ME 323: Mechanics of Materials **Homework 1**

Spring 2026 **Due: January 23, 2026**

Problem 1 (10 points):

While walking through the streets of Osaka, you come across a unique design of a streetlamp (*figure 1*). In this design, a streetlamp of weight W is suspended from the post OABC. A load 4P is also applied at point C in the X direction.

(a) Calculate the reaction forces and moments developed at points O and D of the structure.
(b) In words, suggest how the reaction forces vary as you move along the length of the post i.e. from O to C.

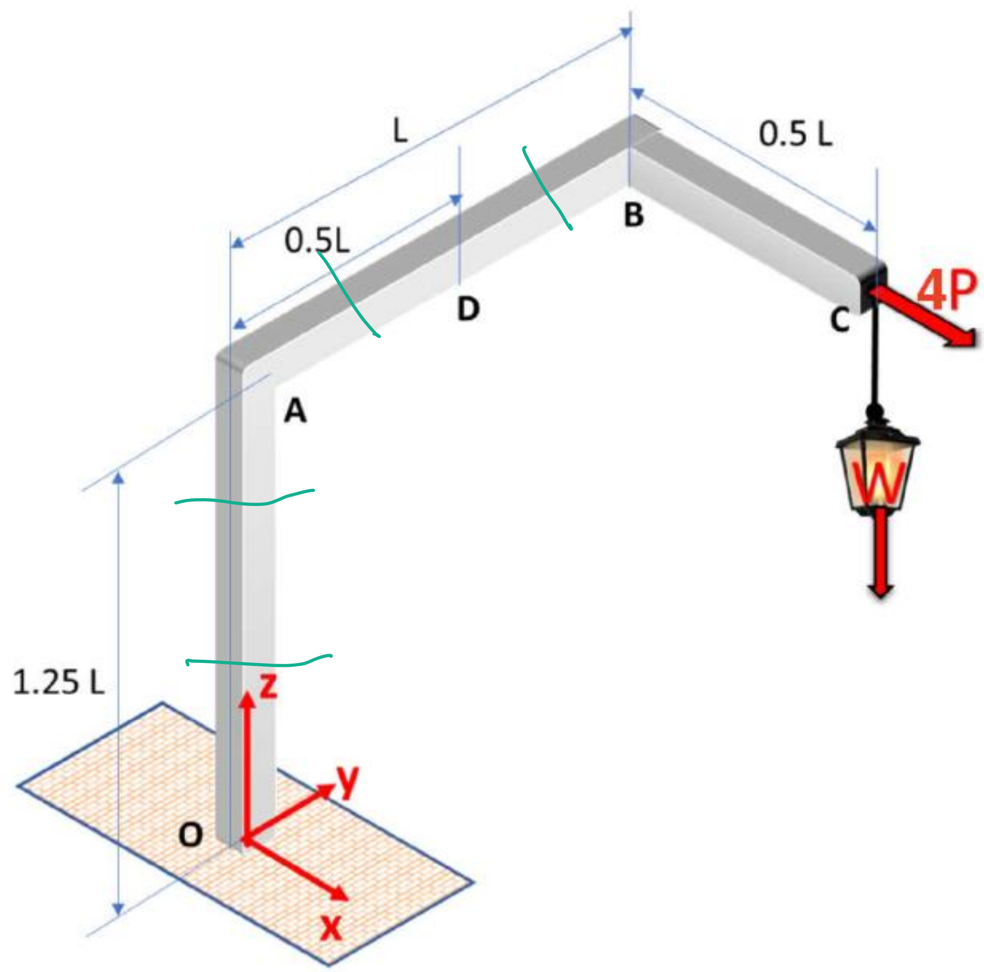


Figure 1: Streetlamp design for Problem 1

Given: L, P, W

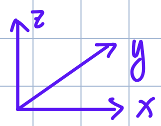
(20 min, mosty Math)

Find: (a) Reaction @ O, D

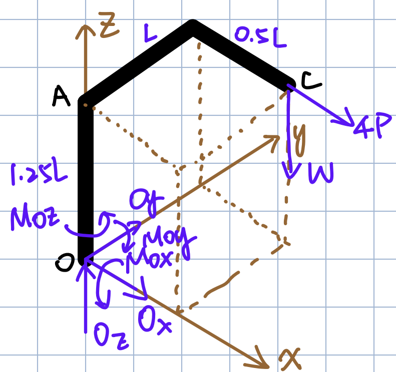
(b) how reaction vary O → C.

