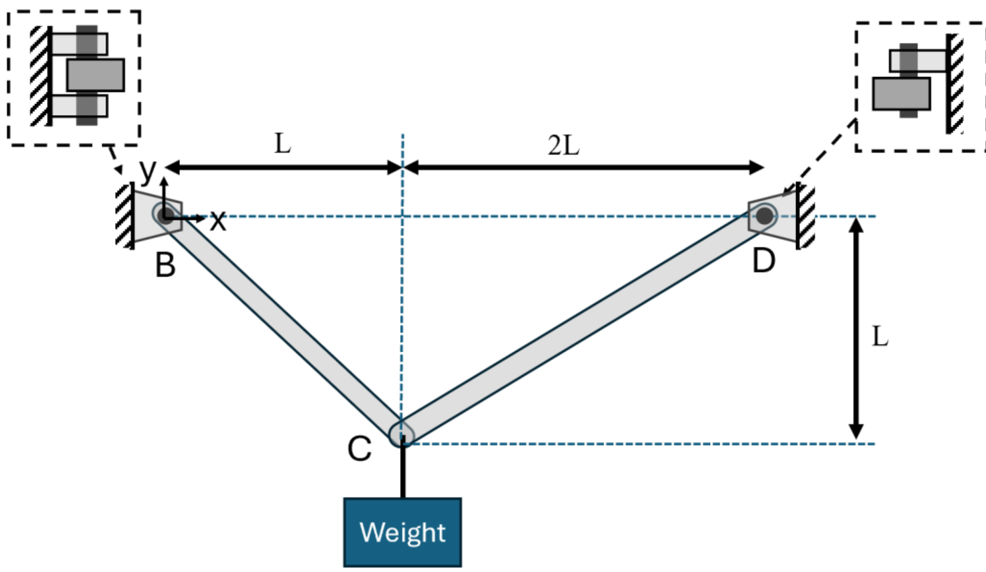
ME 323: Mechanics of Materials **Homework 2**

Spring 2026 **Due: January 30, 2026**

Problem 1 (10 points): Set up 5min. Calculation 25 min

Structure BCD is composed of two axial members BC and CD. BC and CD both have a cross-sectional area of 10 mm^2 and are made of a material with a yield stress (σ_Y) of 180 MPa. The pin at B is double sided and has a diameter of 3.5 mm and a shear yield stress (τ_y) of 75 MPa. The pin at D is single sided and has a diameter of 5 mm and a shear yield stress (τ_y) of 75 MPa.



a) Draw the free body diagram of the node C and write the equilibrium equations.
b) Determine the normal stress in members BC and CD as a function of the weight W.
c) Determine the shear stresses in the pins at B and D as a function of the weight W.
d) What is the maximum value of weight (W) that can be applied to achieve an overall factor of safety of 3. What component (member BC, member CD, pin at B, or pin at D) is the limiting factor for the safety of the structure?
e) What are the factors of safety for the other components at the value of W found in part (d)?

Given: $A_{BC} = A_{CD} = 10\,mm^2$

$d_B = 3.5mm.$

$d_D = 5mm$

$\tau_y = 75MPa$

$\sigma_y = 180MPa.$

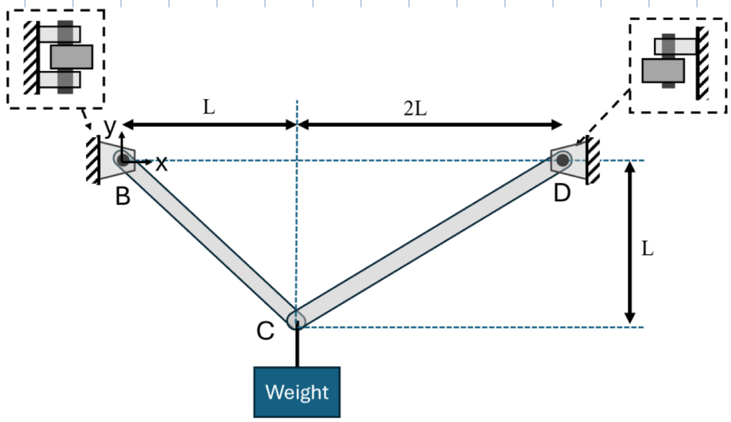
$FS = 3$

Find: σ_{BC}, σ_{CD}.

τ_B, τ_D.

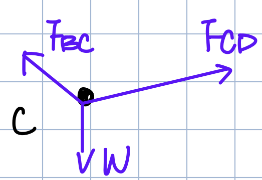
W w/ FS.

