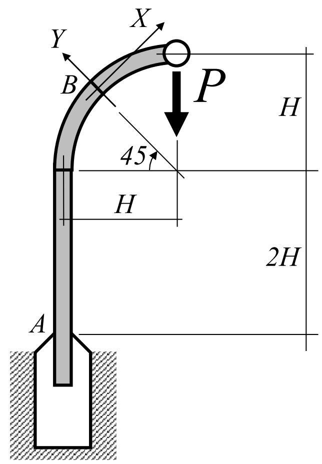
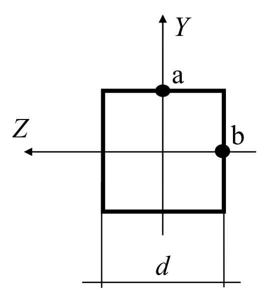
ME 323: Mechanics of Materials
Fall 2025

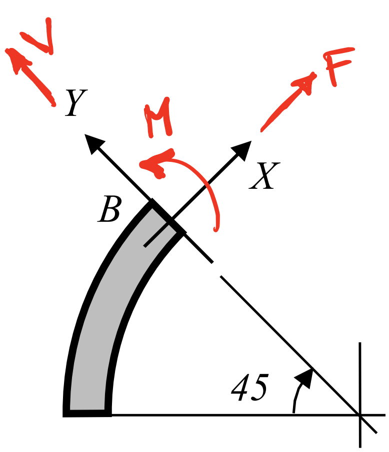
Problem ## (25 points)

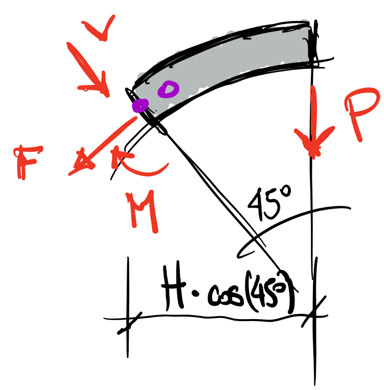
A small boat is suspended by a davit-like lifting system. The boat hangs from two curved davits (arched metal arms), one on each side. The davits support the boat through vertical hanging cables ending in ring-shaped connectors. The load on each cable is equal to *P*. The davits are fixed to a platform at *A* and have a solid squared cross-section of side *d*.





a) Determine the internal reactions at the cut labeled *B*. Classify the reaction forces as either axial, or shear forces, and the reaction moments as either bending moments, or torsion.





$\oplus \nearrow \; \sum F_x = 0 = -F - \frac{\sqrt{2}}{2}P$

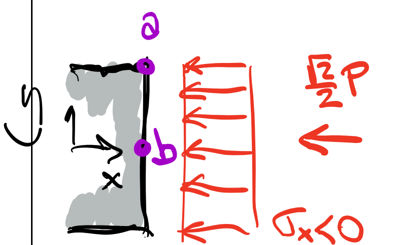
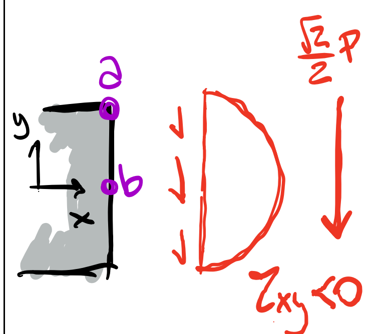
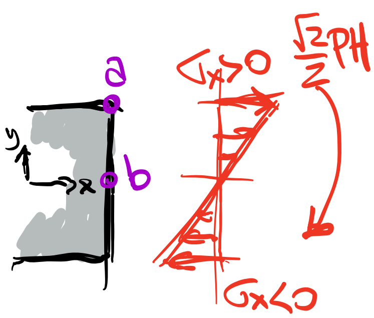
$\Rightarrow F = -\frac{\sqrt{2}}{2}P$

$\oplus \nwarrow \; \sum F_y = 0 = -V - \frac{\sqrt{2}}{2}P$

$\Rightarrow V = -\frac{\sqrt{2}}{2}P$

$\overset{\oplus}{\circlearrowleft} \; \left(\sum M\right)_o = 0 = M + P \cdot \frac{\sqrt{2}}{2}H \Rightarrow M = -\frac{\sqrt{2}}{2}PH$

b) Determine and list the stresses induced at locations "*a*", and "*b*" on the cross section at B due to each reaction calculated in a).
Leave your answer in terms of *P*, *H*, and *d*.

