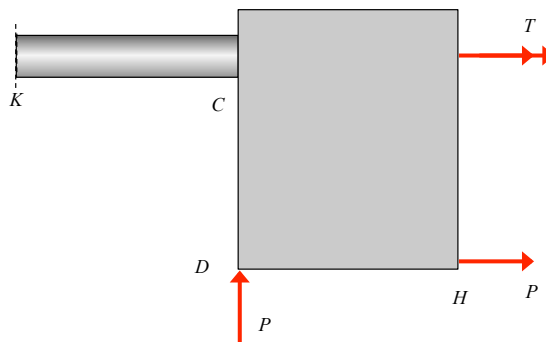
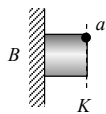


PROBLEM NO. 4 – 20 points max.

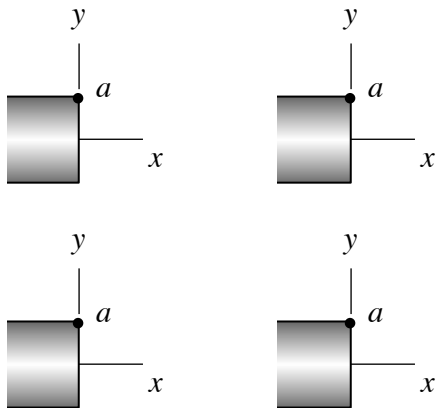
A circular shaft is mounted on a fixed wall at end B. A rigid plate is welded to the shaft at end C. Two loads, both of magnitude P , act on the plate at locations D and H, as shown. A torque T also acts on the plate, as shown in the figure. Ignore the weight of the shaft and plate. It is desired to know the maximum shear stress at point “a” on the cross-section K of the shaft. To this end:

- Determine the resultants acting on the left side of the cut at the cross-section K of the shaft. Show these on the figure provided below.
- Show the different components of stress at point “a” on the cross-section K. Use the figures provided on the next page.
- Make a list of the components of stress from b), and provide the equations for these components as related to the resultants found in (a). Use the table provided on the next page.
- Show the stress components on the stress element provided on the next page.
- Determine the maximum shear stress at point “a” corresponding to $T = 3PL$. Express your answer in terms of, at most, P , L and d .

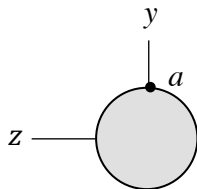


PROBLEM NO. 4 (continued)

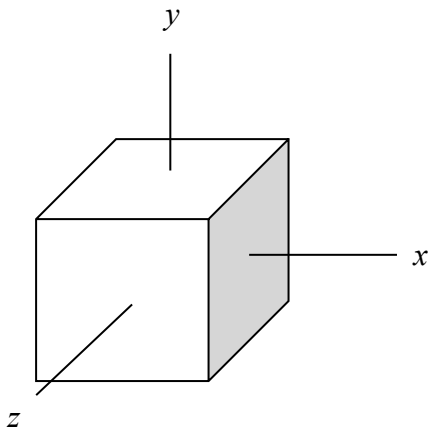
side view



end view



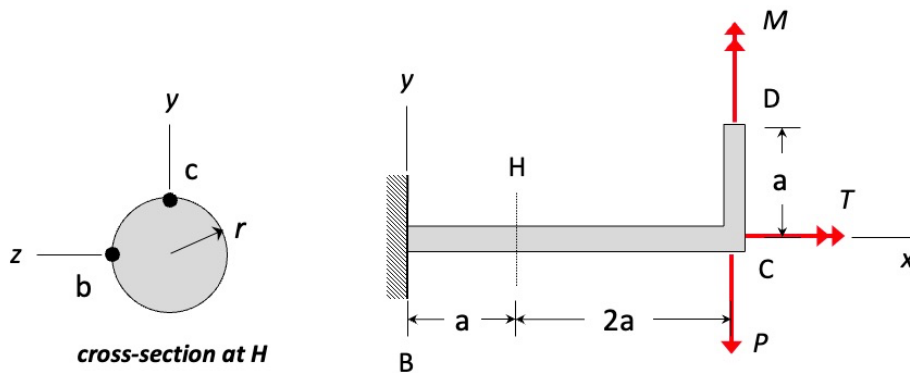
resultant	stress at "a"



