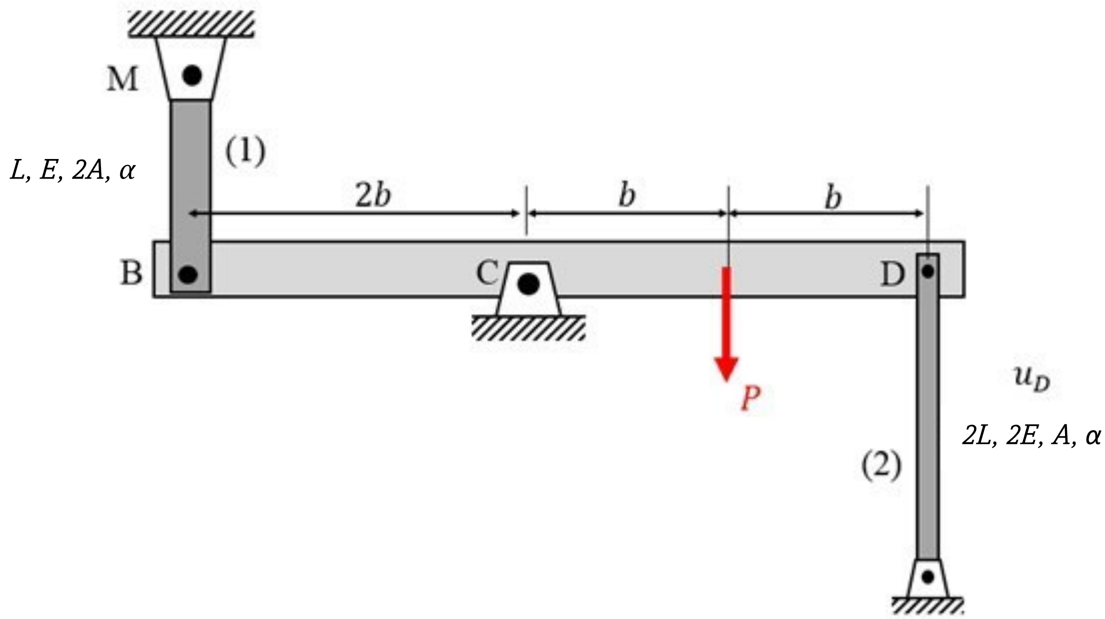
Set up. 10 min   cal. by hand 15 min.
w/ matlab 10 min

Problem 3.1 ***(10 points)***

Rigid bar BD is pinned to ground at end C. The bar is supported by thin rods (1) and (2), where rod (1) has a Young's modulus of E and a cross-sectional area of 2A and rod (2) has a Young's modulus of 2E and a cross-sectional area of A. Each rod possesses a coefficient of thermal expansion α. A load P acts midway between pins C and D on the bar. Assume that the angle of rotation of the bar as a result of the applied load is small and the temperature of both rods is simultaneously increased by ΔT. Ignore the weight of all components of the structure.

(a) Determine the stress in rods (1) and (2).
(b) Determine the angle of rotation of bar BD.



Given: (1) $L, E, 2A, \alpha$

(2) $2L, 2E, A, \alpha$

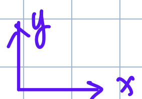
$\Delta T \uparrow$, P, b

Find: σ_1, σ_2

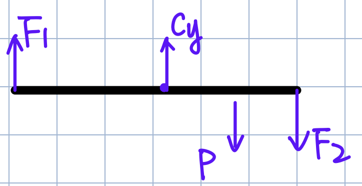
θ_{BD}

Solution:

1. coor.