FS of nonlimiting load.



Solution. 1. coor. y ↑ → x

2. FBD



3. Equilibrium Eq.

$\Sigma F_x: -\frac{1}{\sqrt{2}} F_{BC} + \frac{2}{\sqrt{5}} F_{CD} = 0$ ①

$\Sigma F_y: \frac{1}{\sqrt{2}} F_{BC} + \frac{1}{\sqrt{5}} F_{CD} - W = 0$ ②

4. Stress Strain Eq.

$\sigma_{BC} = F_{BC}/A_{BC}$ ③

$\sigma_{CD} = F_{CD}/A_{CD}$ ④

$\tau_B = \frac{F_{BC}}{2 \cdot \frac{1}{4}\pi d_B^2}$ ⑤

$\tau_D = \frac{F_{CD}}{\frac{1}{4}\pi \cdot d_C^2}$ ⑥

5. Solve.

① – ⑥ → $F_{BC} = \frac{2\sqrt{2}}{3} W$, $F_{CD} = \frac{\sqrt{5}}{3} W$

$\sigma_{BC} = \frac{\sqrt{2}}{15} W$ [MPa] $\approx 0.0943W$

$\sigma_{CD} = \frac{\sqrt{5}}{30} W$ [MPa] $= 0.0745W$

$\tau_B = \frac{16\sqrt{2}}{147\pi} W = 0.0490W$

$\tau_D = \frac{4\sqrt{5}}{75\pi} W = 0.0380W$

6. FS.

W/ FS = 3

$\sigma_{max} = \sigma_Y / FS = 60MPa$

$\tau_{max} = \tau_Y / FS = 25MPa$

for $\sigma_{BC} = \sigma_{max}$, $W = 636.39\,N$

$\sigma_{CD} = \sigma_{max}$, $W = 804.98\,N$

$\tau_B = \tau_{max}$, $W = 510.23N$

$\tau_D = \tau_{max}$, $W = 658.57N$

τ at B is limiting fac.

w/ W = 510.23N.

$FS_{BC} = \frac{\sigma_Y}{\sigma_{BC}} = 3.74$

$FS_{CD} = \frac{\sigma_Y}{\sigma_{CD}} = 4.73$

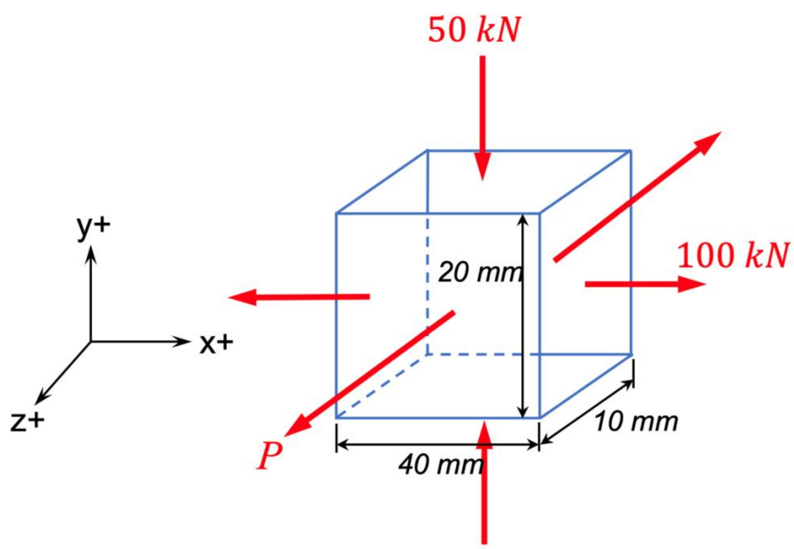
$\tau_D = \frac{\tau_Y}{\tau_D} = 3.87$

Problem 2 (10 points): Set up 5min Calculation 10 min w/ matlab.

An elastic cuboid made of steel ($E = 90\ GPa$, $\nu = 0.15$), is subjected to uniformly distributed net axial forces $100\ kN$, $50\ kN$, and an unknown force P (kN) on the x, y, and z faces respectively, as shown in Fig 1. The loads are applied such that uniform axial stresses are induced throughout the volume of the cuboid.