Solution:

1. Coor



2. FBD.



3. Equilibrium Eq

ΣF_x: $O_x + 4P = 0$ ①

ΣF_y: $O_y = 0$ ②

ΣF_y: $O_z - W = 0$ ③

$\Sigma \vec{M}_O$: $(M_{Ox}\hat{i} + M_{Oy}\hat{j} + M_{Oz}\hat{k})$ ④–⑥

$+ (0.5L\hat{i} + L\hat{j} + 1.25L\hat{k}) \times (4P\hat{i} - W\hat{k}) = 0$

unknown: $O_x, O_y, O_z, M_{Ox}, M_{Oy}, M_{Oz}$.

4. Solve:

$O_x = -4P$, $O_y = 0$. $O_z = W$

$M_{Ox}\hat{i} + M_{Oy}\hat{j} + M_{Oz}\hat{k}$

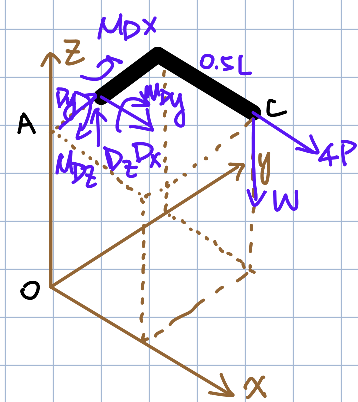
$+ 0.5WL\hat{j} - 4PL\hat{k} - WL\hat{i} + 5PL\hat{j} = 0$

$M_{Ox} = WL$, $M_{Oy} = -5PL - 0.5WL$

$M_{Oz} = 4PL$

$\vec{F}_O = -4P\hat{i} + W\hat{k}$

$\vec{M}_O = WL\hat{i} - (5PL + 0.5WL)\hat{j} + 4PL\hat{k}$



3. Equilibrium Eq.

ΣF_x: $D_x + 4P = 0$ ①

ΣF_y: $D_y = 0$ ②

ΣF_z: $D_z - W = 0$ ③

$\Sigma \vec{M}_D$: $(M_{Dx}\hat{i} + M_{Dy}\hat{j} + M_{Dz}\hat{k})$ ④–⑥

$+ (0.5L\hat{i} + 0.5L\hat{j}) \times (4P\hat{i} - W\hat{k}) = 0$

unknown: $D_x, D_y, D_z, M_{Dx}, M_{Dy}, M_{Dz}$.

4. Solve.

$D_x = -4P$, $D_y = 0$. $D_z = W$

$\vec{M}_D + 0.5WL\hat{j} - 2PL\hat{k} - 0.5WL\hat{i} = 0$

$M_{Dx} = 0.5WL$, $M_{Dy} = -0.5WL$

$M_{Dz} = 2PL$

$\vec{F}_D = -4P\hat{i} + W\hat{k}$

$\vec{M}_D = 0.5WL\hat{i} - 0.5WL\hat{j} + 2PL\hat{k}$

Reaction force stay the same from base O to end C.

Problem 2 (10 points):

Your toddler nephew glues together two wooden blocks of identical square cross section (10 mm x 10 mm) along the plane AC as shown in *figure 2*. The dimension of the entire setup is 10 mm x 10 mm x 100 mm.

a) If the glue can withstand a maximum shear stress of 10 kPa, calculate the maximum force P that can be applied at the end of the wooden blocks so that the blocks stay together (or the minimum force to take them apart).
b) If the glue can withstand a maximum normal stress of 5 kPa and shear stress of 3.5 kPa. What is the maximum load that can be applied?

