Name (Print)__Solution.____

(Last) **(First)**

ME 323 - Mechanics of Materials
Exam # 1
Date: February 18, 2026 Time: 8:00 – 10:00 PM

Instructions:

Circle your lecturer's name and your class meeting time.

Chortos	Shah	Zhao	Han
8:30-9:20AM	12:30-1:20PM	1:30-2:20PM	3:30-4:20PM

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ____

Begin each problem in the space provided on the examination sheets.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Submission to Gradescope: **10:00-10:20PM**.

Prob. 1 ________________________

Prob. 2 ________________________

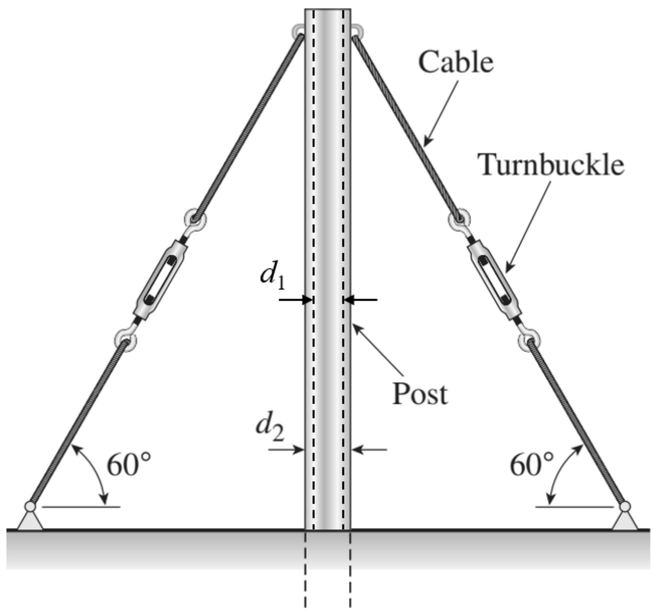
Prob. 3 ________________________

Prob. 4 ________________________

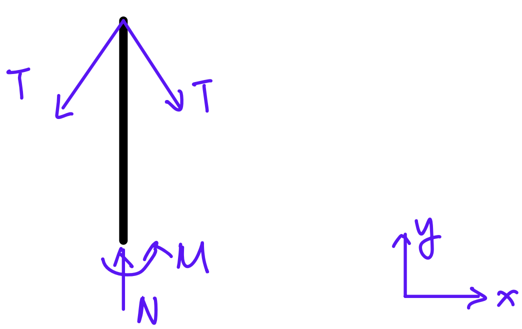
Total __________________________

PROBLEM # 1 (25 points)

A tubular post of inner diameter $d_1 = 100$ mm and outer diameter $d_2 = 130$ mm, is held in place by two cables fitted with turnbuckles, as shown in the figure. The cables are tightened by rotating the turnbuckles, thus producing tension in the cables and compression in the post. Both cables are tightened to the same tensile force T (N) and the angle between each cable and the ground is 60°.



(a) Draw the free body diagram of the post



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(b) Write the static equilibrium equation(s) and determine the reactions at the ground which supports the post in terms of T

$\Sigma F_x: \frac{1}{2}T - \frac{1}{2}T = 0$

$\Sigma F_y: -\frac{\sqrt{3}}{2}T - \frac{\sqrt{3}}{2}T + N = 0$

$\Sigma M_{Base} = \cancel{\frac{1}{2}T \cdot h} - \cancel{\frac{1}{2}T \cdot h} + M = 0$ (h is height of post)

$\Rightarrow$ $N = \sqrt{3}\,T$

(c) If the allowable compressive stress in the post is $\sigma_c = 40$ MPa, and the factor of safety $FS = 2$, determine the maximum tensile force T that can be applied at the cables.