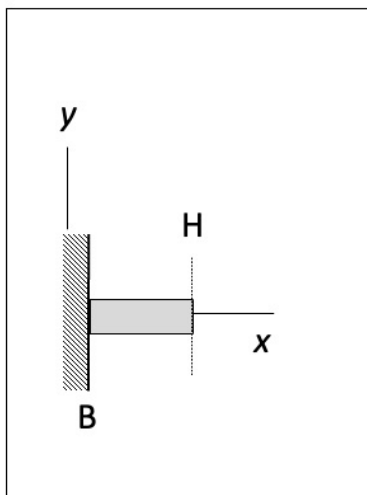
Problem No. 3 – 20 pts

The L-shaped bracket BCD is fixed to ground at end B. Couples T and M act at C and D in the positive x - and y -directions, respectively, whereas force P acts at location C in the negative y -direction. You are asked to determine the states of stress at points “b” and “c” on the cross-section of the bracket at location H. At location H, the cross-section of the bracket is a solid circular section of outer radius r . To this end, do the following:

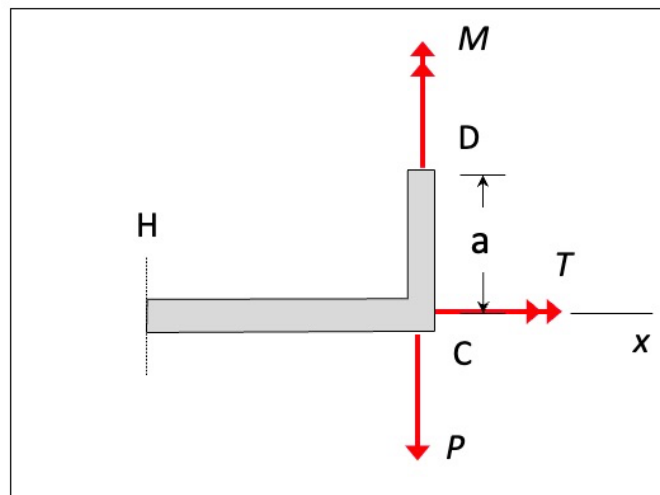
- Complete the free body diagram below of section HCD.
- Draw the internal resultants on the left face of the cut of H in the figure provided below. Be sure to show these resultants in their actual true directions.
- Show the stress distributions for each of the resultants in the stress distribution figures on the worksheet.
- Fill in the table on the worksheet with equations for the stresses at points “b” and “c” on the cross-section. Do not include signs on these equations.
- Draw the stress elements on the worksheet.



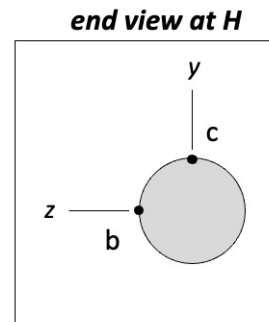
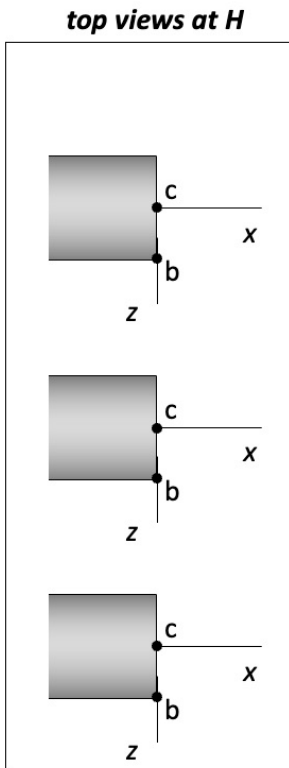
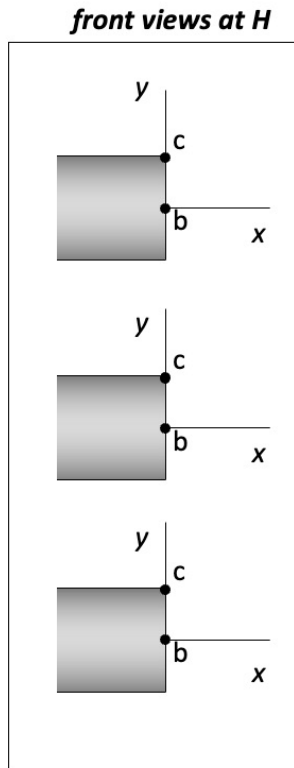
resultants on BH