2. F.B.D. * Assume (1)(2) in tension.



3. Equilibrium Eq.

ΣF_y: $F_1 + C_y - P - F_2 = 0$

ΣM_C: $(-2b)F_1 - (b)P - (2b)F_2 = 0$ ①

4. Stress Strain Disp. Eq.

$e_1 = \frac{F_1 L}{2EA} + L\alpha\Delta T = \frac{LF_1}{2EA} + L\alpha\Delta T$ ②

$e_2 = \frac{F_2(2L)}{2EA} + 2L\alpha\Delta T = \frac{LF_2}{EA} + 2L\alpha\Delta T$ ③

$\sigma_1 = \frac{F_1}{2A}$ ④

$\sigma_2 = \frac{F_2}{A}$ ⑤

5. Compatibility Eq.

$\frac{e_1}{e_2} = \frac{2b}{2b} \Rightarrow e_1 = e_2$. ⑥

$\theta = \tan^{-1}[e_1/(2b)]$ ⑦

6. Solve:

* ΣF_y is not helpful since it includes C_y.

Step ①: ②, ③, ⑥ → Relate F_1, F_2

Step ②: ① → Solve F_1, F_2

Step ③: ④, ⑤ → Solve σ_1, σ_2.

→ ② ③ ⑥ →

$\frac{L}{\cancel{2}EA}[F_1 + \cancel{2}EA\alpha\Delta T] = \frac{L}{\cancel{2}EA}[2F_2 + \overset{2}{\cancel{4}}EA\alpha\Delta T]$

$F_1 = 2F_2 + 2EA\alpha\Delta T$

① → $2F_1 + 2F_2 = -P$

→ $4F_2 + 4EA\alpha\Delta T + 2F_2 = -P$

→ $6F_2 = -P - 4EA\alpha\Delta T$

→ $F_2 = -\frac{1}{6}P - \frac{2}{3}EA\alpha\Delta T$

→ $F_1 = -\frac{1}{3}P + \frac{2}{3}EA\alpha\Delta T$

④ → $\sigma_1 = -\frac{1}{6}\frac{P}{A} + \frac{1}{3}E\alpha\Delta T$

⑤ → $\sigma_2 = -\frac{1}{6}\frac{P}{A} - \frac{2}{3}E\alpha\Delta T$

② → $e_1 = -\frac{PL}{6EA} + \frac{L\alpha\Delta T}{3} + L\alpha\Delta T$

(sanity check)

③ → $e_2 = -\frac{PL}{6EA} - \frac{2L\alpha\Delta T}{3} + 2L\alpha\Delta T$

$= -\frac{PL}{6EA} + \frac{4}{3}L\alpha\Delta T$

$e_1 = e_2$. math check out.

$\theta = \tan^{-1}\left[\frac{e_1}{2b}\right]$

$\theta = \tan^{-1}\left[-\frac{PL}{12EAb} + \frac{2L\alpha\Delta T}{3b}\right]$

setup 5min   cal. by hand 15min
w/ Matlab 5min

Problem 3.2 ***(10 points)*** A rod is made up of elements (1), (2) and (3), as shown below. Element (2) is hollow with outer and inner diameters of 2d and d, respectively, whereas elements (1) and (3) are solid with diameters of d and 3d, respectively. Elements (1) and (2) are joined by rigid connector D, elements (2) and (3) are joined by rigid connector C, and elements (1) and (3) are connected to ground at ends H and B, respectively. The modulus of elasticity for all three elements is E. The weights of connectors D and C are 2W and 3W, respectively, whereas the weights of the rod elements (1), (2) and (3) are to be considered negligible.

(a) Determine the stress in element (3) resulting only from the weights of the connectors (refer to Fig A).
(b) Suppose that an axial load P is applied to connector D in a way that the magnitude of the stress in element (3) is reduced (refer to Fig B). Determine the load value for P such that the magnitude of the compressive stress in (3) reduced by 50 percent from that found in (a).