$$\sigma = \frac{N}{A} = \frac{\sqrt{3}\,T}{\pi\left[\left(\frac{130mm}{2}\right)^2 - \left(\frac{100mm}{2}\right)^2\right]} = \frac{\sigma_c}{FS}$$

$$T = \frac{40MPa}{2} \cdot \frac{\pi\left[(65mm)^2 - (50mm)^2\right]}{\sqrt{3}} = 6.258 \times 10^4 N = 62.58\,kN$$

$[MPa \cdot mm^2] = [N]$

(d) Based on the result in (c), if the cable diameter is $d_c = 35$ mm, and the allowable tensile stress in the cable is $\sigma_{max} = 150$ MPa, what is the factor of safety for the cable?

$T = 62.58\ kN$

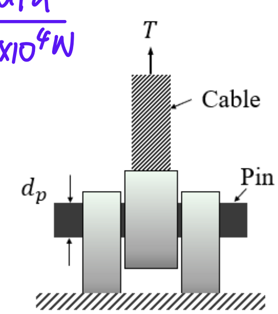
$d_c = 35mm$

$$FS = \frac{\sigma_{max}}{\frac{4T}{\pi \cdot d_c^2}} = \frac{\pi \cdot (35mm)^2 \cdot 150MPa}{4 \times 6.258 \times 10^4 N} = 2.31$$

(e) The diameter of the pins that attach the cable to the ground is $d_p = 20$ mm. Using the result in (c), if the allowable shear stress in the pins is $\tau_{max} = 250$ MPa, what is the factor of safety for the pins?

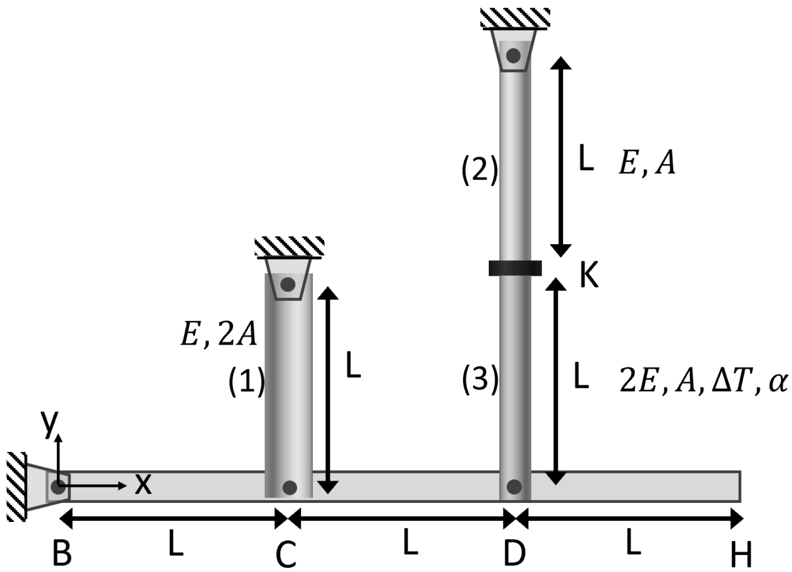
$$FS = \frac{\tau_{max}}{\frac{T}{2\pi \cdot (d_p/2)^2}} = \frac{\pi\, d_p^2\, \tau_{max}}{2T} = \frac{\pi \cdot (20mm)^2 \times 250MPa}{2 \times 6.258 \times 10^4 N}$$

$= 2.51$

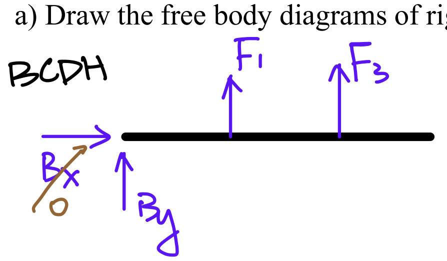


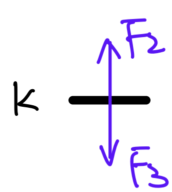
PROBLEM # 2 (25 points)

Axial assembly BCDHK is composed of one rigid member BCDH and three deformable members. The moduli and areas of the different members are described in the diagram below. All members are initially stress-free, but then member (3) is heated by a temperature change of ΔT.



a) Draw the free body diagrams of rigid bar BCDH and rigid connector K.





b) Write the equilibrium equations.