1. Calculate the unknown force P such that length of the cube along the x axis remains unchanged.

2. Calculate the corresponding deformed lengths along y and z axes due to the applied loads.



Given: $P_x = 100\,kN$

$P_y = -50\,kN$

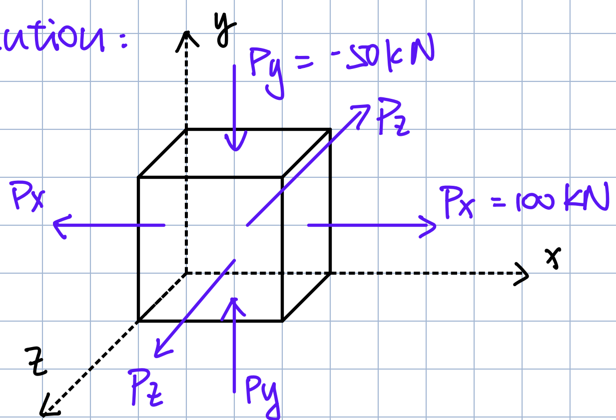
$E = 90\,GPa$

$\nu = 0.15$, $L_x = 40mm$, $L_y = 20mm$, $L_z = 10mm$

Find: P_z so that $\epsilon_x = 0$

e_y, e_z

Solution:



$\sigma_x = P_x / A_x = \frac{100kN}{200mm^2} = 500\,MPa$ ①

$\sigma_y = P_y / A_y = \frac{-50kN}{400mm^2} = -125\,MPa$ ②

$\sigma_z = P_z / A_z = \frac{P_z}{800mm^2}$ ③

$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] = 0$ ④

$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$ ⑤

$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$ ⑥

① ~ ④ $\Rightarrow 0 = \sigma_x - \nu(\sigma_y + \sigma_z)$

$\sigma_z = \frac{\sigma_x}{\nu} - \sigma_y = \frac{500MPa}{0.15} + 125MPa$

$= 3458.33\,MPa$

$P_z = \sigma_z \cdot 800mm^2 = 2766.67\,kN$

⑤ $\rightarrow \epsilon_y = \frac{1}{90GPa}[-125MPa - 0.15(500MPa + 3458.33MPa)] = -7.9861 \times 10^{-3}$

$e_y = \epsilon_y L_y = 20mm \times (-7.9861 \times 10^{-3}) = -0.1597mm$

$L_y' = L_y + e_y = 20mm - 0.1597mm = 19.8403mm$

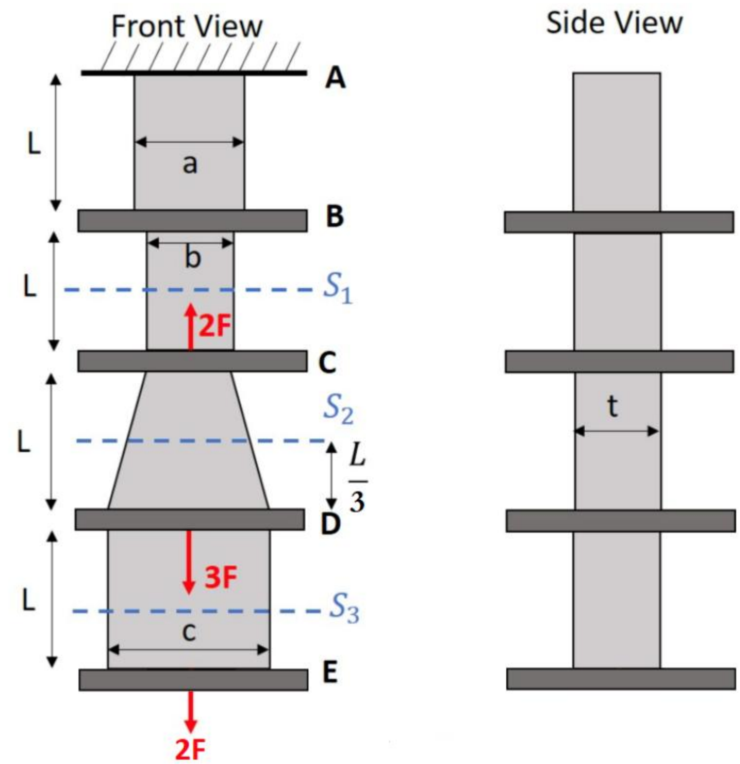
⑥ $\rightarrow \epsilon_z = \frac{1}{90GPa}[3458.33MPa - 0.15(500MPa - 125MPa)] = 37.80 \times 10^{-3}$

$e_z = \epsilon_z L_z = 10mm \times 37.80 \times 10^{-3} = 0.378mm$

$L_z' = L_z + e_z = 10mm + 0.378mm = 10.378mm$

Problem 3 (10 points): 8min set up. 15 min calculation.