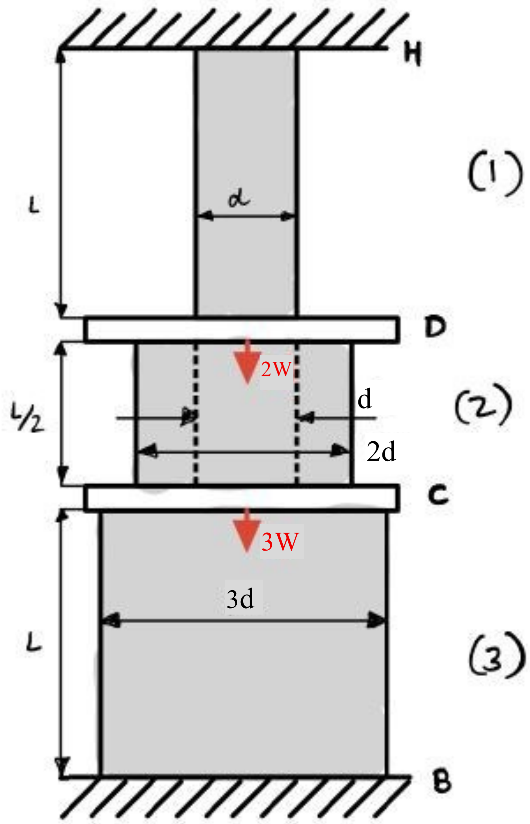


Fig A

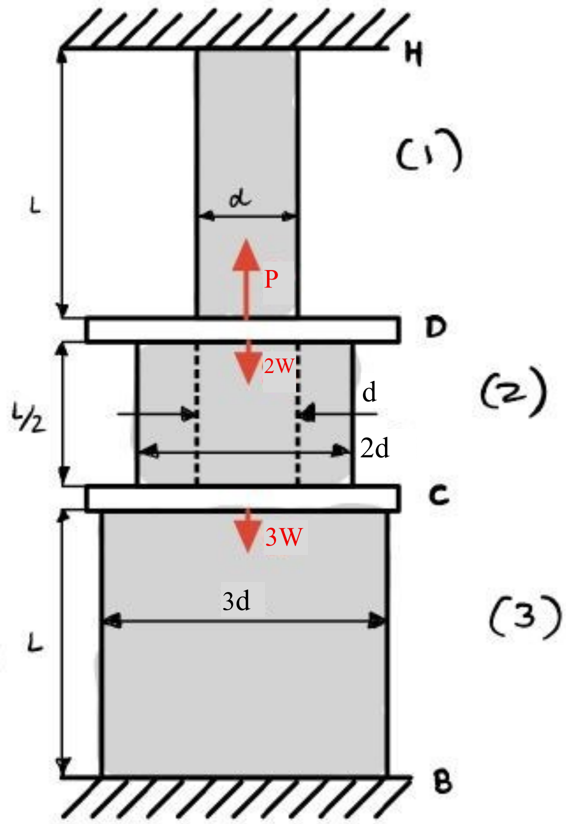


Fig B

Given: (1): d, L

(2): d, $2d$, $L/2$

(3): $3d$, L

E, W

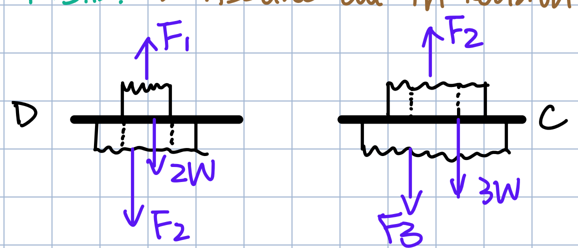
Find: σ_3 w/ only W.

P so that σ_3 is halfed.