$B_y + F_1 + F_3 = 0$ 1

$L F_1 + 2L F_3 = 0$ 2

$F_2 - F_3 = 0$ 3

c) Is the system determinate or indeterminate?

Indeterminate (3 Equilibrium Eqs. 4 unknowns. B_y. F_1, F_2, F_3)

d) Solve for the forces in all of the members in terms of $\alpha, \Delta T, E, and\ A$.

From Equilibrium Eq.

③ → $F_2 = F_3$

② → $F_1 = -2F_3$

① → $B_y = -F_1 - F_3 = 2F_3 - F_3 = F_3$

Stress Strain Disp. Eq.

$$e_1 = \frac{F_1 L}{2EA} = \frac{-F_3 L}{EA}$$

$$e_2 = \frac{F_2 L}{EA} = \frac{F_3 L}{EA}$$

$$e_3 = \frac{F_3 L}{2EA} + L\alpha\Delta T$$

Compatibility Eq.

$\delta_D = 2\delta_C$ (from BCDH).

$\delta_D = e_2 + e_3$.

$\delta_C = e_1$

$$-\frac{2F_3 \cancel{L}}{EA} = \frac{F_3 \cancel{L}}{EA} + \frac{F_3 \cancel{L}}{2EA} + \cancel{L}\alpha\Delta T$$

$$-2F_3 = F_3 + \frac{1}{2}F_3 + EA\alpha\Delta T$$

$$-\frac{7}{2}F_3 = EA\alpha\Delta T$$

$$F_3 = -\frac{2}{7}EA\alpha\Delta T$$

$$F_2 = -\frac{2}{7}EA\alpha\Delta T$$

$$F_1 = \frac{4}{7}EA\alpha\Delta T$$

(2) (3) compression

(1) tension.

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e) Solve for the vertical displacement of node H.

$$\delta_H = 3\delta_C = 3e_1 = \frac{3 \cdot \cancel{\frac{4}{7}}^{2} \cancel{EA} \alpha \Delta T L}{\cancel{2}\cancel{EA}} = \frac{6}{7} \alpha \Delta T L \quad \text{(downwards)}$$

f) Solve for the vertical displacement of node K.

$$\delta_K = e_2 = \frac{-\frac{2}{7} EA \alpha \Delta T L}{EA} = -\frac{2}{7} \alpha \Delta T L \qquad \text{(upwards)}$$

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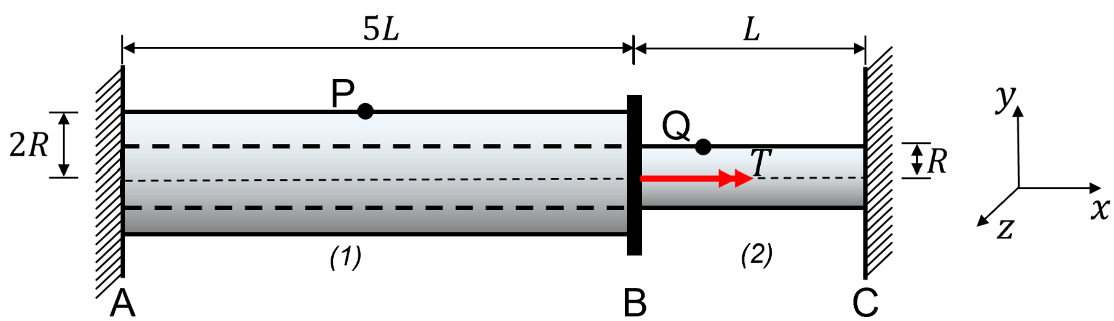
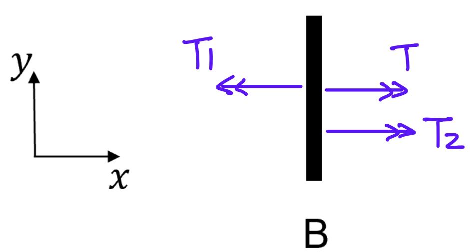
PROBLEM # 3 (25 points)