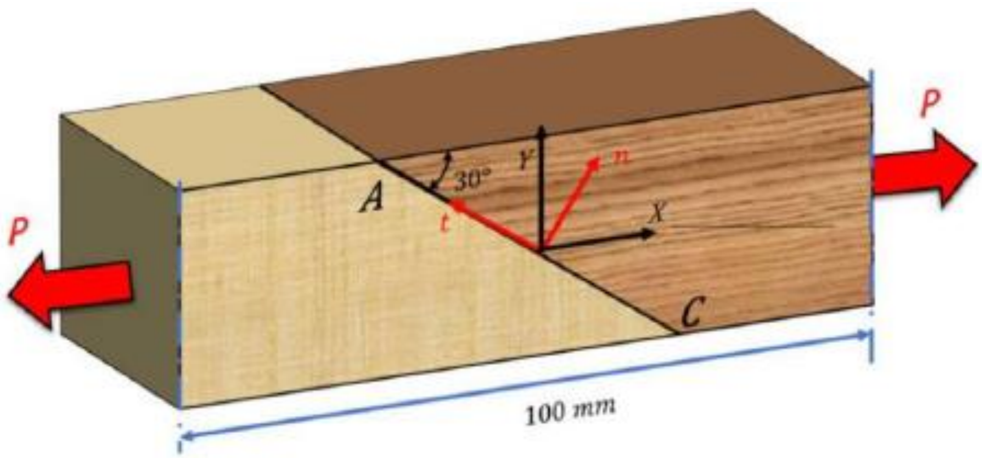


Figure 2: Structure for Problem 2

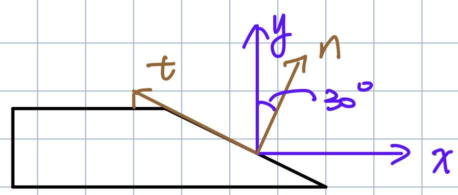
Given: Block 10mm x 10mm x 100 (20min, Vector translation calculation).

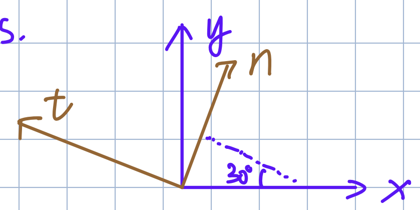
$\tau_{max} = 10\,kPa$

$\sigma_{max} = 5\,kPa$, $\tau_{max} = 3.5\,kPa$

Find: P_{max} based on two conditions.

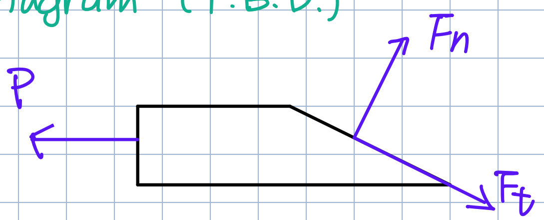
1. coor.





$\hat{i} = \frac{1}{2}\hat{e}_n - \frac{\sqrt{3}}{2}\hat{e}_t$

2. Diagram (F.B.D.)



3. Equilibrium.

* Easier to translate $\vec{P}$ into n and t direction than translate both $\vec{F_n}$ and $\vec{F_t}$ into x, y direction.

$-P\hat{i} = -P(\frac{1}{2}\hat{e}_n - \frac{\sqrt{3}}{2}\hat{e}_t)$

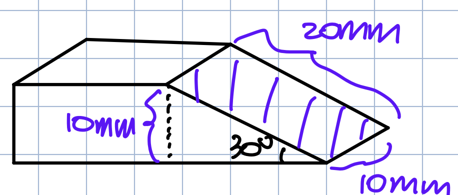
$\vec{P} = -\frac{1}{2}P\hat{e}_n + \frac{\sqrt{3}}{2}P\hat{e}_t$

$\Sigma F_n: \quad F_n - \frac{1}{2}P = 0$ ①

$\Sigma F_t: -F_t + \frac{\sqrt{3}}{2}P = 0$ ②

4. Stress.

* Need to find Area of glue.



$A = 10mm \times 20mm = 200\,mm^2 = 2\times10^{-4}\,m^2$

(a) $\tau = \frac{|F_t|}{A} = \frac{\sqrt{3}P}{2A} \Rightarrow P_{max} = \tau_{max}\cdot A \cdot \frac{2}{\sqrt{3}} = 10^4 Pa \times 2\times10^{-4}m^2 \cdot \frac{2}{\sqrt{3}} = 2.31\,N$

(b)

$P_{\tau max} = 3.5\times10^3 Pa \times 2\times10^{-4} m^2 \times \frac{2}{\sqrt{3}} = 0.81\,N$

$\sigma = \frac{|F_n|}{A} = \frac{1P}{2A} \Rightarrow P_{\sigma max} = \sigma_{max}\cdot A \cdot 2 = 5\times10^3 Pa \times 2\times10^{-4} m^2 \times 2 = 2N$

For (b), limit based on τ is lower $\Rightarrow P_{max} = 0.81N$

Problem 3 (10 points):

The truss shown below (*figure 3*) is supported by pin joints at B and D. Each member of the truss is made of the same material but has different cross-sectional areas: Member (1) - A, Member (2) - 2A, Member (3) - 2A, and Member (4) - A. If a force P acts downward at H,

(a) Determine the load carried by each member of the truss.

