ME 323: Mechanics of Materials **Quiz 5**

Name:______________________

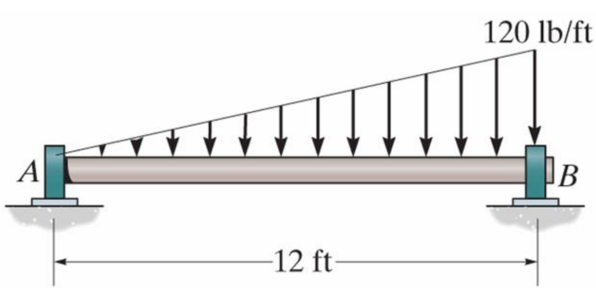
Q1 (10 Points): A beam of a T-shape cross section is subject to a linearly distributed load shown below. b=2 in., h=6 in. Determine the moment in the middle section of the beam (i.e., 6 ft from A). Also, within this cross section, find:

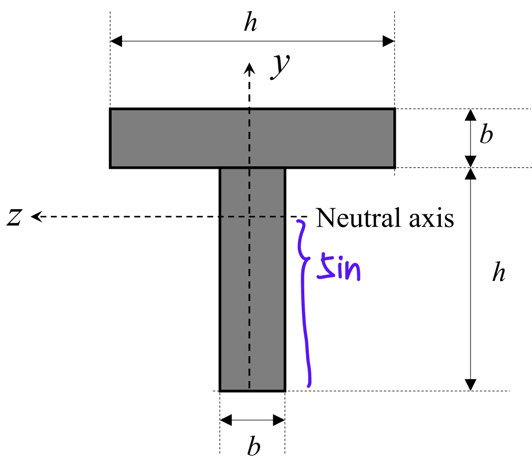
(i) The location of the neutral axis of the cross section.
(ii) The second area moment of the cross section.
(iii) The largest magnitude of stress in the cross section

Given: $b = 2in$, $h = 6in$

Find: $\bar{y}$, I_y, $|\sigma_{max}|$

Solution:



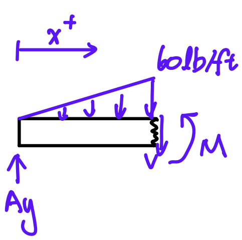


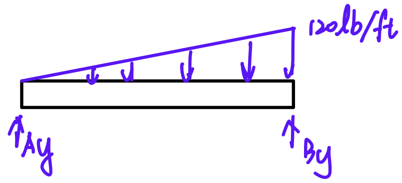
$\uparrow$ $y=0$

$$\bar{y} = (A_1\bar{y}_1 + A_2\bar{y}_2)\cdot\frac{1}{A_1+A_2} = [12in^2\cdot 3in + 12in^2\cdot 7in]\cdot\frac{1}{24in^2} = 5in$$

$$I_y = I_1 + d_1^2A_1 + I_2 + d_2^2A_2 = \frac{1}{12}bh^3 + d_1^2A_1 + \frac{1}{12}hb^3 + d_2^2A_2$$

$$= \frac{1}{12}(2in)(6in)^3 + (2in)^2\cdot(12in^2) + \frac{1}{12}(6in)(2in)^3 + (2in)^2(12in^2) = 136in^4$$





$W(x) = 10lb/ft\cdot x$ $\quad x\in[0, 12in]$

$$\int_0^{12} 120x - 10x^2 dx = 60x^2 - \frac{10}{3}x^3\Big|_{x=0}^{x=12} = 2880 lbf ft$$

Equilibrium:

$$\vec{M}_{AB} @ B: \; -A_y\cdot(12ft) + \int_0^{12ft} W(x)\cdot(12ft - x)dx = 0 \Rightarrow A_y = 240 lbf.$$

$$\vec{M} \text{ of left @ cut}: \; -A_y\cdot(6ft) + \int_0^{6ft} W(x)\cdot(6ft - x)dx + M = 0 \qquad \int_0^6 60x - 10x^2 dx = 30x^2 - \frac{10}{3}x^3\Big|_{x=0}^{x=6} = 360 lbf ft.$$

$$-240 lbf\cdot 6ft + 360 lbf\cdot ft + M = 0 \Rightarrow M = 1080 lbf ft = 12960 lbf in$$

$$|\sigma_{max}| = \frac{|M y_{max}|}{I_y} = \frac{12960 lbf in\cdot 5in}{136 in^4} = 476.47 lbf/in^2 = 476.47 psi$$