As shown in the figure below, a two-segment structure is fixed at the two ends, A and C and is connected by rigid plate B. Member (1) between A and B is tubular with inner radius of R and outer radius of $2R$ and has a length of $5L$. Member (2) between B and C is a solid cylinder with radius of R and length of L. Torque T is applied to the rigid plate B. The material for the member (1) has a shear modulus of G_1 and the material for member (2) has an unknown shear modulus of G_2. Point P and Q are located at the top of member (1) and (2).



a. In the provided shape below, draw the Free Body Diagram of rigid plate B. Use T_1 and T_2 to express the torque carried by member (1) and (2).



b. Based on the F.B.D. above, write down Equilibrium Equation(s).

$$-T_1 + T + T_2 = 0$$

c. List the Compatibility Equation(s) based on the set up. Write the expression(s), at most, in terms of L, R, G_1, G_2, T_1 and T_2.

$$J_1 = \frac{\pi}{2}\left[(2R)^4 - R^4\right] = \frac{15}{2}\pi R^4$$

$$J_2 = \frac{\pi}{2} R^4 = \frac{1}{2}\pi R^4$$

Stress strain Disp Eq.

$$\phi_1 = \frac{T_1(5L)}{J_1 G_1}$$

$$\phi_2 = \frac{T_2(L)}{J_2 G_2}$$

Compatibility Eq.

End to End torsion

$$\phi_1 + \phi_2 = 0$$

$$\frac{\cancel{5}\,T_1 \cancel{L}}{3\,\cancel{\tfrac{15}{2}}\cancel{\pi R^4} G_1} = -\frac{T_2 \cancel{L}}{\cancel{\tfrac{1}{2}}\cancel{\pi R^4} G_2}$$

$$\frac{T_1}{3G_1} = -\frac{T_2}{G_2}$$

d. If it is given that member (1) and (2) carry the same magnitude of torque ($|T_1| = |T_2|$), find the shear modulus of member (2), G_2. Write the expression, at most, in terms of T, L, R and G_1.

Since $|T_1| = |T_2|$, $T_1 = -T_2$

Compatibility Eq $\rightarrow \frac{\cancel{T_1}}{3G_1} = \frac{\cancel{T_1}}{G_2} \Rightarrow G_2 = 3G_1$

e. Based on part e, find the expression of angle of twist at B, ϕ_B. Write the expression, at most, in terms of T, L, R and G_1.

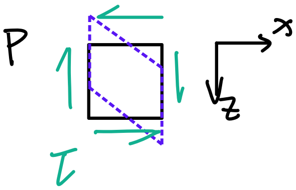
Equilibrium Eq $\Rightarrow -T_1 + T + (-T_1) = 0 \Rightarrow T_1 = \frac{1}{2}T$

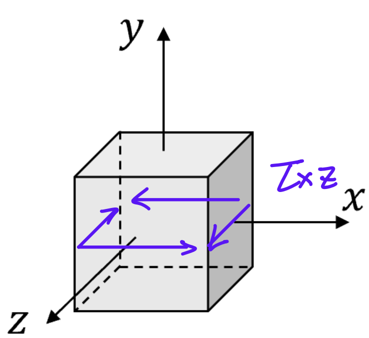
$$\phi_B = \phi_1 = \frac{T_1\,(5L)}{J_1\,G_1} = \frac{\frac{5}{2}TL}{\frac{15}{2}\pi R^4 G_1} = \frac{TL}{3\pi R^4 G_1}$$ (same direction as T)

f. Show the stress states at points P and Q on the given member (1) and (2) in the diagrams below. Please make the stresses shown in the diagram below are under equilibrium and in the correct direction.