Solution:

1. coor. $\downarrow x^+$

2. F.B.D. * Assume all in tension



3. Equilibrium Eq

D: $-F_1 + 2W + F_2 = 0$ (1)

C: $-F_2 + 3W + F_3 = 0$ (2)

4. Stress Strain Disp. Eq.

$\sigma_1 = F_1/A_1$ (3) $A_1 = \frac{1}{4}\pi d^2$

$\sigma_2 = F_2/A_2$ (4) $A_2 = \frac{3}{4}\pi d^2$

$\sigma_3 = F_3/A_3$ (5) $A_3 = \frac{9}{4}\pi d^2$

$e_1 = \frac{F_1 L}{E A_1}$ (6)

$e_2 = \frac{F_2 L}{2 E A_2}$ (7)

$e_3 = \frac{F_3 L}{E A_3}$ (8)

5. Compability Eq.

$e_1 + e_2 + e_3 = 0$ (9)

6. Solve.

(6) – (9):

$$\frac{\cancel{4} F_1 \cancel{L}}{\cancel{E}\cancel{\pi}\cancel{d^2}} + \frac{\cancel{4} F_2 \cancel{L}}{6\cancel{E}\cancel{\pi}\cancel{d^2}} + \frac{\cancel{4} F_3 \cancel{L}}{9\cancel{E}\cancel{\pi}\cancel{d^2}} = 0$$

$F_1 = -\frac{1}{6}F_2 - \frac{1}{9}F_3$

(1) $\rightarrow \frac{7}{6}F_2 + \frac{1}{9}F_3 = -2W$

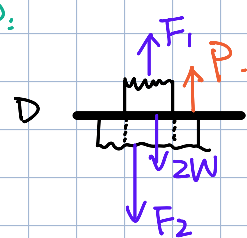
$\rightarrow F_2 + \frac{2}{21}F_3 = -\frac{12}{7}W$

(2) $\rightarrow \frac{23}{21}F_3 = -\frac{33}{7}W$

$F_3 = -\frac{21}{23} \cdot \frac{33}{7} W \rightarrow F_3 = -\frac{99}{23}W$

(5) $\rightarrow \sigma_3 = -\frac{99}{23} W \cdot \frac{4}{9\pi d^2} \rightarrow \sigma_3 = -\frac{44W}{23\pi d^2}$

7. Add P:



$F_3 = -\frac{99}{46}W.$

Equilibrium Eq.

$-F_1 + 2W + F_2 - P = 0$ (10)

$-F_2 + 3W + F_3 = 0$ (11)

σ and e Eq. stay the same.

(11) $\rightarrow F_2 = 3W + F_3 = (3 - \frac{99}{46})W = \frac{39}{46}W$

From prev. $F_1 = -\frac{1}{6}F_2 - \frac{1}{9}F_3$

$= -\frac{13}{92}W + \frac{11}{46}W$

$= \frac{9}{92}W$

(10) $\rightarrow P = -F_1 + F_2 + 2W$

$= \frac{-9}{92}W + \frac{39}{46}W + 2W$

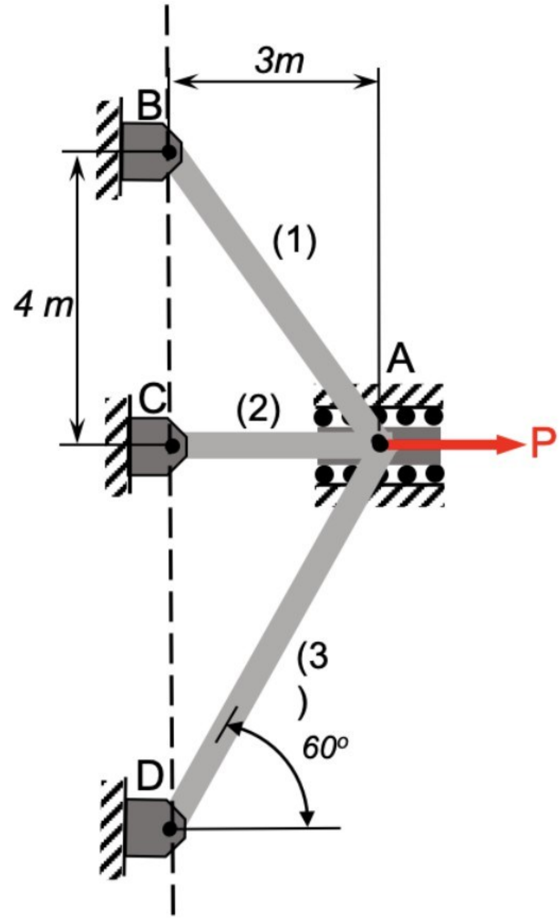
$= (\frac{-9+78}{92} + 2)W = \frac{11}{4}W$