(b) Determine the stress developed in each member of the truss. State whether it is in tension or compression.

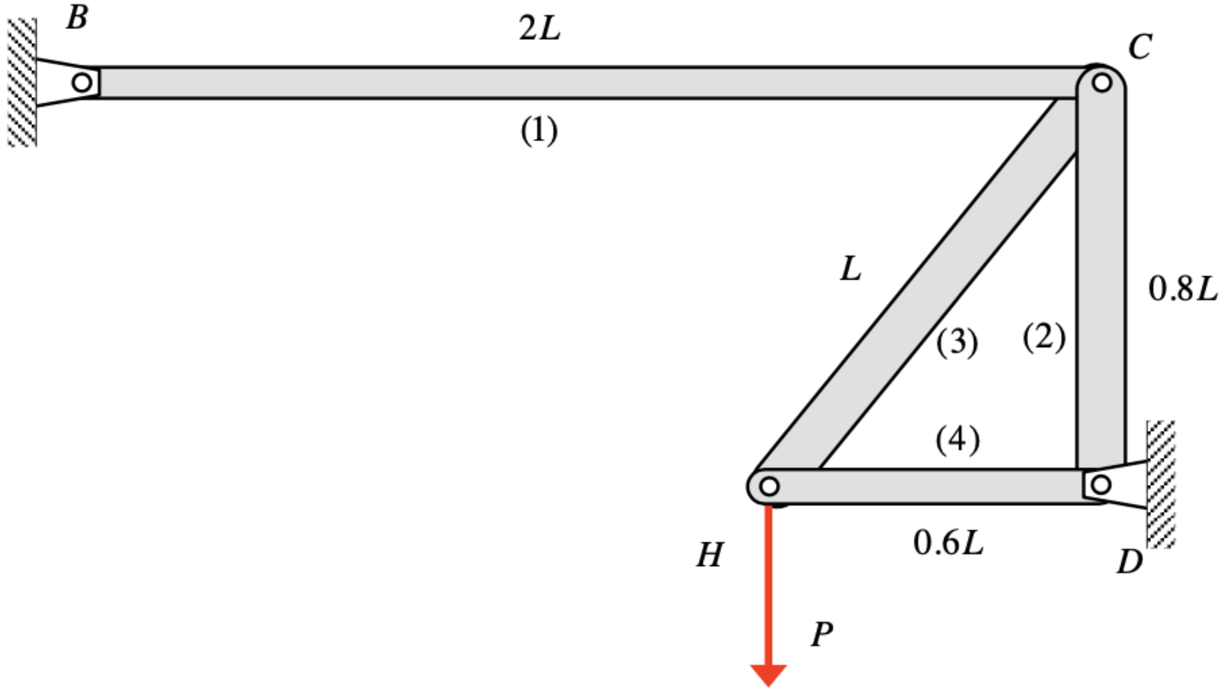


Figure 3: Truss for Problem 3

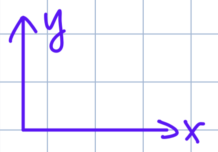
Given: dimension, load P (15 min. FBD. math).