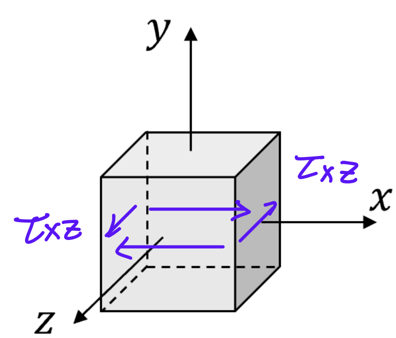
$T_1 > 0$

Top view





P



Q

g. Which direction is the angle of twist at B, ϕ_B, in? (circle one option)

i. ϕ_B is in the same direction as T
ii. ϕ_B is in the opposite direction as T

Extra Space for PROBLEM # 3

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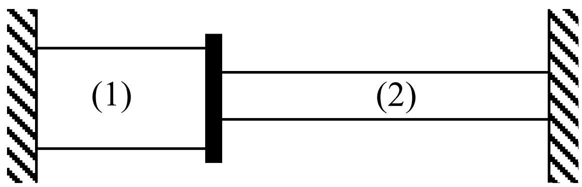
PROBLEM # 4 (25 points – No need to justify your answers, partial credit may not be granted)

PROBLEM # 4 – PART A (3 points)

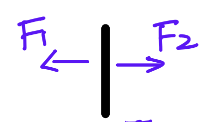
A two-segment structure is fixed at the two ends. The length, Young's modulus, cross section area, and thermal expansion coefficient of the two segments are denoted by L, E, A, and α, respectively. Segment (1) is heated by ΔT, while the temperature of segment (2) is held constant.



L_1, E_1, A_1, α_1 $\quad$ L_2, E_2, A_2, α_2

Circle the correct answer about the internal forces, elongations, and axial strains:

i. $F_1 = F_2, e_1 = e_2, |\varepsilon_1| = |\varepsilon_2|$
ii. $F_1 = F_2, e_1 = -e_2, |\varepsilon_1| > |\varepsilon_2|$
iii. $F_1 = -F_2, e_1 = -e_2, |\varepsilon_1| < |\varepsilon_2|$
iv. $F_1 = -F_2, e_1 = e_2, |\varepsilon_1| = |\varepsilon_2|$
v. None of the above



$-F_1 + F_2 = 0 \Rightarrow F_1 = F_2$

$e_1 + e_2 = 0 \Rightarrow e_1 = -e_2$

$|\epsilon_1| = e_1/L_1$ $\quad$ $L_1 < L_2$

$|\epsilon_2| = e_2/L_2$ $\quad$ $|\epsilon_1| > |\epsilon_2|$

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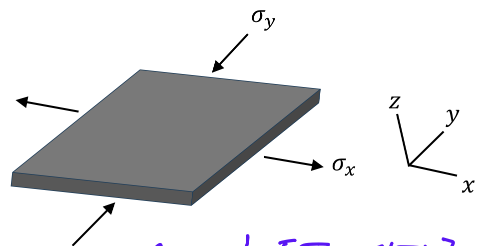
PROBLEM # 4 – PART B (6 points)

(1) A thin plate is subject to the load $\sigma_x = -\sigma_y$ (σ_x is tension and σ_y is compression), while the stress in the z direction is negligible ($\sigma_z = 0$). Such a state is called plane stress state in elasticity. The plate is made of a linear elastic material with Young's modulus $E > 0$, and Poisson's ratio $-1 < v < 0.5$. Circle TRUE or FALSE for the following statements.



$\epsilon_x = \frac{1}{E}[\sigma_x + \nu\sigma_x]$
$= \frac{1}{E}[1+\nu]\sigma_x > 0$

i. **TRUE** or **FALSE**: The strain in the x direction ε_x is positive.

