= -300x^2 + 475x - 6.25$

$x \in (0.5, 0.75]$

$V(0.5) = 175N$. $M(0.5) = 156.25NM$

$V(x) = 175N$

$M(x) = M(0.5) + \int_{0.5}^x 175\,dx$

$= 156.25 + 175x - 175\cdot 0.5 = 175x + 68.75$

$x \in (0.75, 1]$

$V(0.75^-) = 175N$, $V(0.75^+) = 0N$. $M(0.75) = 200N\cdot m$.

$V(x) = 0$

$M(x) = M(0.75) + 0 = 200 N\cdot M$

10 min (mostly calculation)

Problem 3 (10 points):

A design decision is to be taken based on whether a beam can support higher normal stresses in a box configuration or an H-configuration (see the two figures below), considering plane of bending is the XY plane. Assuming that both cross sections are symmetric about the origins placed at the respective centroids:

(a) Determine the second area moment of inertia of the two cross sections, $I_{ZZ}{}^{BOZ}$ and $I_{ZZ}{}^{H}$

(b) Determine which cross section would experience a lower magnitude of maximum normal stress for the same loading conditions.

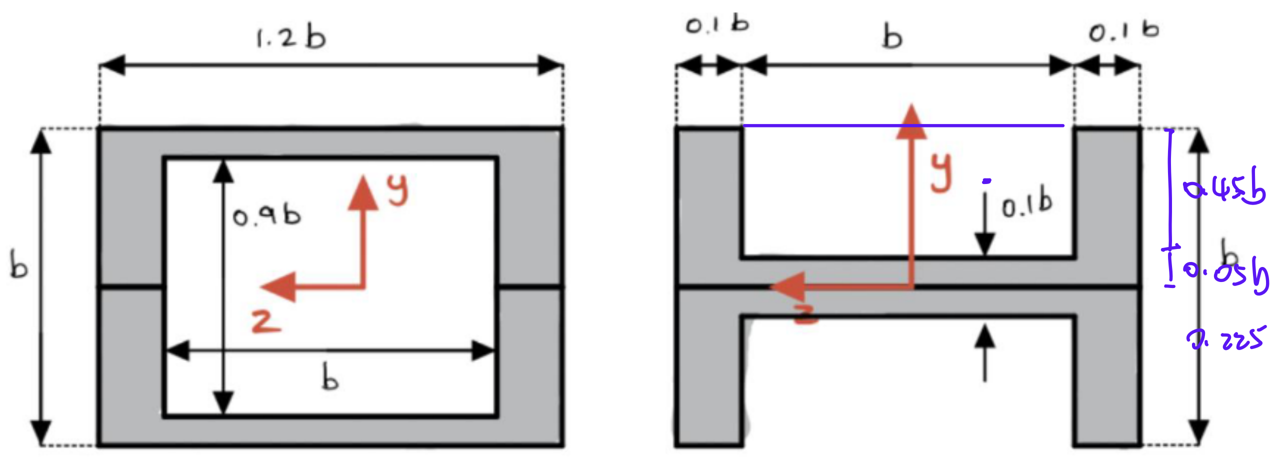


Figure 3: Cross-sections for Problem 3

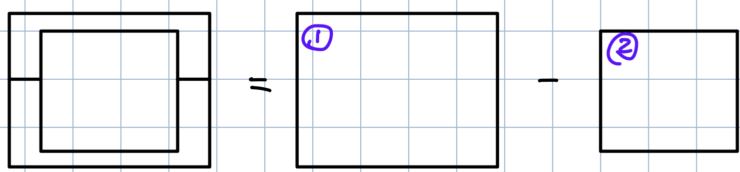
Given: b.