Member	loc.	len.	Area
(1)	BC	2L	A
(2)	CD	0.8L	2A
(3)	CH	L	2A
(4)	DH	0.6L	A

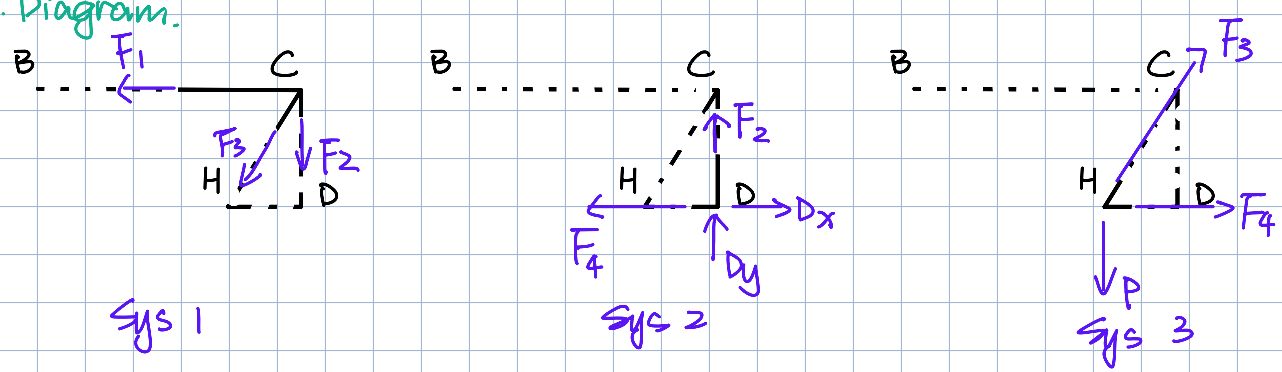
Find: load of each member.
stress of each member.
tension / compression.

Solution:

1. Coor.



2. Diagram.



3. Equilibrium Eq.

Sys 1 $\Sigma F_x: -F_1 - 0.6F_3 = 0$ ①
$\Sigma F_y: -F_2 - 0.8F_3 = 0$ ②

Sys 2: $\Sigma F_x: -F_4 + D_x = 0$
$\Sigma F_y: F_2 + D_y = 0$

Sys 3: $\Sigma F_x: 0.6F_3 + F_4 = 0$ ③
$\Sigma F_y: -P + 0.8F_3 = 0$ ④

4. Solve

④ → $F_3 = 1.25P$
③ → $F_4 = -0.75P$
① → $F_1 = -0.75P$
② → $F_2 = -P$

5. Stress:

(1) $\sigma_1 = -0.75\,P/A$, comp.
(2) $\sigma_2 = -0.5\,P/A$, comp.
(3) $\sigma_3 = 0.625\,P/A$, tens
(4) $\sigma_4 = -0.75\,P/A$, comp.

5 min (set up sys. of 2 Eq. w/ 2 unknowns).

Problem 4 (2.5 points + 2.5 points):

A component is known to be in a state of plane stress ($\sigma_z = 0$). The folowing values are also known for stresses and strains:

$\sigma_x = 20 \times 10^6$ N/m^2

$\sigma_y = 120 \times 10^6$ N/m^2

$\varepsilon_x = -1 \times 10^{-3}$

$\varepsilon_y = 6 \times 10^{-3}$

4.1 The numerical value for Poisson's ratio of the material is:

(a) $\nu = -0.2956$

(b) $\nu = 0.1853$

(c) $\nu = 0.5722$

(d) $\nu = 0.3243$

4.2 The numerical value for Young's Modulus of the material is:

(a) $E = 1.89 \times 10^{10}$ Pa

(b) $E = 2.39 \times 10^{15}$ Pa

(c) $E = 5.45 \times 10^{8}$ Pa

(d) $E = 6.07 \times 10^{5}$ Pa

$\sigma_z = 0$ (plane stress)

$\sigma_x = 20\times10^6$ Pa. σ_y: 120×10^6 Pa. $\varepsilon_x = -10^{-3}$, $\varepsilon_y = 6\times10^{-3}$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \cancel{\sigma_z}^{\,0})] \quad ①$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \cancel{\sigma_z}^{\,0})] \quad ②$$

$$\Rightarrow \begin{cases} -10^{-3}E = 20\times10^6 - \nu\cdot120\times10^6 \\ 6\times10^{-3}E = 120\times10^6 - \nu\cdot20\times10^6 \end{cases}$$

$$\Rightarrow \begin{cases} E = 120\times10^9\nu - 20\times10^9 \\ E = 20\times10^9 - \frac{10}{3}\times10^9\nu \end{cases}$$

$$120\nu - 20 = 20 - \frac{1}{3}\nu \Rightarrow \left(120 + \frac{10}{3}\right)\nu = 40 \qquad \nu = 0.3243$$

$$E = 10^9(0.3243\times120 - 20)$$

$$= 1.892\times10^{10}\text{Pa}$$