Internal resultant	"*a*"	"*b*"
	$\sigma_x = -\frac{\sqrt{2}}{2}P \cdot \frac{1}{d^2}$	$\sigma_x = -\frac{\sqrt{2}}{2}P \cdot \frac{1}{d^2}$
	$\tau_{xy} = 0$	$\tau_{xy} = -\frac{3}{2}\frac{\sqrt{2}}{2}P \cdot \frac{1}{d^2}$
	$\sigma_x = \frac{\sqrt{2}}{2}PH \cdot \frac{d/2}{d^4/12}$	$\sigma_x = 0$

@A: $\sigma_x = -\frac{\sqrt{2}}{2}P\frac{1}{d^2} + \frac{\sqrt{2}}{2}P\frac{6H}{d^3} = \frac{\sqrt{2}}{2}\frac{P}{d^2}\left[-1 + 6H/d\right] > 0$

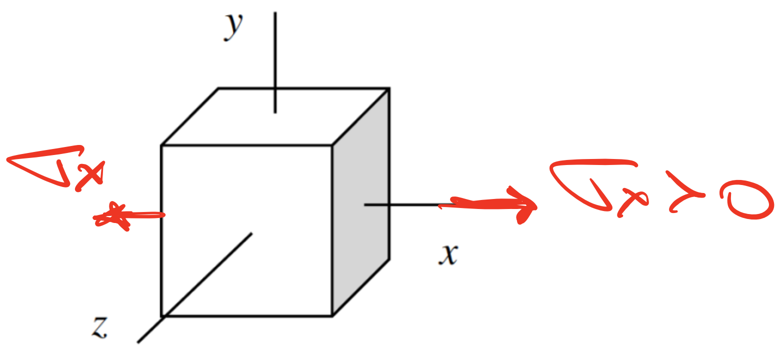
$\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0$

@B: $\sigma_x = -\frac{\sqrt{2}}{2}\frac{P}{d^2}$; $\tau_{xy} = -\frac{\sqrt{2}}{2}\frac{P}{d^2}\cdot\frac{3}{2}$; $\sigma_y = \sigma_z = \tau_{yz} = \tau_{xz} = 0$

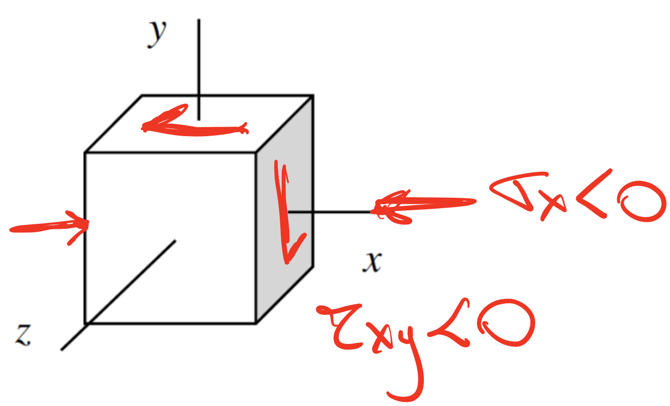
ME 323: Mechanics of Materials
Fall 2025

c) Draw separate stress elements for "*a*" and "*b*", and indicate both direction, and magnitude of the stresses induced.

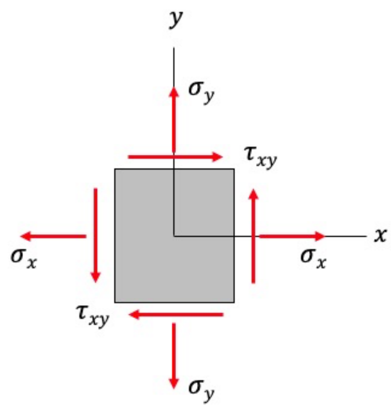
stress element at '*a*'

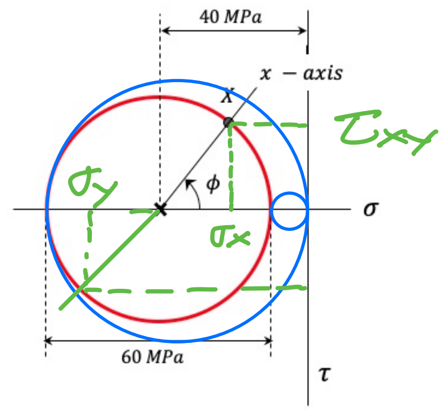


stress element at '*b*'



PROBLEM NO. 2 – 25 points max.