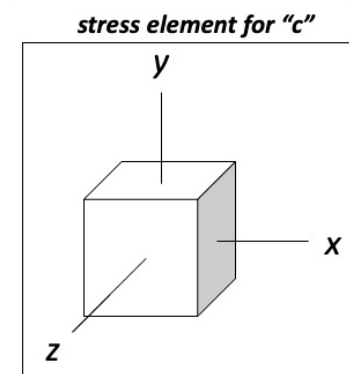
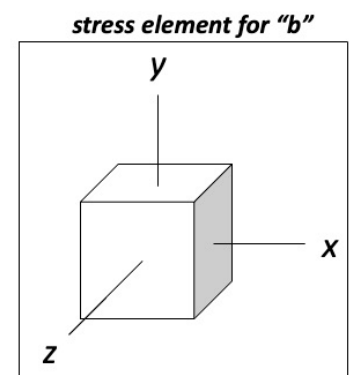
FBD of section HCD



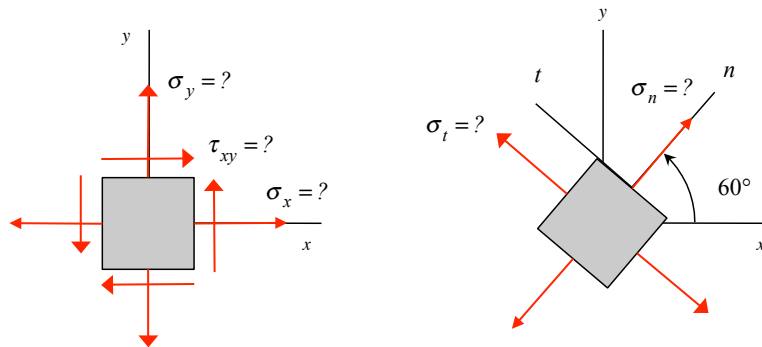
Problem No. 3 - worksheet



resultant	stress @ "b"	stress @ "c"



August 1, 2019

**PROBLEM NO. 4 – 20 points max.**

A structural component is made up of a material having a yield strength of $\sigma_Y = 60 \text{ ksi}$. A state of plane stress at a point in this structural component is represented by the (unknown) stress components σ_x , σ_y and τ_{xy} , as shown in the stress element shown above left that is aligned with the xy -axes.

When the stress element is rotated counterclockwise through an angle of 60° , the shear stress on the surface perpendicular to the n -axis is zero.

The following is known about this state of stress. The magnitudes of the maximum in-plane and maximum absolute shear stress are $|\tau_{max}|_{in-plane} = 10 \text{ ksi}$ and $|\tau_{max}|_{abs} = 20 \text{ ksi}$, respectively. It is also known that the largest principal component of stress is positive.

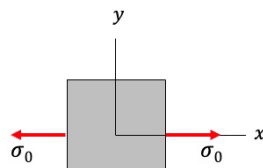
- Determine the principal components of stress.
- Draw the in-plane Mohr's circle for this state of plane stress. Show the location of the x -axis on the in-plane Mohr's circle.
- Determine the σ_x , σ_y and τ_{xy} components of stress.
- If the maximum distortional energy theory is used in the failure analysis, what is the factor of safety against yielding?

Problem No. 4 – 20 pts

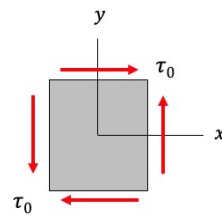
PART 4.A – 4 points

Consider stress states (a) and (b) provided below, where $\tau_0 = 2\sigma_0 \neq 0$. Let $(\sigma_{P1})_a$ and $(\sigma_{P2})_b$ represent the largest of the principal components of stress for states (a) and (b), respectively. Choose the response below that most accurately describes the relative sizes of these principal components of stress. *Justify your answer.*

- i) $0 = |(\sigma_{P1})_a| < |(\sigma_{P1})_b|$
- ii) $0 < |(\sigma_{P1})_a| < |(\sigma_{P1})_b|$
- iii) $0 = |(\sigma_{P1})_b| < |(\sigma_{P1})_a|$
- iv) $0 < |(\sigma_{P1})_b| < |(\sigma_{P1})_a|$



Stress state (a)