The axial bar shown in Figure 3 has four sections. Each section is connected to the neighbouring sections by rigid connectors (at B, C and D). The size and weight of the rigid connectors are negligible. The first section AB, second section BC and the last section DE have uniform rectangular cross sections with widths a, b and c, respectively. The section CD has width varying linearly from b to c. The length of each section is L. All the sections have same thickness t. Three loads 2F, 3F and 2F are applied to the bar as shown in the figure. Assume the Young's Modulus of all the sections is E.

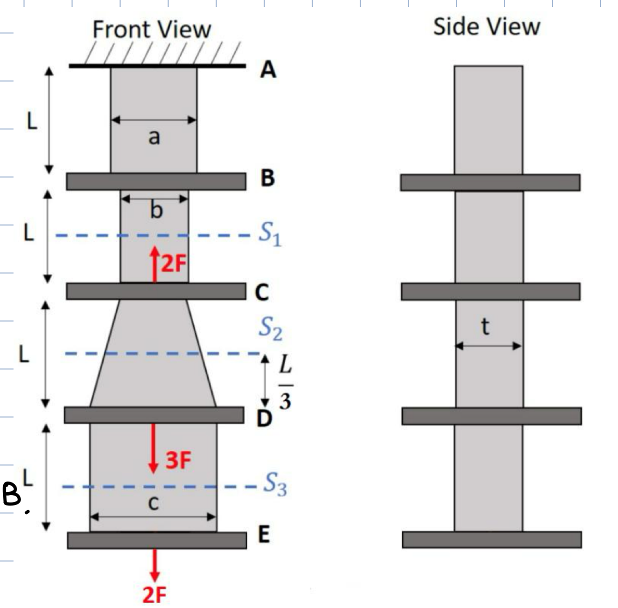


(a) Find expressions for the stresses at sections S1, S2 and S3. (S2 is L/3 above D as shown)

(b) Find expressions for the displacements at points B, C, and D.

Given: a, b, c, t
F, E

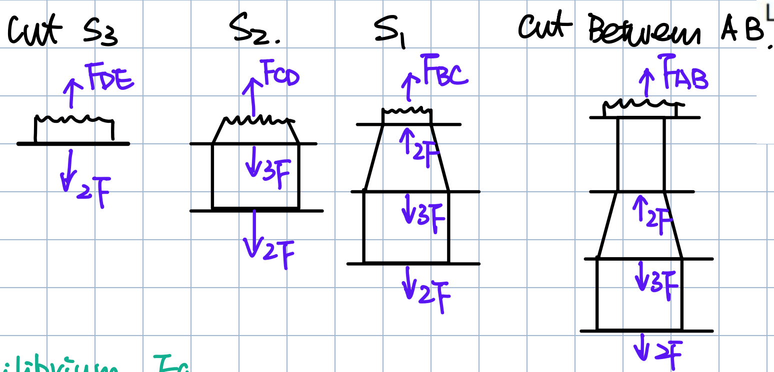
Find: $\sigma_1, \sigma_2, \sigma_3$
u_B, u_C, u_D.



Solution:

1. coor: $\downarrow x^+$

2. FBD:



3. Equilibrium Eq.

@ S_3: $2F - F_{DE} = 0$ ①

@ S_2: $3F + 2F - F_{CD} = 0$ ②

@ S_1: $2F + 3F - 2F - F_{BC} = 0$ ③

@ AB: $2F + 3F - 2F - F_{AB} = 0$ ④

4. Stress Strain. Displacement.

$$\sigma_1 = \frac{F_{BC}}{A_1} = \frac{F_{BC}}{bt} \quad ⑤$$

$$\sigma_2 = \frac{F_{CD}}{A_2} = \frac{F_{CD}}{(\frac{2}{3}c + \frac{1}{3}b)t} \quad ⑥$$

$$\sigma_3 = \frac{F_{DE}}{A_3} = \frac{F_{DE}}{ct} \quad ⑦$$

$$e_{AB} = \frac{F_{AB} L}{E \cdot at} \quad ⑧$$

$$e_{BC} = \frac{F_{BC} L}{E \cdot bt} \quad ⑨$$

$$e_{CD} = \int_0^L \frac{F_{CD}}{E A(x)} dx. \quad ⑩$$

w/ $A(x)$: $A(0) = bt$, $A(L) = ct$.

$$A(x) = \frac{t(c-b)}{L} x + bt$$

5. Solve.

①–④ $\Rightarrow$ $F_{DE} = 2F$. $F_{BC} = 3F$
$F_{CD} = 5F$ $F_{AB} = 3F$.