The xy-components of a state of plane stress are shown in the figure below left. The in-plane Mohr's circle for this state of stress is shown in the figure below right, where $\phi = 53.13°$.

For this problem, you are asked to do the following:

a) Determine the principal components of stress (σ_{P1} *and* σ_{P2}) and the maximum in-plane shear stress.

From figure:

$\sigma_{ave} = -40 MPa$

$R = \frac{60}{2} = 30 MPa$

$\therefore \sigma_{P1} = \sigma_{ave} + R = -10 MPa$

$\sigma_{P2} = \sigma_{ave} - R = -70 MPa$

b) Draw the two out-of-plane Mohr's circles on the Mohr's circle figure provided above.

c) Determine the absolute maximum shear stress for this state of stress.

$\tau_{max,abs}$ = radius of the largest Mohr's circle

$= \frac{40+30}{2} = 35 MPa$

d) Determine numerical values for the xy stress components of σ_x, σ_y and τ_{xy}.

From figure:

$\sigma_x = \sigma_{ave} + R\cos\phi = -40 + 30(0.6) = 22\,MPa$

$\sigma_y = \sigma_{ave} - R\cos\phi = -40 - (30)(0.6) = -58\,MPa$

$\tau_{xy} = -R\sin\phi = -(30)(0.8) = -24\,MPa$

e) What is the smallest counterclockwise angle through which the stress element must be rotated in order to expose the principal stress σ_{P1} and the principal stress σ_{P2}?

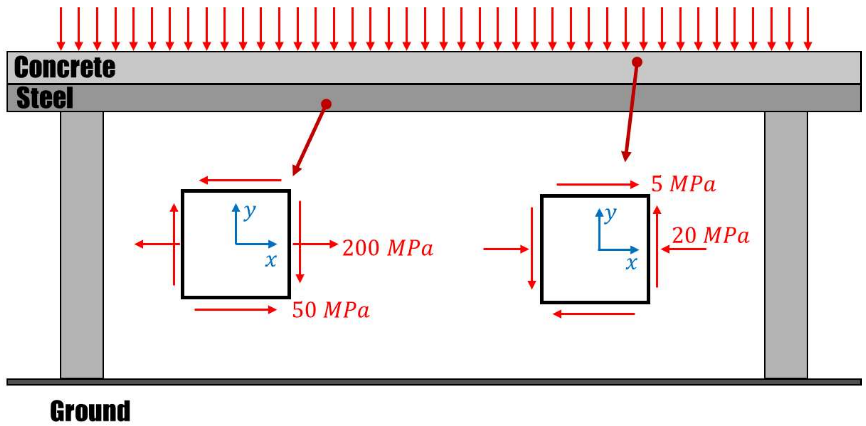
$2\theta_{P1} = 360° - \phi = 360 - 53.13° = 306.87°$

$\hookrightarrow \theta_{P1} = \frac{306.87°}{2} = 153.44°$

$$2\theta_{P2} = 180° - \phi = 180° - 53.13° = 126.87° \quad \Rightarrow \quad \theta_{P2} = 63.44°$$

PROBLEM NO. 3 – 25 points max.

A section of a steel-reinforced concrete bridge subjected to a distributed load is analyzed at two locations: one in the concrete portion and the other in the A36 steel section. For this analysis, plane-stress elements are assumed for both regions, based on the simplifying assumption that the bridge section behaves as a slender beam and that the concrete and steel exhibit comparable Poisson's ratios. The material properties are as follows: A36 steel has a yield strength of $250\ MPa$, while the concrete has an ultimate compressive strength of $30\ MPa$ and an ultimate tensile strength of $5\ MPa$.



a) Determine if the A36 steel section will fail according to the *Maximum Shear Stress Theory*.