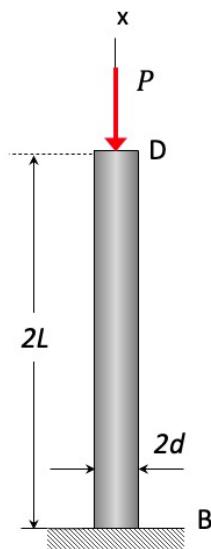


Stress state (b)

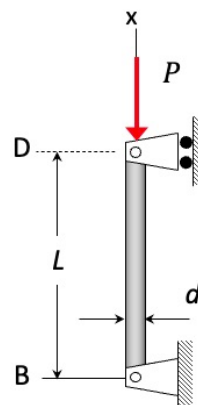
PART 4.B – 4 points

Columns (a) and (b) shown each have a circular cross-section and are made from a material having a Young's modulus of E . Each column carries a compressive axial load of P . Let $(P_{cr})_a$ and $(P_{cr})_b$ represent the critical buckling load for columns (a) and (b), respectively, as predicted by the Euler theory. Choose the response below that most accurately describes the relative sizes of these critical buckling loads. *Justify your answer.*

- i) $(P_{cr})_a > (P_{cr})_b$
- ii) $(P_{cr})_a = (P_{cr})_b$
- iii) $(P_{cr})_a < (P_{cr})_b$



Column (a)

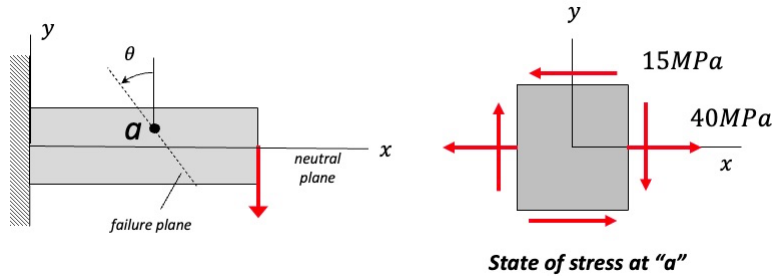


Column (b)

Problem No. 4 (continued)

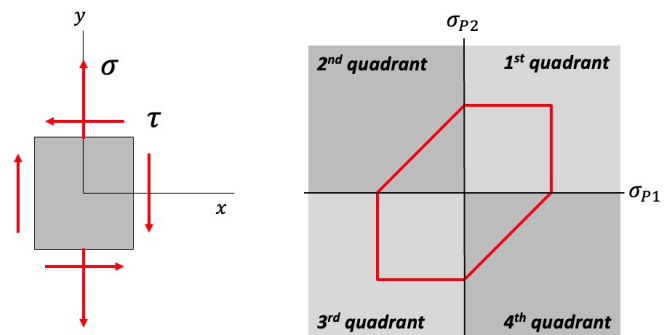
PART 4.C – 4 points

A beam, made from a ductile material, is loaded as shown. This loading creates a state of stress at point “a” on the beam shown. At this loading, the material of the beam fails along the failure plane that is shown. Determine the angle θ that defines the failure plane.

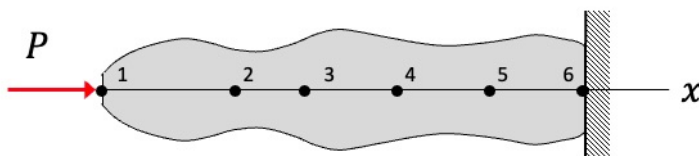
**PART 4.D – 4 points**

Consider the state of stress shown. Let σ_{P1} and σ_{P2} be the two principal components of stress for this state of stress. For this state of stress (regardless of the values for $\sigma \neq 0$ and $\tau \neq 0$), the values of σ_{P1} and σ_{P2} represent a point in the principal stress plane in which quadrant? *Justify your answer.*

- a) 1st quadrant
- b) 2nd quadrant
- c) 3rd quadrant
- d) 4th quadrant

**PART 4.E – 4 points**

A variable cross-section rod is described using a six-node finite element model. The stiffness matrix for this model after the enforcement of boundary conditions is provided below. Unfortunately, information on four terms in the stiffness matrix (K_{11} , K_{21} , K_{23} and K_{22}) has been lost. What are the values of these four missing stiffness matrix terms?