Find: I^{Box} I^{H}

Pick option w/ lower $|\sigma_{max}|$

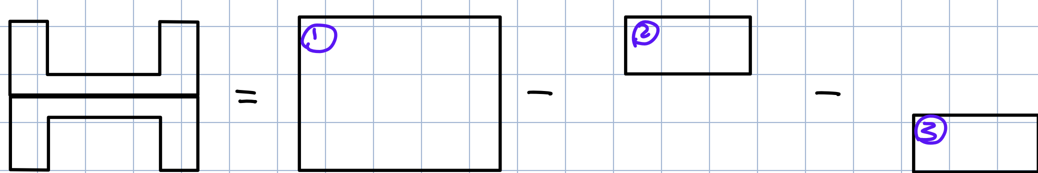
Solution:

1. For Box.



$$I = I_1 - I_2 = \frac{1}{12}(1.2b)\,b^3 - \frac{1}{12}\,b\,(0.9b)^3 = 0.03925\,b^4$$

2. For H.



$$I = I_1 - \left[I_2 + A_2 \cdot (0.275b)^2\right] - \left[I_3 + A_3\,(0.275b)^2\right]$$

$$= \frac{1}{12}(1.2b)\,b^3 - \left[\frac{1}{12} \cdot b \cdot (0.45b)^3 - b\,(0.45b)(0.275b)^2\right] \times 2$$

$$= 0.01675\,b^4$$

$I^{Box} = 0.03925\,b^4$. $I^{H} = 0.01675\,b^4$

$I^{Box} > I^{H} \Rightarrow$ Box cross section will have lower $|\sigma_{max}|$

Problem 4 (2.5 + 2.5 points): (5min + 2min)

Problem 4.1

A shear force V and bending moment M act at a cross section of a trapezoidal cross-sectioned beam. Consider the five points (i), (ii), (iii), (iv) and (v) on the beam cross section, as shown above. Match up the state of stress at each of these five points with the stress elements (a) through (o) shown below. If you choose "(o) NONE of the above", provide a sketch of the correct state of stress for your answer.