(6 Pts)

$$\sigma_{Avg} = \frac{200\ MPa}{2} = 100\ MPa$$

$$R = \sqrt{\left(\frac{200}{2}\right)^2 + (50)^2} = 111.8\ MPa$$

$$\sigma_{P_1} = \sigma_{Avg} + R = 211.8\ MPa$$

$$\sigma_{P_2} = \sigma_{Avg} - R = -11.8\ MPa$$

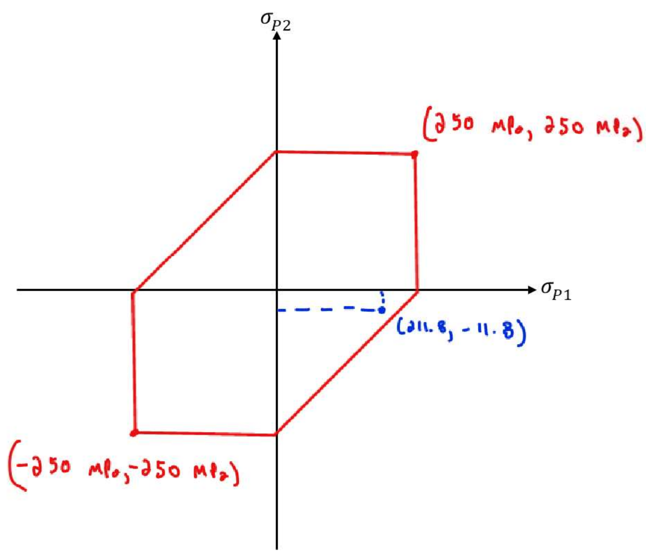
$$\tau_{max}^{Abs} = 111.8\ MPa < \frac{\sigma_Y}{2} = \frac{250\ MPa}{2} = 125\ MPa$$

$$111.8\ MPa < 125\ MPa$$

won't fail according to MSS.

b) Draw the failure boundary for the maximum shear stress theory. Indicate the values at the vertices of the failure boundary and indicate with a dot the state of stress in the A36 steel section.

(4 Pts)



c) Determine the Factor of Safety for the A36 steel section.

$$F.S = \frac{\frac{\sigma_y}{2}}{\tau^{max}_{abs}} = \frac{125}{111.8} = 1.118$$

(2 Pts)

d) Determine if the concrete section will fail according to the *Mohr's* failure criteria.

(6 Pts)

$$\sigma_{Avg} = \frac{-20}{2} = -10 \text{ MPa}$$

$$R = \sqrt{\left(\frac{-20}{2}\right)^2 + (5)^2} = 11.18$$

$$\sigma_{P_1} = \sigma_{Avg} + R = 1.18 \text{ MPa}$$

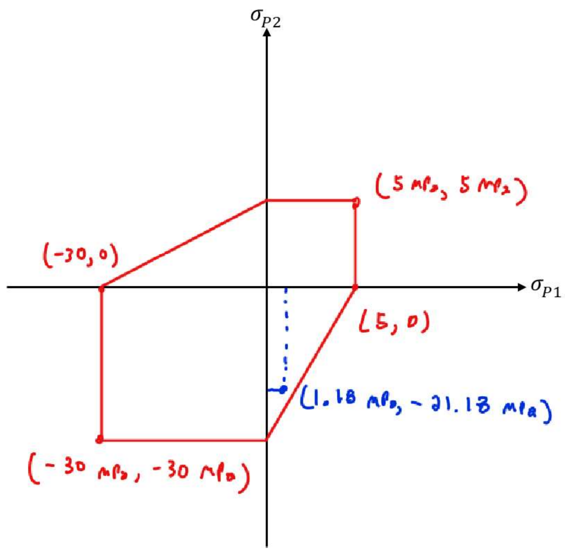
$$\sigma_{P_2} = \sigma_{Avg} - R = -21.18 \text{ MPa}$$

$$\frac{\sigma_{P_1}}{\sigma_{UT}} - \frac{\sigma_{P_2}}{\sigma_{UC}} = \frac{1.18}{5} - \frac{(-21.18)}{30} = 0.236 + 0.706 = 0.942 < 1$$

won't fail according to Mohr's failure criteria!

e) Draw the failure boundary for the Mohr's failure criteria. Indicate the values at the vertices of the failure boundary and indicate with a dot the state of stress in the concrete section.

(4 Pts.)



f) Determine the Factor of Safety for the concrete section (use Mohr's failure criteria).

(2 Pts)

$$F.S. = \frac{1}{\frac{\sigma_{P1}}{\sigma_{UT}} - \frac{\sigma_{P2}}{\sigma_{UC}}} = \frac{1}{0.942} = 1.062$$

g) Indicate what is the most critical region of the bridge (Steel section or Concrete section).