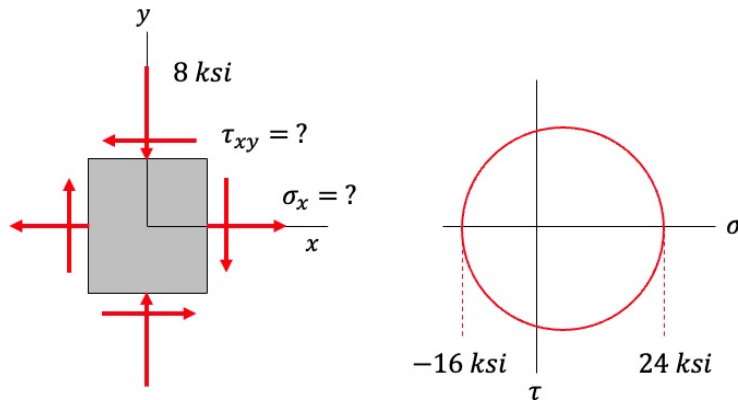


$$[K] = \frac{EA}{L} \begin{bmatrix} K_{11} & -2 & 0 & 0 & 0 \\ K_{21} & K_{22} & K_{23} & 0 & 0 \\ 0 & -1 & 3 & -2 & 0 \\ 0 & 0 & -2 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

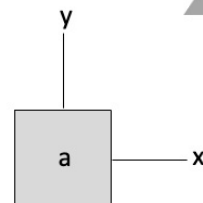
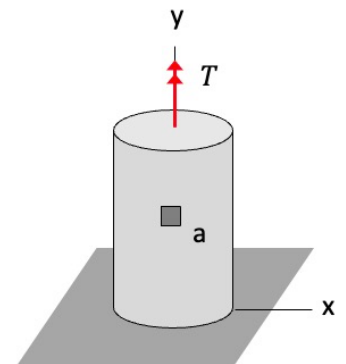
Problem No. 5 – 20 pts

PART 5.A – 5 points

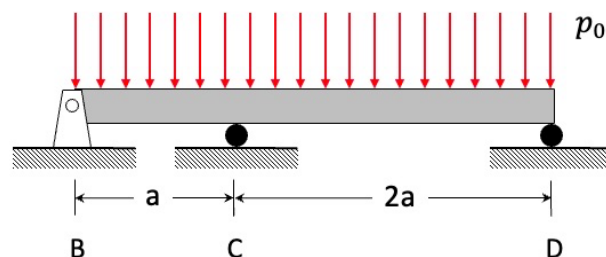
Consider the state of stress and the corresponding in-plane Mohr's circle shown, where σ_x and τ_{xy} are unknown. Determine numerical values for σ_x and τ_{xy} .

**PART 5.B – 5 points**

A thin-walled cylindrical tank having an inside radius of r and a wall thickness of t contains a gas under a pressure of p . A torque T is applied at the end along the longitudinal axis of the tank. Determine the xy -components of stress in the sidewall of the tank for point “a” on the outside surface of the tank. Show these components on a stress element. Consider the weight of the tank to be negligible.

**PART 5.C – 5 points**

A line load acts on the beam shown. Use superposition to determine the reaction at support C on the beam.

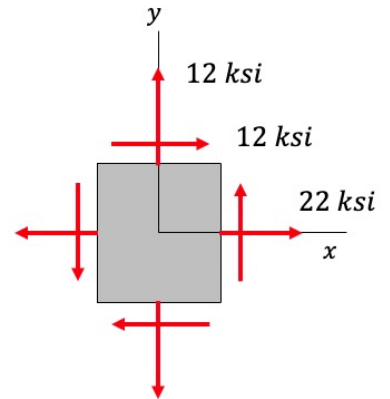


Problem No. 5 (continued)

