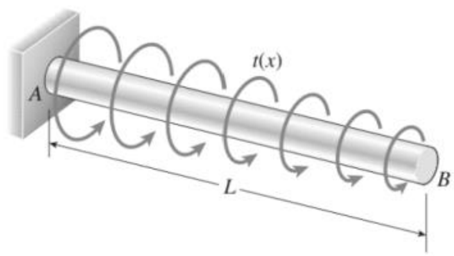
HW 4 Solution

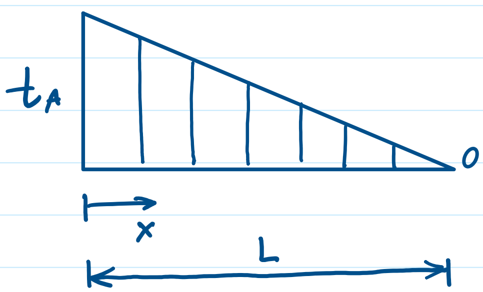
Tuesday, February 3, 2026 8:55 AM

Problem 1.

A solid prismatic bar AB of uniform circular cross section (diameter d) is loaded by a distributed torque (see figure). The intensity of the distributed torque, that is, the torque per unit distance, is denoted $t(x)$ (Nm/m) and varies linearly from a maximum value t_A at end A to zero at end B. Also, the length of the bar is L and the shear modulus of elasticity of the material is G.

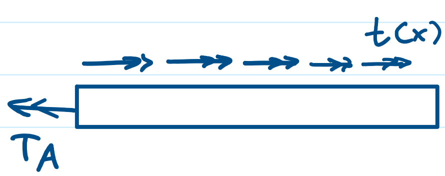
(a) Determine the maximum shear stress τ_{max} in the bar.
(b) Determine the angle of twist ϕ between the ends of the bar.





$$t(x) = t_A - \frac{t_A x}{L} \quad \left(\frac{Nm}{m}\right)$$

FBD:



$$\Sigma T = 0 \; : \quad -T_A + \int_0^L t(x)\,dx = 0$$

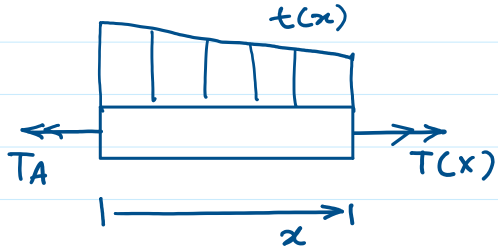
$$-T_A + \int_0^L \left(t_A - \frac{t_A x}{L}\right) dx = 0$$

$$T_A = \frac{t_A L}{2} \qquad (N \cdot m)$$

Section at x.

$t(x)$

Section at x.



$$\Sigma T = 0$$

$$-T_A + \int_0^x t(\xi)\,d\xi + T(x) = 0$$

$$T(x) = \frac{t_A x^2}{2L} - t_A x + \frac{t_A L}{2}$$

$$T_{max} = \frac{t_A L}{2} \quad @\ x = 0$$

(a). $$\tau_{max} = \frac{T_{max} \cdot r}{I_p}$$

$$I_p = \frac{\pi r^4}{2} \text{ or } \frac{\pi d^4}{32}$$

$$= \frac{t_A L \cdot d}{4 \cdot \frac{\pi \cdot d^4}{32}}$$

$$\boxed{\tau_{max} = \frac{8 t_A L}{\pi d^3}}$$

(b) ϕ :

$$\frac{d\phi}{dx} = \frac{T(x)}{G I_p}$$

$$\phi = \int_0^L \frac{T(x)}{G I_p}\,dx = \frac{1}{G I_p} \int_0^L \left(\frac{t_A x^2}{2L} - t_A x + \frac{t_A L}{2} \right) dx$$

$$\phi = \frac{32}{G \cdot \pi d^4} \cdot \frac{t_A L^2}{6}$$

$$\boxed{\phi = \frac{16 t_A L^2}{3 \pi G d^4}}$$

Problem 2:

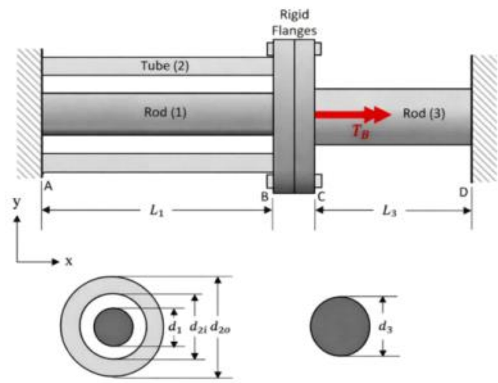
A solid rod (1) of diameter d_1 is enclosed by a concentric tube (2) of inner diameter d_{2i} and outer diameter d_{2o}, and both are attached to a rigid support at A and a rigid flat flange at B. A solid rod (3) of diameter d_3 is similarly attached to a rigid support at D and a rigid flat flange at C. The flanges are securely connected. Subsequently, an external torque T_B is applied at B.

(a) Determine the internal torques T_1, T_2, T_3 in three elements resulting from the load T_B
(b) Determine the maximum shear stresses $\tau_{max\,(1)}, \tau_{max\,(2)}, \tau_{max\,(3)}$ in the three elements.
(c) Determine the angle of twist at the rigid flanges (B or C)

Given : $d_1 = 3$ cm, $d_{2i} = 4$ cm, $d_{2o} = 5$ cm, $d_3 = 3.5$ cm,

$G_1 = 20$ GPa, $G_2 = 10$ GPa, $G_3 = 30$ GPa,

$L_1 = 200$ cm, $L_3 = 100$ cm, $T_B = 100$ N m



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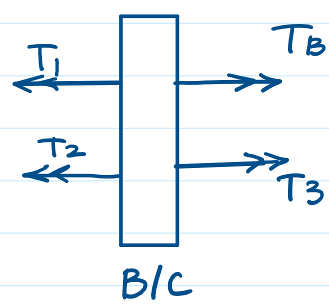


$$\Sigma T: \quad T_1 + T_2 = T_3 + T_B \rightarrow (1)$$

3 unknowns, 1 equation.

Compatibility conditions:

$$\Delta\phi_1 = \Delta\phi_2 \rightarrow (2)$$

$$\Delta\phi_1 + \Delta\phi_3 = 0 \rightarrow (3)$$

$\Delta\phi_1 = \frac{T_1 L_1}{}$ $\quad \Delta\phi_2 = \frac{T_2 L_2}{}$ $\quad \Delta\phi_3 = \frac{T_3 L_3}{}$

$$\Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{p1}} \qquad \Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{p2}} \qquad \Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{p3}}$$

Substituting into 2 and 3

$$\frac{T_1 L_1}{G_1 I_{p1}} = \frac{T_2 L_2}{G_2 I_{p2}} \longrightarrow (4)$$

$$\frac{T_1 L_1}{G_1 I_{p1}} = -\frac{T_3 L_3}{G_3 I_{p3}} \longrightarrow (5)$$

Solve for T_1, T_2, T_3 using 1, 4 and 5

$$I_{p1} = \frac{\pi d_1^4}{32} = 7.952 \times 10^{-8} \ m^4$$

$$I_{p2} = \frac{\pi (d_{2o}^4 - d_{2i}^4)}{32} = 3.623 \times 10^{-7} \ m^4$$

$$I_{p3} = \frac{\pi d_3^4}{32} = 1.473 \times 10^{-7} \ m^4$$

Substituting the given values of L, G and I_p into 4 and 5 we get:

$$2.278 \ T_1 = T_2$$

$$-5.557 \ T_1 = T_3$$

Substituting into 1

$$T_1 + 2.278 T_1 + 5.557 T_1 = T_B$$

$$\boxed{\begin{aligned} T_1 &= 11.319 \text{ N}\cdot\text{m} \\ T_2 &= 25.785 \text{ N}\cdot\text{m} \\ T_3 &= -62.9 \text{ N}\cdot\text{m} \end{aligned}}$$

(b) τ_{max} is at the outer surface of all rods.