(1 Pto)

F.S. concrete = 1.062 < F.S. steel = 1.118

concrete is the most critical

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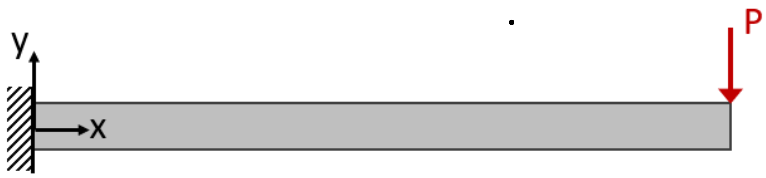
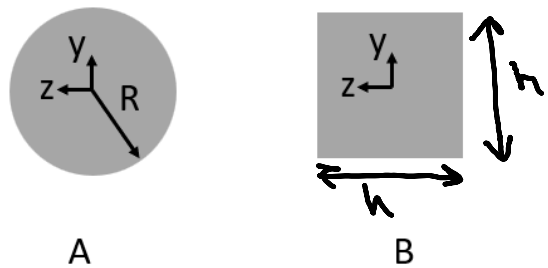
November 5, 2025 *(Last)* *(First)*

PROBLEM NO. 4
Part 4A (4 points)

Two cantilevers have the same length L and have a transverse load applied at the end. The two beams have the same cross-sectional area A, but one has a circular cross-section and the other has a square cross-section.





4A.i Which beam has a larger maximum shear stress?

$\tau_{max,A} > \tau_{max,B}$

$\tau_{max,A} = \tau_{max,B}$

$\boxed{\tau_{max,A} < \tau_{max,B}}$

$$\tau_{max,circle} = \frac{4V}{3A} = \tau_{max,A}$$

$$\tau_{max,rectangle} = \frac{3V}{2A} = \tau_{max,B}$$

4A.ii Which beam has a larger maximum flexural stress?

$\boxed{\sigma_{max,A} > \sigma_{max,B}}$

$\sigma_{max,A} = \sigma_{max,B}$

$\sigma_{max,A} < \sigma_{max,B}$

$$\pi R^2 = h^2$$

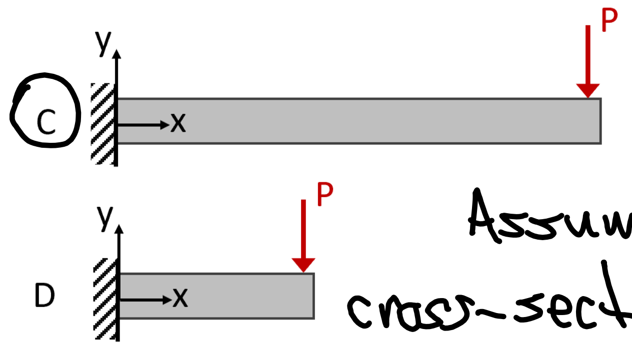
$$h = \sqrt{\pi}\, R$$

$$\sigma_{max,A} = \frac{|M|R}{I_p} = \frac{4|M|}{\pi R^3} = 1.27\frac{|M|}{R^3}$$

$$\sigma_{max,B} = \frac{|M|\left(\frac{h}{2}\right)}{\frac{h^4}{12}} = \frac{6|M|}{h^3} = \frac{6|M|}{\pi^{3/2}R^3} = 1.08\frac{|M|}{R^3}$$

Part 4B (1 point)

Two cantilevers have a transverse load applied at the end. Both have the same square cross-sections, but they have different lengths. For which beam is the equation for shear stress ($\tau = VA^*\bar{y}^*/It$) the most accurate? (circle one)



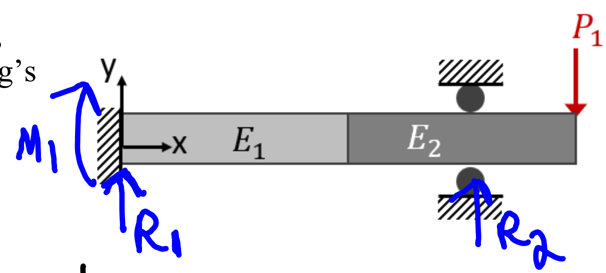
Assumption of plane cross-sections is most accurate for long thin beams.

Part 4C (4 points)

A beam is made of two materials that have different Young's moduli E_1 and E_2.

4C.i When a transverse load is applied to the beam (P_1), the external reaction(s) depend on the value of the Young's modulus E_2.