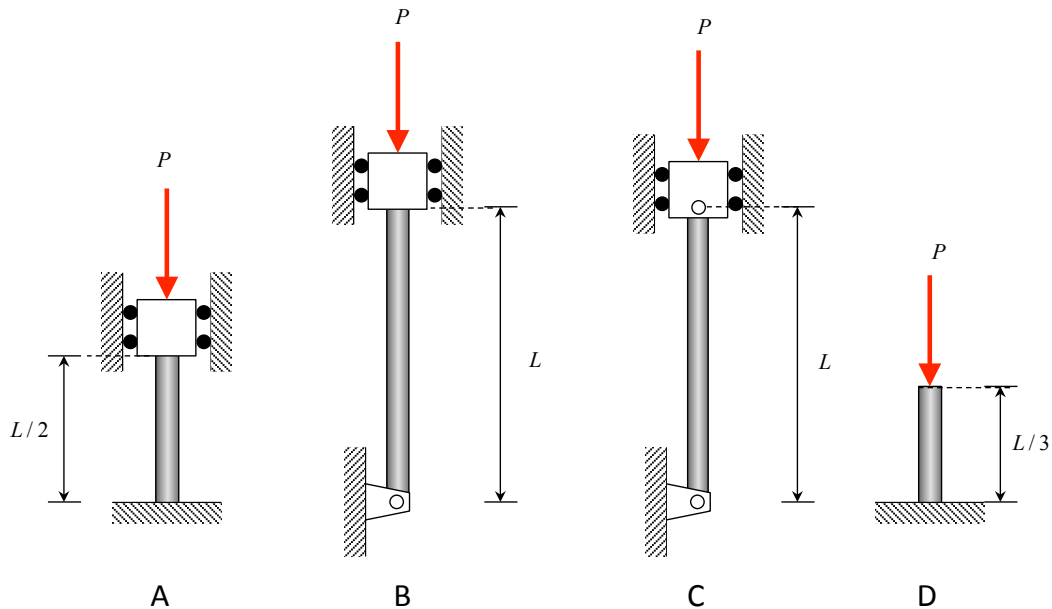
PART 5.D – 5 points

The state of stress at a point on a structural member is shown, with the ductile material of the member being ASTM-A36 structural steel (yield strength of $\sigma_Y = 36 \text{ ksi}$).

- Using the maximum shear stress (MSS) theory, determine the factor of safety against yielding.
- Without performing any calculations, state whether the factor of safety against yielding using the maximum distortional energy (MDE) theory will be greater than, less than or equal to the factor of safety found using MSS.



August 3, 2023

PROBLEM NO. 4 - PART D – 3 points max.

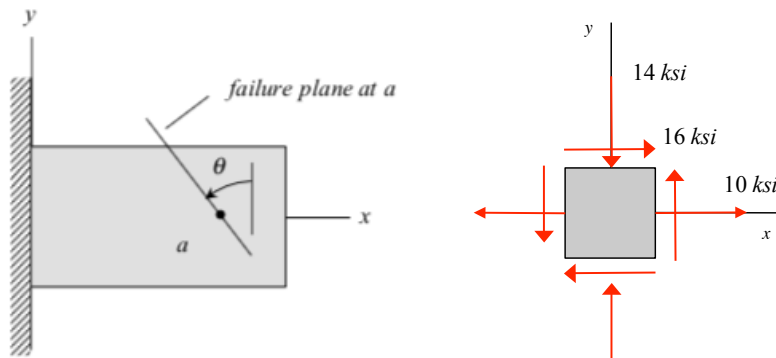
Consider the four columns (A, B, C and D) shown above with differing boundary conditions and lengths. The loading is the same for each column, each column is made up of the same material having a Young's modulus and each column has the same circular cross section.

- Which column has the largest critical Euler buckling load? A B C D
- Which column has the second largest critical Euler buckling load? A B C D
- Which column has the smallest critical Euler buckling load? A B C D

August 3, 2023

PROBLEM NO. 4 - PART E – 6 points max.

The ductile material making up the structural component shown is known to have failed in yielding at point “a”, where the state of stress at “a” is shown below right. What is the angle θ that defines the plane of yielding in the material?



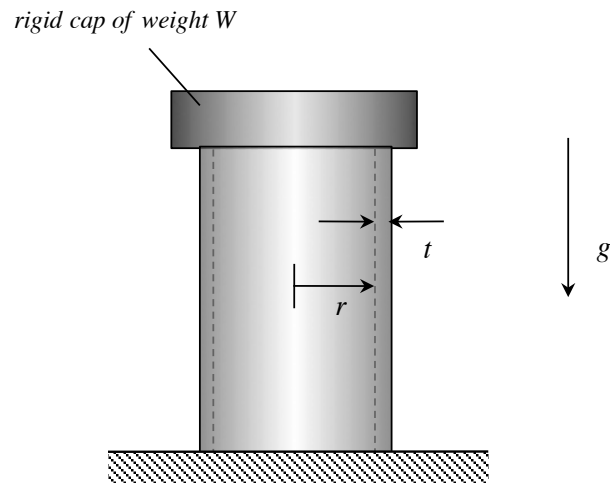
PROBLEM NO. 5 - PART A – 4 points max.