$P = \frac{11}{4}W$

Problem 3.3 ***(10 points)*** Setup 5min Calculation 10min

Three elastic members form the pin-jointed truss in the figure below. The joint at A is constrained by a slider block to move only in the horizontal direction (i.e., $v_A = 0$). All members have the same cross-sectional area A = 400 mm^2 and a modulus of elasticity E = 150 GPa. The pin-joints B, C and D are colinear.

(a) Determine the member forces F_1, F_2 and F_3 which result in moving the joint A to the right by 15 mm (i.e., u_A = 15 mm).
(b) Determine the horizontal load P.
(c) Determine the vertical reaction between the slider block at A and the track.



Given: $A = 400mm^2$ $L_1 = 5m$

$E = 150 GPa$ $L_2 = 3m$

$U_A = 15mm$ $L_3 = 6m$

$V_A = 0$

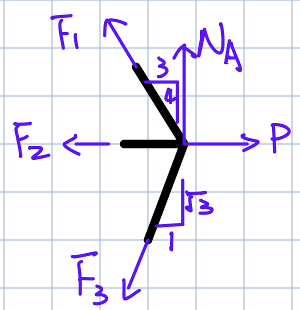
Find: F_1, F_2, F_3,

P, N_A.

Solution:

1. coor. $\uparrow y$, $\rightarrow x$

2. FBD. (Assume tension)



3. Equilibrium Eq.

$\Sigma F_x: \quad P - \frac{3}{5}F_1 - F_2 - \frac{1}{2}F_3 = 0$ ①

$\Sigma F_y: \quad \frac{4}{5}F_1 - \frac{\sqrt{3}}{2}F_2 + N_A = 0$ ②

4. Displacement Eq.

$e_1 = \frac{F_1 L_1}{EA}$ ③

$e_2 = \frac{F_2 L_2}{EA}$ ④

$e_3 = \frac{F_3 L_3}{EA}$ ⑤

5. Compability Eq.

$e_1 = \frac{3}{5} U_C$ ⑥

$e_2 = U_C$ ⑦

$e_3 = \frac{1}{2} U_C$ ⑧

6. Solve.

③, ⑥:

$\frac{3}{5} U_C = \frac{F_1 L_1}{EA}$

$\Rightarrow F_1 = \frac{3 U_C EA}{L_1} = \frac{3\,(15mm)\,(150\frac{kN}{mm^2})\,(400mm^2)}{5 \times 5 \times 10^3 mm}$

$F_1 = 108kN$

④, ⑦:

$U_C = \frac{F_2 L_2}{EA}$

$\Rightarrow F_2 = \frac{U_C EA}{L_2} = \frac{(15mm)(150GPa)(400mm^2)}{3 \times 10^3 mm}$

$F_2 = 300kN$

⑤, ⑧:

$\frac{1}{2} U_C = \frac{F_3 L_3}{EA}$

$\Rightarrow F_3 = \frac{U_C EA}{2 L_3} = \frac{(15mm)(150GPa)(400mm^2)}{2 \times 6 \times 10^3 mm}$