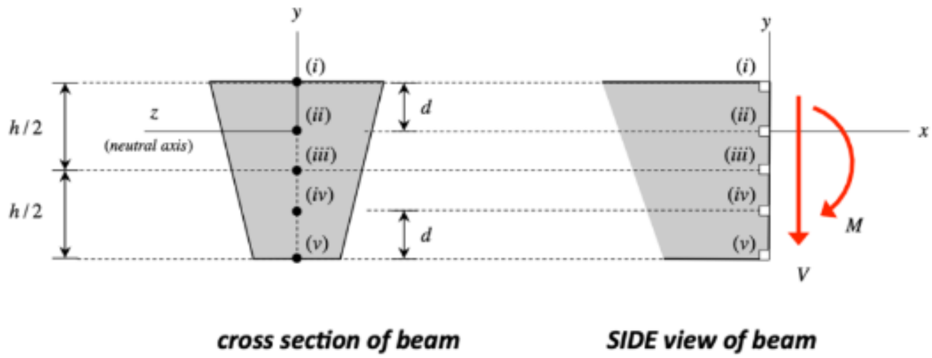


Figure 4.1: Cross-section and side view of beam problem

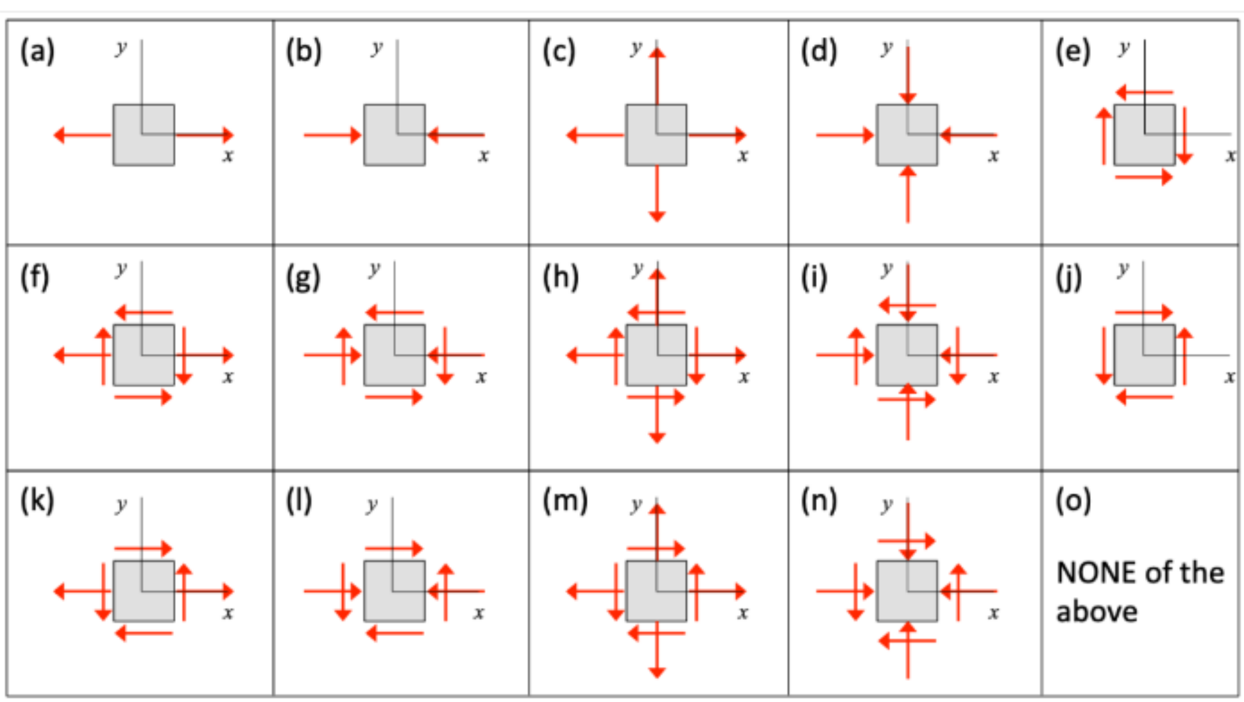
The state of stress at point (i) is

The state of stress at point (ii) is

The state of stress at point (iii) is

The state of stress at point (iv) is

The state of stress at point (v) is



Problem 4.2

A beam has the T-shaped cross section shown below. The internal bending moment in the beam is known to be M.

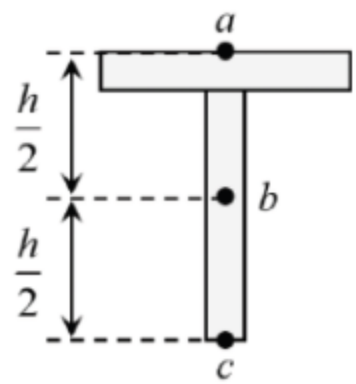


Figure 4.2: T-shaped cross section

Let $|\sigma_a|$, $|\sigma_b|$, $|\sigma_c|$ be the magnitudes of the normal stress at points a, b, and c, respectively. It is desired to determine $|\sigma_{max}|$, the maximum magnitude of the normal stress on the cross section.

Choose the correct statement:

a) $|\sigma_{max}| = |\sigma_a|$
b) $|\sigma_{max}| = |\sigma_b|$
c) $|\sigma_{max}| = |\sigma_c|$
d) $|\sigma_{max}| = |\sigma_a| = |\sigma_c|$
e) $|\sigma_{max}| = |\sigma_a| = |\sigma_b| = |\sigma_c|$

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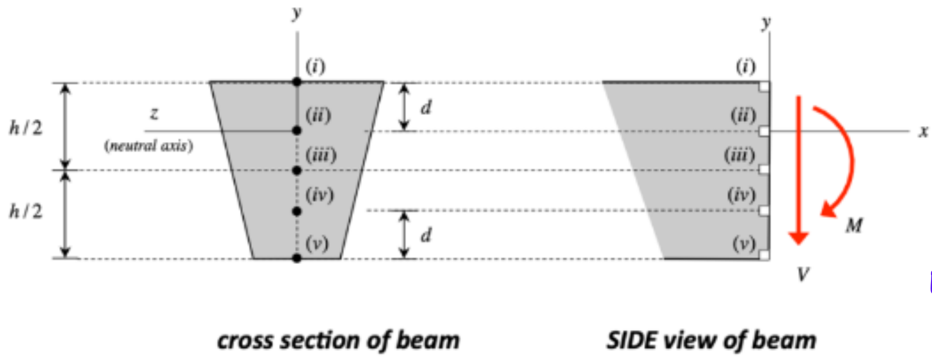


Figure 4.1: Cross-section and side view of beam problem

w/ M.
(i) $+\sigma_x$
(ii) 0 σ_x
(iii) (iv)(v) $-\sigma_x$

W/ V.
(i) (V) 0 τ_{xy}.