$$\tau_{max(1)} = \frac{T_1 (d_1/2)}{I_{p1}}$$

$$\tau_{max(2)} = \frac{T_2 (d_{20}/2)}{I_{p2}}$$

$$\tau_{max(3)} = \frac{T_3 (d_3/2)}{I_{p3}}$$

$$\boxed{\begin{aligned} \tau_{max(1)} &= 2.135 \text{ MPa} \\ \tau_{max(2)} &= 1.779 \text{ MPa} \\ \tau_{max(3)} &= 7.473 \text{ MPa} \end{aligned}}$$

(c) $\phi_B = \Delta\phi_1 = \dfrac{T_1 L_1}{G_1 I_{p1}}$

$$\boxed{\phi_B = 0.01423 \text{ rad}}$$

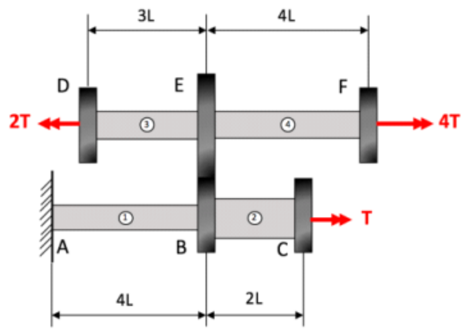
Problem 3.

The gear shaft system consists of two parallel shafts, ABC and DEF. The shafts are interconnected by gears at B and E. The diameters of members 1, 2, 3, and 4 are d_1, d_2, d_3 and d_4, respectively. Both shafts have the same shear modulus G. Determine the angle of twist at C and F

Given : $d_1 = D, d_2 = 2D, d_3 = 1.5D, d_4 = 1.5D,$

$d_B = 4D, d_E = 6D;\ D = 20$ mm,

$L = 500$ mm, $G = 100$ GPa, $T = 1500$ N m



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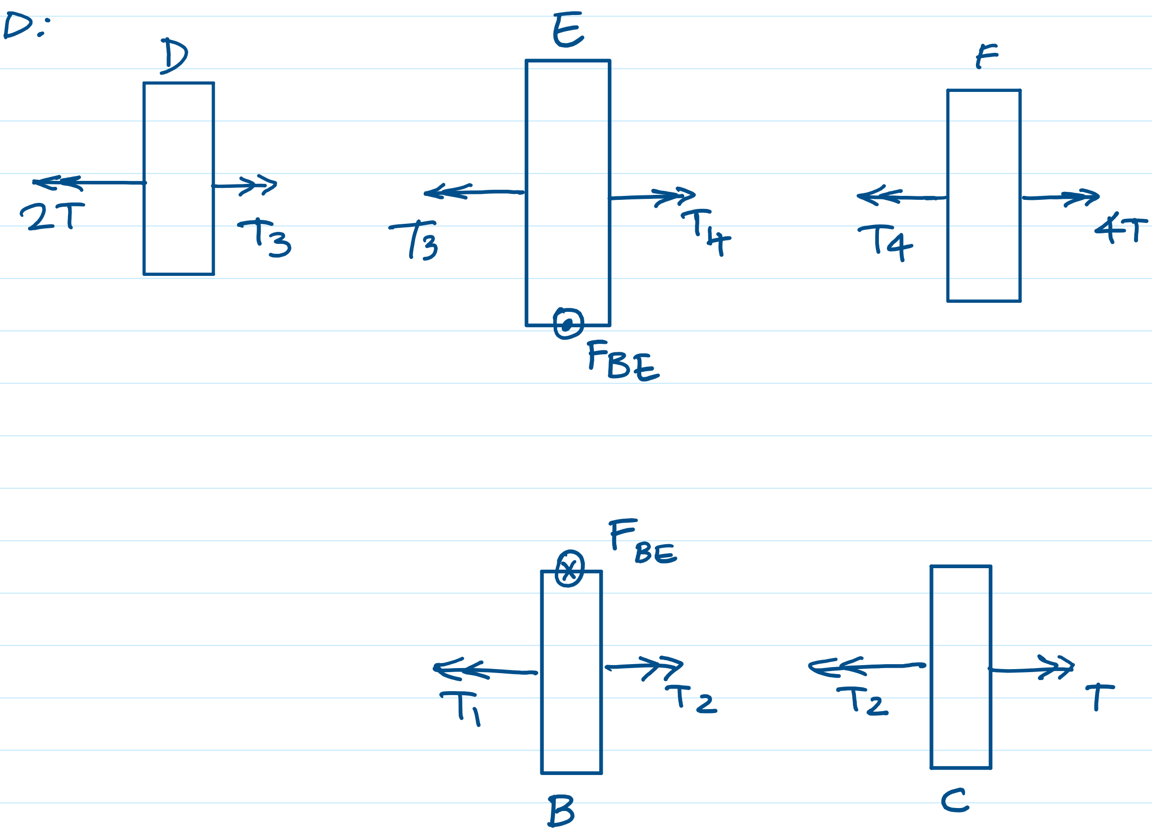