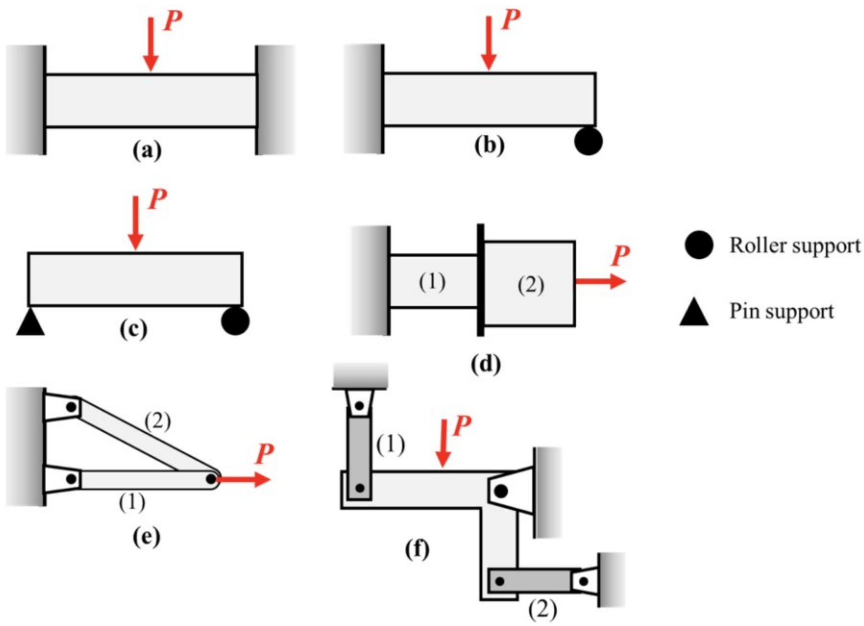
$F_3 = 75kN.$

② $\Rightarrow N_A = \frac{\sqrt{3}}{2}F_2 - \frac{4}{5}F_1 = -21.45kN$

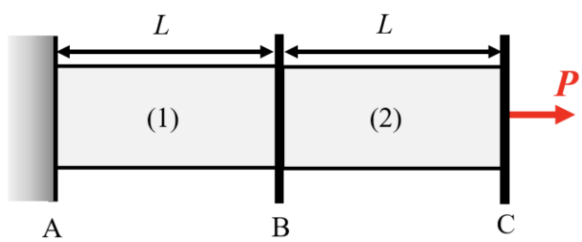
① $\Rightarrow P = \frac{3}{5}F_1 + F_2 + \frac{1}{2}F_3 = 402.3kN$

Problem 3.4 Conceptual *(2.5+2.5 points)* 5min

(a) Out of the six structures (a)-(f) below, how many are statically indeterminate? Circle them and briefly justify your position.



(b) The rod consists of elements (1) and (2) and rigid connectors B and C. Both elements have length L and cross-sectional area A. Element (1) has Young's modulus E_1, and element (2) has Young's modulus E_2, with $E_1 < E_2$.



Let F_1 and F_2 represent the axial forces in members (1) and (2). Choose the correct option and explain why.