(ii)(iii)(iv)

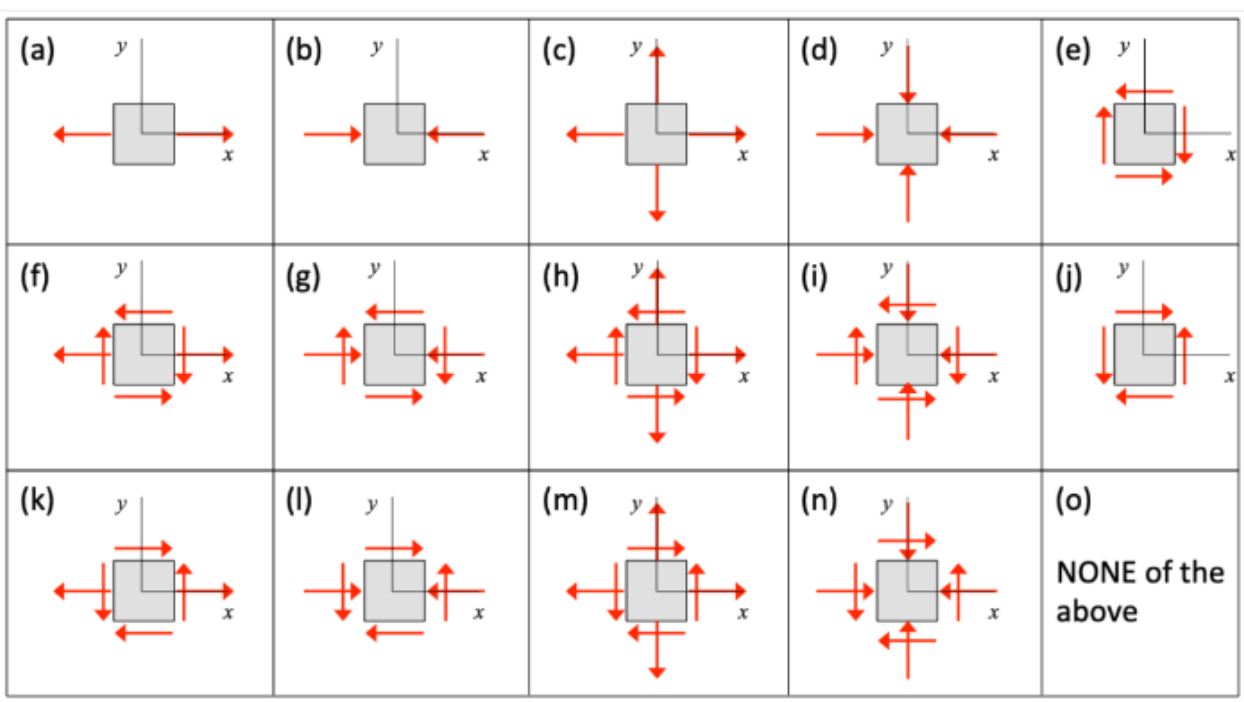
The state of stress at point (i) is (a)

The state of stress at point (ii) is (e)

The state of stress at point (iii) is (g)

The state of stress at point (iv) is (g)

The state of stress at point (v) is (b)



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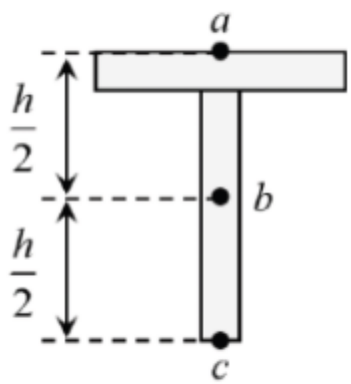

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Choose the correct statement:

a) $|\sigma_{max}| = |\sigma_a|$
b) $|\sigma_{max}| = |\sigma_b|$
c) $|\sigma_{max}| = |\sigma_c|$
d) $|\sigma_{max}| = |\sigma_a| = |\sigma_c|$
e) $|\sigma_{max}| = |\sigma_a| = |\sigma_b| = |\sigma_c|$

N.A Between a and b.
c has max $|\bar{y}|$