Useful Equations

$$\sigma_{avge} = \frac{F_N}{A} \qquad FS = \frac{\sigma_{fail}}{\sigma_{allow,member}}$$

$$\tau_{avge} = \frac{V}{A} \qquad FS = \frac{\tau_{fail}}{\tau_{allow,member}}$$

Generalized Hooke's law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \qquad G = \frac{E}{2(1+\nu)}$$

Axial deformations:

$$e_{AB} = u_B - u_A \qquad e = \int_0^L \frac{F}{A\,E}\,dx + \int_0^L \alpha\,\Delta T\ dx, \qquad e = \frac{F\,L}{A\,E} + \alpha\,\Delta T\,L$$

Planar trusses: $e = u\cos(\theta) + v\sin(\theta)$

Torsional deformations:

$$\phi_{AB} = \phi_B - \phi_A \qquad \phi_{AB} = \int_0^L \frac{T_{AB}(x)}{G(x)\,I_p(x)}\,dx \qquad \phi_{AB} = \frac{T_{AB}L}{G\,I_P}$$

$$\gamma = \rho\frac{d\phi}{dx} \qquad \tau = G\rho\frac{d\phi}{dx} \qquad \gamma = \frac{\rho T}{G\,I_p} \qquad \tau = \frac{\rho T}{I_p}$$

with $I_p = \int_A \rho^2\,dA,$ $\quad I_p = \frac{\pi r^4}{2}$ (solid), $\quad I_p = \frac{\pi}{2}\left(r_0^4 - r_i^4\right)$ (hollow)

Bending deformation

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x) \qquad M = EIv'' \qquad \Delta V = P \qquad \Delta M = -M_0$$

$$\sigma(x,y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}}$$

$I_{zz} = \frac{bh^3}{12}$ (rectangle), $\quad I_{zz} = \frac{\pi r^4}{4}$ (circle)

$$\tau(x,y) = \frac{VQ}{I_{zz}t} = \frac{VA^*y^*}{I_{zz}t},$$

$\tau_{\max} = \frac{3V}{2A}$ (rectangle), $\quad \tau_{\max} = \frac{4V}{3A}$ (circle)

Parallel axis theorem: $\quad I_B = I_O + Ad_{OB}^2$

Strain energy density:

$$\bar{u} = \frac{1}{2}[\sigma_x(\varepsilon_x - \alpha\Delta T) + \sigma_y(\varepsilon_y - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods:

$$U = \frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx \qquad U = \frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx \qquad U = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}dx$$

Castigliano's 2nd theorem:

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \qquad \theta_{M_i} = \frac{\partial U}{\partial M_i} \qquad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

$$\delta_{P_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial P_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial P_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial P_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial P_i}dx$$

$$\theta_{M_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial M_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial M_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial M_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial M_i}dx$$

$$\phi_{T_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial T_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial T_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial T_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial T_i}dx$$

f_s=6/5 (rectangular cross section), f_s=10/9 (circular cross section)