At connector D:

ΣT : $\quad T_3 = 2T \quad \rightarrow$ ①

At connector F:

ΣT : $\quad T_4 = 4T \quad \rightarrow$ ②

At gear E:

$$\Sigma T: \quad T_4 = d_E F_{BE} + T_3$$

substituting ① and ②

$$F_{BE} = \frac{2T}{d_E} = \frac{T}{3D} \rightarrow ③$$

At connector C:

$$\Sigma T: \quad T_2 = T \rightarrow ④$$

At gear B:

$$T_1 + d_B F_{BE} = T_2$$

$$T_1 = T - \frac{T}{3D} \cdot d_B$$

$$T_1 = -\frac{T}{3} \rightarrow ⑤$$

Angle of twist of (1)

$$\Delta\phi_1 = \frac{T_1 L_1}{G I_{p_1}} = \frac{-T \cdot 4L}{3 \cdot G \cdot \frac{\pi D^4}{32}}$$

$$\Delta\phi_1 = -0.637 \text{ rad}$$

Angle of twist of (2):

$$\Delta\phi_2 = \frac{T_2 L_2}{G I_{p_2}} = \frac{2TL}{G \frac{\pi (2D)^4}{32}}$$

$$\Delta\phi_2 = 0.06 \text{ rad}$$

Angle of twist of (4):

$$\Delta\phi_4 = \frac{T_4 L_4}{G I_{P4}} = \frac{16TL}{G \cdot \frac{\pi(1.5D)^4}{32}}$$

$$\Delta\phi_4 = 1.509 \text{ rad}$$

Angle of twist at C:

$$\Delta\phi_C = \Delta\phi_1 + \Delta\phi_2$$

$$\boxed{\Delta\phi_C = -0.577 \text{ rad}}$$

Angle of twist at F:

$$\Delta\phi_F = \Delta\phi_E + \Delta\phi_4$$

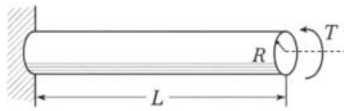
$$= -\frac{d_B}{d_E}\Delta\phi_1 + \Delta\phi_4$$

$$\boxed{\Delta\phi_F = 1.934 \text{ rad}}$$

Problem 4.

Consider a circular shaft of radius R, length L, and shear modulus G, subjected to torque T at the free end, as shown in the figure. Determine the effect the following changes would have:

4.1 Increasing the shaft radius, R, would ________ the maximum shear stress



(a) Increase

(b) Decrease ✓

(c) Not change

$\tau_{max} = \frac{TR}{I_p} = \frac{2T}{\pi R^3}$

4.2 Increasing the shaft length, L, would ________ the maximum shear stress

(a) Increase

(b) Decrease

(c) Not change ✓

Independent of L

4.3 Increasing the shaft shear modulus, G, would ________ the maximum shear stress

(a) Increase

(b) Decrease

(c) Not change ✓

Ind. of G

4.4 Increasing the shaft radius, R, would ________ the angle of rotation at the free end

(a) Increase

(b) Decrease ✓

(c) Not change

$\Delta\phi = \frac{TL}{GI_p} = \frac{2TL}{G\pi R^4}$

4.5 Increasing the shaft length, L, would ________ the angle of rotation at the free end

(a) Increase ✓

(b) Decrease

(c) Not change