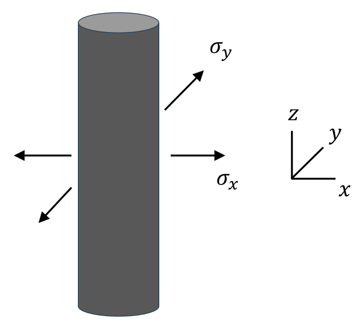
ii. **TRUE** or **FALSE**: The sign of strain in the y direction ε_y is unknown which depends on the specific value of v.

$\epsilon_y = \frac{1}{E}[-\sigma_x - \nu\sigma_x]$
$= \frac{-1}{E}[1+\nu]\sigma_x < 0$

iii. **TRUE** or **FALSE**: The strain in the z direction ε_z is zero.

$\epsilon_z = \frac{1}{E}[-\nu(\cancel{\sigma_x} - \cancel{\sigma_x})] = 0$

(2) A prismatic bar is subject to the load $\sigma_x = \sigma_y > 0$, while its deformation in the z-direction is negligible ($\varepsilon_z = 0$). Such a state is called plane strain state in elasticity. The rod is made of a linear elastic material with Young's modulus $E > 0$, and Poisson's ratio $v = 0.3$. Circle TRUE or FALSE for the following statements.



iv. **TRUE** or **FALSE**: The stress in the z direction σ_z is zero.

v. **TRUE** or **FALSE**: The stress in the z direction σ_z is negative.

vi. **TRUE** or **FALSE**: The strain in the x direction ε_x is positive.

$\sigma_x = \sigma_y > 0, \quad \epsilon_z = 0$

$\cancel{\epsilon_z}^{0} = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \Rightarrow \sigma_z = \nu(\sigma_x + \sigma_y) \Rightarrow \sigma_z = 0.6\sigma_x$

$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$

$= \frac{1}{E}\sigma_x[1 - 0.3(1+0.6)] > 0$

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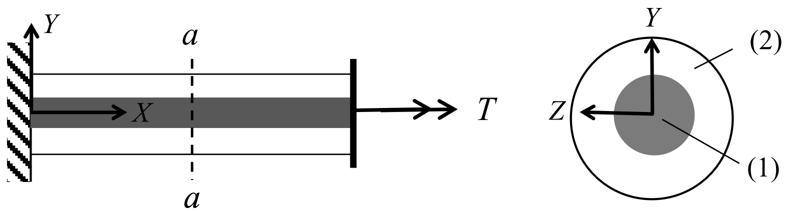
PROBLEM # 4 – PART C (7 points)

A welded bimetallic bar is composed of a core material (1) and a shell material (2), and is subjected to a torque T. The shear moduli of the core and shell are known to be $G_1 = 2G_2$, and polar moment of inertia $I_{P1} = 2\, I_{P2}$.



Cross section

Circle the correct answer for the internal torques of the two materials:

(i) $T_1 > T_2$

(ii) $T_1 < T_2$

(iii) $T_1 = T_2$

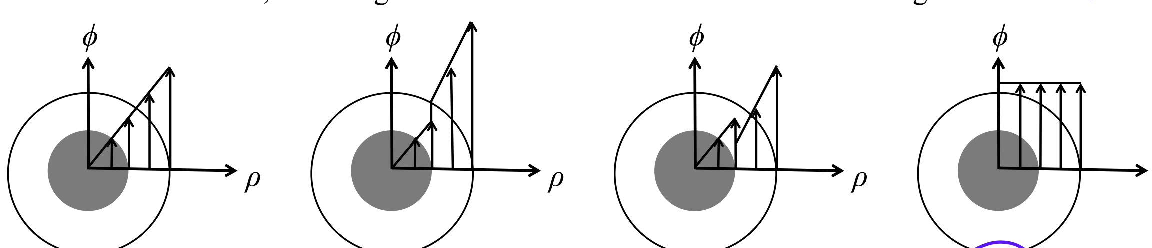
(iv) More information is needed

$$\frac{T_1 L}{I_{p1} G_1} = \frac{T_2 L}{I_{p2} G_2}$$

$$\frac{T_1 \cancel{L}}{4 \cancel{I_{p2}} G_2} = \frac{T_2 \cancel{L}}{\cancel{I_{p2}} G_2} \Rightarrow \frac{T_1}{4} = T_2 \qquad T_1 > T_2$$

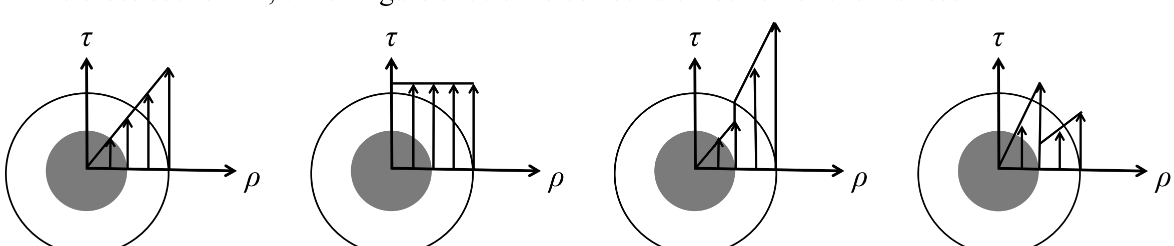
In the cross section *aa*, which figure shows the correct distribution of twist angle?

ϕ is the same in cross-sec area



(i) (ii) (iii) (iv)

In the cross section *aa*, which figure shows the correct distribution of shear stress?



(i) (ii) (iii) (iv)

(1): $\tau = \frac{T_1 c}{2 I_{p2}} = \frac{2 T_2 c}{I_{p2}} = \frac{2 T_2}{I_{p2}} \cdot c$ $\Rightarrow$ slope @ (1) is 2x slope @ (2)

(2): $\tau = \frac{T_2 c}{I_{p2}} = \frac{T_2}{I_{p2}} \cdot c$

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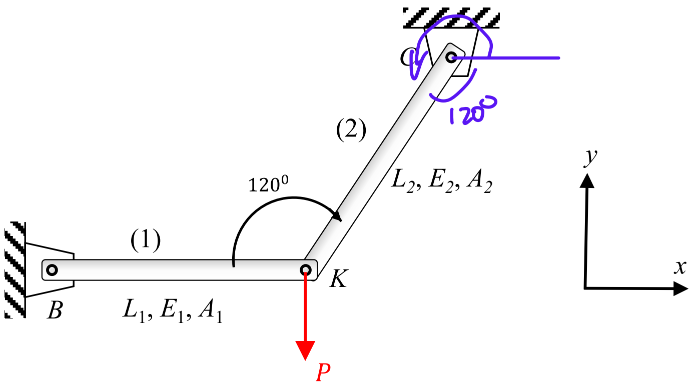
PROBLEM # 4 – PART D (8 points)

The geometry of deformation in planar truss is given by $e = u\cos\theta + v\sin\theta$, where e represents the elongation of a truss member, u and v are the displacement of the joint K in the x and y directions, respectively. The truss is subject to a vertical force P in the negative y direction.



(a) Determine the θ value, in **radian**, for (1) and (2)

$\boldsymbol{\theta_1} =$ 0

$\boldsymbol{\theta_2} =$ $4\pi/3$ rad

$\theta_1 = 0° = 0$ rad

$\theta_2 = 240° = \frac{2}{3} \cdot 2\pi$ rad $= \frac{4}{3}\pi$ rad

(b) The length, Young's modulus, and cross section area of the two members are denoted by L, E, A, respectively, as shown in figure. Assuming Young's modulus E_1 increases by 50% while other parameters remain the same, how will that change the internal reaction force in (1)? Circle the correct answer.