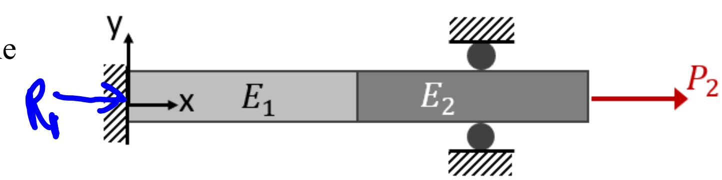
$\boxed{\text{TRUE}}$ FALSE



3 unknowns
2 eqns
Indeterminate.

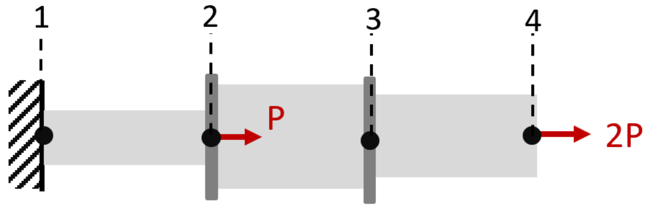
4C.ii When an axial load is applied to the beam (P_2), the external reaction(s) depend on the value of the Young's modulus E_2.

TRUE $\boxed{\text{FALSE}}$



1 unknown
1 equns
Determinate.

Part 4D (6 points)
The axial system below will be analyzed using the finite element method using 4 nodes.



4D.i Fill in the missing terms of the stiffness matrix prior to the enforcement of the boundary conditions:

$$[K] = \frac{EA}{3L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 4 & -3 & 0 \\ 0 & -3 & 5 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \quad \begin{matrix} k_1 = 1 \\ k_2 = 3 \\ k_3 = 2 \end{matrix}$$

4D.ii Fill in the force vector prior to the enforcement of the boundary conditions:

$$\{F\} = \begin{bmatrix} R_1 \\ P \\ 0 \\ 2P \end{bmatrix}$$

R_1 is a reaction (e.g. F_x, P_A can also be used).

4.D.iii If the number of nodes was increased to 8, the accuracy of the finite element calculations would improve (circle one).

TRUE $\boxed{\text{FALSE}}$

Assumption behind FEM is $k = \frac{(EA)_{avg}}{L}$. When E and A are uniform within the elements, the solution is already exact and will not improve by adding more nodes.

Part 4E (4 points)

A column is long and slender so that it undergoes Euler buckling. The column is pinned on both ends and has a diameter of D, a length of L, and a Young's modulus of E.

4E.i If the column is fixed on both sides, how does the critical buckling force change? (circle one)

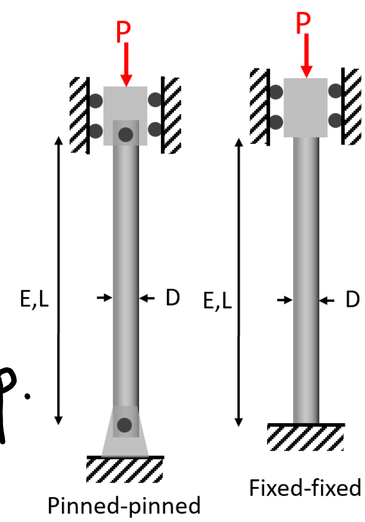
$P_{cr,fixed-fixed} = 4P_{cr,pinned-pinned}$

$P_{cr,fixed-fixed} = 2P_{cr,pinned-pinned}$

$P_{cr,fixed-fixed} = P_{cr,pinned-pinned}/2$

$P_{cr,fixed-fixed} = P_{cr,pinned-pinned}/4$

None of the above



$$P_{cr,f\text{-}f} = \frac{\pi^2 EI}{(0.5L)^2} = \frac{4\pi^2 EI}{L^2} = 4P_{cr,p\text{-}p}.$$

4E.ii If the Young's modulus of the pinned-pinned column is doubled to 2E, how does the critical buckling force change? (circle one)