(a) $F_1 > F_2$

(b) $F_1 = F_2$

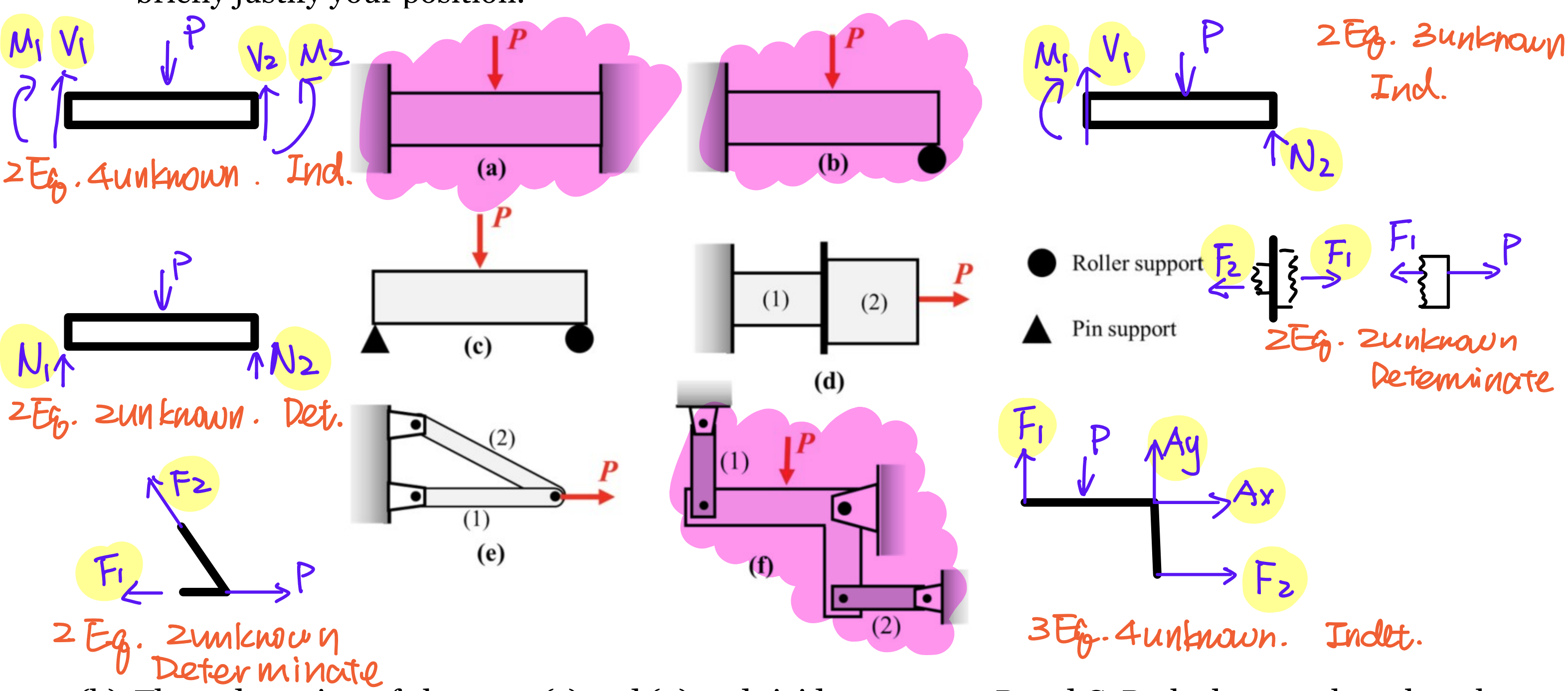
(c) $F_1 < F_2$

(d) Insufficient information

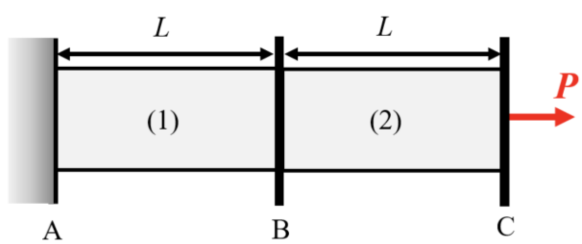
Problem 3.4 Conceptual *(2.5+2.5 points)*

3 (highlighted).

(a) Out of the six structures (a)-(f) below, how many are statically indeterminate? Circle them and briefly justify your position.



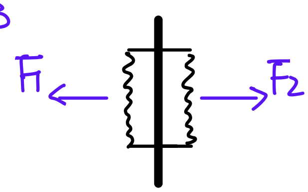
(b) The rod consists of elements (1) and (2) and rigid connectors B and C. Both elements have length L and cross-sectional area A. Element (1) has Young's modulus E_1, and element (2) has Young's modulus E_2, with $E_1 < E_2$.



Let F_1 and F_2 represent the axial forces in members (1) and (2). Choose the correct option and explain why.

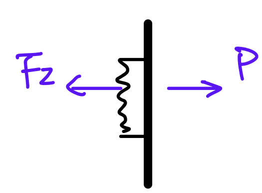
(a) $F_1 > F_2$
(b) $F_1 = F_2$
(c) $F_1 < F_2$
(d) Insufficient information

FBD of B



$F_1 = F_2$

FBD of C



$F_2 = P$