(i) F_1 becomes larger

(ii) F_1 becomes smaller

(iii) F_1 remains the same

Statically determinate.

$\Rightarrow$ FBD calculate F_1 F_2.

(c) Assuming the temperature of (1) increases by 50°, while the temperature of (2) is held constant, and other parameters remain the same. How will that change the elongation of (1)? Circle the correct answer.

(i) e_1 becomes larger

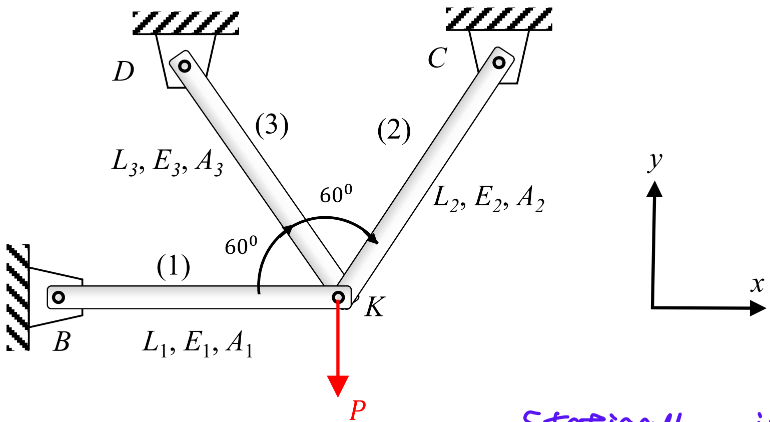
(ii) e_1 becomes smaller

(iii) e_1 remains the same

(iv) More information is needed

$\Delta T > 0 \Rightarrow e_1 \uparrow$

(d) Another truss member (3) is added into the structure as shown below. The length, Young's modulus, and cross section area of the three members are denoted by L, E, A, respectively. In this configuration, how does the internal reaction force in (1) depend on E_1 and A_1?



(i) F_1 depends on both E_1 and A_1

(ii) F_1 is independent of E_1 and A_1

(iii) F_1 depends on E_1 but is independent of A_1

(iv) More information is needed

Statically indeterminate.

need compatibility Eq. (E_1, A_1) for F_1.

Useful Equations

$$\sigma_{avge} = \frac{F_N}{A} \qquad FS = \frac{\sigma_{fail}}{\sigma_{allow,member}}$$

$$\tau_{avge} = \frac{V}{A} \qquad FS = \frac{\tau_{fail}}{\tau_{allow,member}}$$

Generalized Hooke's law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \qquad G = \frac{E}{2(1+\nu)}$$

Axial deformation:

$$e_{AB} = u_B - u_A \qquad e = \int_0^L \frac{F}{A\,E}\,dx + \int_0^L \alpha\,\Delta T\ \,dx, \qquad e = \frac{F\,L}{A\,E} + \alpha\,\Delta T\,L$$

$$e = u\cos(\theta) + v\sin(\theta)$$

Torsional deformation:

$$\phi_{AB} = \phi_B - \phi_A \qquad \phi_{AB} = \int_0^L \frac{T_{AB}(x)}{G(x)\,I_p(x)}\,dx \qquad \phi_{AB} = \frac{T_{AB}L}{G\,I_P}$$

$$\gamma = \rho\frac{d\phi}{dx} \qquad \tau = G\rho\frac{d\phi}{dx} \qquad \gamma = \frac{\rho T}{G\,I_p} \qquad \tau = \frac{\rho T}{I_p}$$

with $I_p = \int_A \rho^2\,dA,$ $\quad I_p = \dfrac{\pi r^4}{2}$ (solid), $\quad I_p = \dfrac{\pi}{2}\left(r_0^4 - r_i^4\right)$ (hollow)