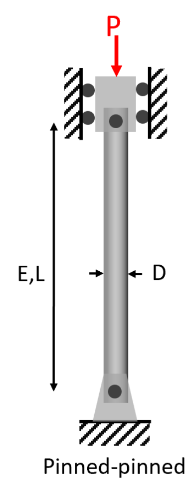
$P_{cr,2E} = 4P_{cr,E}$

$P_{cr,2E} = 2P_{cr,E}$

$P_{cr,2E} = P_{cr,E}/2$

$P_{cr,2E} = P_{cr,E}/4$

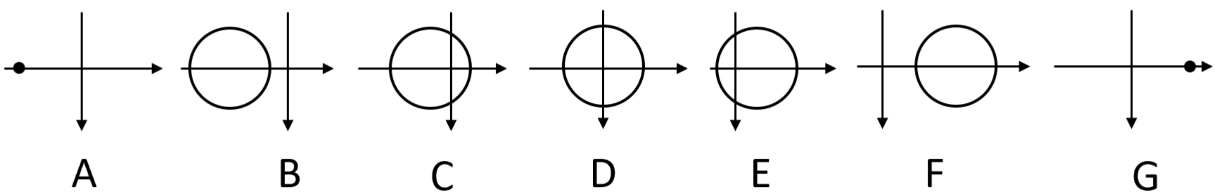
None of the above



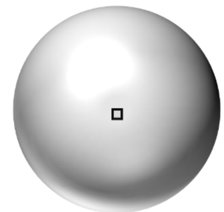
$$P_{cr,2E} = \frac{\pi^2 (2E) I}{L^2} = 2\frac{\pi^2 EI}{L^2} = 2P_{cr,E}$$

ME 323 Final Exam **Name** *(Print)* ______________________________
November 5, 2025 *(Last)* *(First)*

Part 4F (6 points)



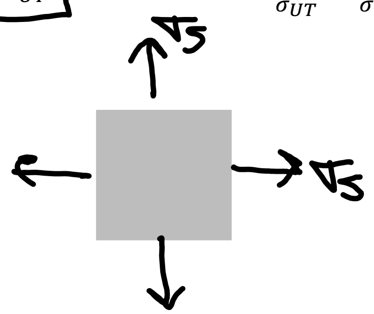
4F.a A spherical pressure vessel has positive pressure inside and is made of a *brittle* material with $\sigma_{UC} > \sigma_{UT}$.



4F.a(i) Circle the correct Mohr's circle for this stress state:
A B C D E F G

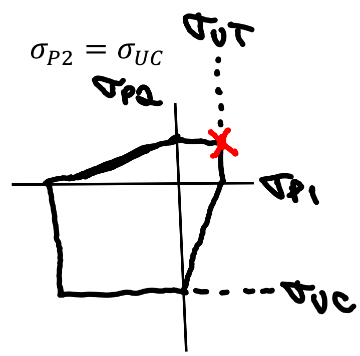
4F.a(ii) What is the condition for failure? (circle one)

$\boxed{\sigma_{P1} = \sigma_{UT}}$ $\qquad \frac{\sigma_{P1}}{\sigma_{UT}} = \frac{\sigma_{P2}}{\sigma_{UC}} + 1 \qquad$ $\sigma_{P2} = \sigma_{UC}$

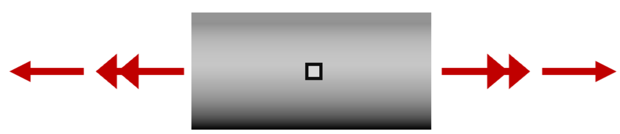


$\sigma_{P1} = \sigma_s$

$\sigma_{P2} = \sigma_s$



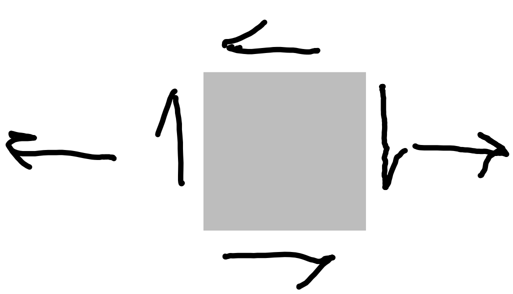
4F.b A circular bar has a tensile force and a torque applied and is made of a *brittle* material with $\sigma_{UC} > \sigma_{UT}$.



4F.b(i) Circle the correct Mohr's circle for this stress state:
A B C D E F G

4F.b(ii) What is the condition for failure? (circle one)

$\sigma_{P1} = \sigma_{UT}$ $\qquad \boxed{\frac{\sigma_{P1}}{\sigma_{UT}} = \frac{\sigma_{P2}}{\sigma_{UC}} + 1} \qquad$ $\sigma_{P2} = \sigma_{UC}$



$\sigma_{avg} = \frac{\sigma_x}{2}$

$R > \sigma_{avg}$

$\Rightarrow \sigma_{P1} = \text{positive}$

$\sigma_{P2} = \text{negative}$

$|\sigma_{P1}| > |\sigma_{P2}|$