⑤–⑦: $\sigma_1 = \frac{3F}{bt}$

$$\sigma_2 = \frac{15F}{(b+2c)t}$$

$$\sigma_3 = \frac{2F}{ct}$$

⑧ $\rightarrow e_{AB} = \frac{3FL}{Eat}$ ⑨ $\rightarrow e_{BC} = \frac{3FL}{Ebt}$

⑩ $\rightarrow e_{CD} = \frac{5F}{Et} \int_0^L \frac{1}{(\frac{c-b}{L}x + b)} dx.$

let $u = (\frac{c-b}{L})x + b$ $\Rightarrow \frac{du}{dx} = \frac{c-b}{L}$

$\Rightarrow dx = \frac{L}{c-b} \cdot du$, lim: $x = 0 \Rightarrow u = b$
$x = L \rightarrow u = c$

$$e_{CD} = \frac{5F}{Et} \int_b^c u^{(-1)} \cdot \frac{L}{c-b} du$$

$$= \frac{5F}{Et} \cdot \frac{L}{c-b} \cdot \ln u \Big|_{u=b}^{u=c}$$

$$= \frac{5FL}{Et(c-b)} \ln\left(\frac{c}{b}\right)$$

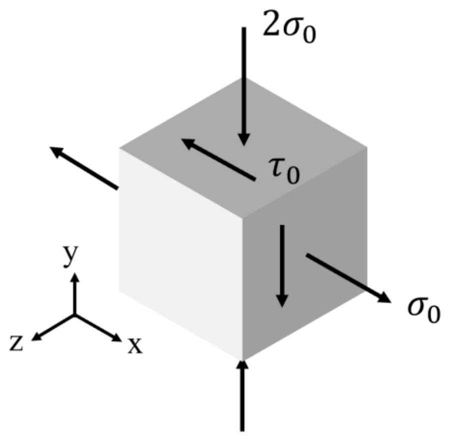
$$u_B = e_{AB} = \frac{3FL}{Eat}$$

$$u_C = e_{AB} + e_{BC} = \frac{FL}{Et}\left(\frac{3}{a} + \frac{3}{b}\right)$$

$$u_D = e_{AB} + e_{BC} + e_{CD} = \frac{FL}{Et}\left[\frac{3}{a} + \frac{3}{b} + \frac{5}{c-b} \ln\left(\frac{c}{b}\right)\right]$$

Problem 4 (5 points) 3min.

Conceptual: You do not need to give an explanation for the answers. The state of stress at a given point on a structure is shown above in terms of its xyz components. The material has a Young's modulus of E and a Poisson's ratio of ν, where $0 < \nu < 0.5$. State whether each of the xyz components of strain ($\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}$) is <0, =0, or >0.

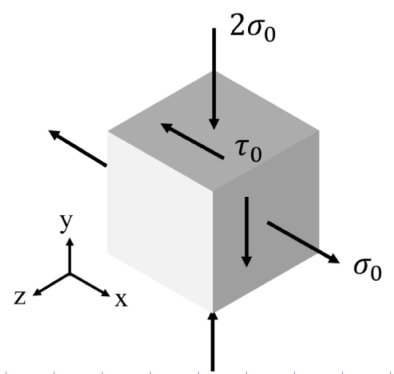


	<0, =0, or >0
ϵ_x	
ϵ_y	
ϵ_z	
γ_{xy}	
γ_{xz}	
γ_{yz}	

Given: E, $0<\nu<0.5$

$\sigma_x = \sigma_0$, $\sigma_y = -2\sigma_0$, $\sigma_z = 0$

$\tau_{xy} = -\tau_0$, $\tau_{xz} = \tau_{yz} = 0$



Find: ϵ and γ +, 0 or −

$\gamma_{xy} < 0$

$\gamma_{xz} = 0$

$\gamma_{yz} = 0$

$$\epsilon_x = \frac{1}{E}\left[\sigma_0 - \underbrace{\nu(-2\sigma_0)}_{}\right] \quad > 0$$

magnitude $< \sigma_0$

$$\epsilon_y = \frac{1}{E}\left[-2\sigma_0 - \nu(\sigma_0)\right] \quad < 0$$

$$\epsilon_z = \frac{1}{E}\underbrace{\left[-\nu\,(\overbrace{\sigma_0 - 2\sigma_0}^{<0})\right]}_{>0} \quad > 0$$