A given state of plane stress in a ductile material with a yield strength of $\sigma_Y = 175 \text{ MPa}$ has principal stress components of $\sigma_{p1} = 180 \text{ MPa}$ and $\sigma_{p2} = 20 \text{ MPa}$.

TRUE or FALSE: The *maximum shear stress theory* predicts that the material will fail.

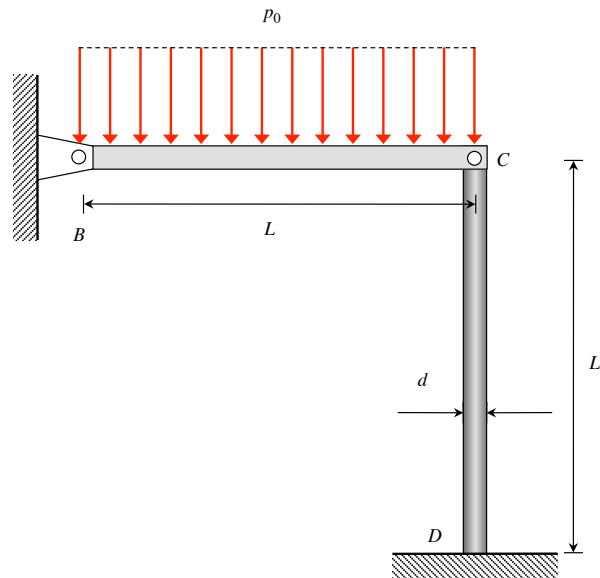
TRUE or FALSE: The *maximum distortional energy theory* predicts that the material will fail.

PROBLEM NO. 5 - PART B – 4 points max.



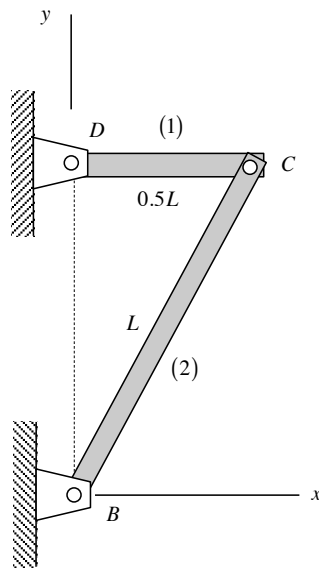
A thin-walled tank (having an inner radius of r and wall thickness t) constructed of a ductile material contains a gas with a pressure of p . A rigid cap of weight $W = 2\pi pr^2$ rests on top of a seal on the tank (i.e., the cap is not attached to the tank). Ignore the weight of the tank. If the tank fails under this loading, determine on which plane will the failure occur (i.e., describe the orientation of this plane relative to the horizontal). With your answer, include a sketch of the Mohr's circle for the stress state in the tank wall.

PROBLEM NO. 5 - PART C – 4 points max.



A structure is made up of a horizontal rigid bar BC and a vertical elastic member CD, where CD has a circular cross section of diameter d and is made up of a material having a Young's modulus of E . A distributed loading having a transverse force/length of p_0 acts on bar BC. Ignore the weight of the members. Determine the critical load value p_0 that corresponds to the buckling of member CD using the Euler theory of buckling.

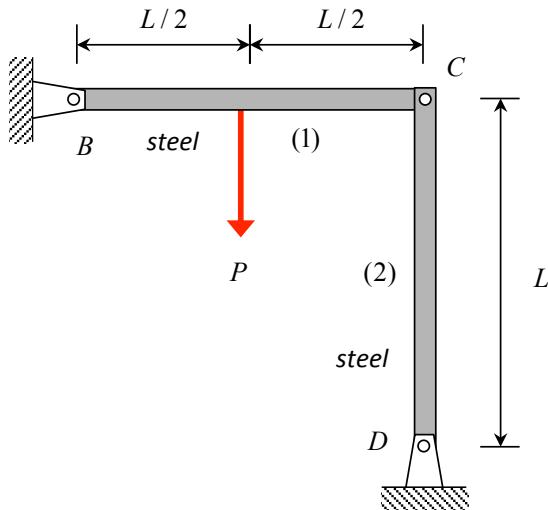
PROBLEM NO. 5 - PART D – 4 points max.



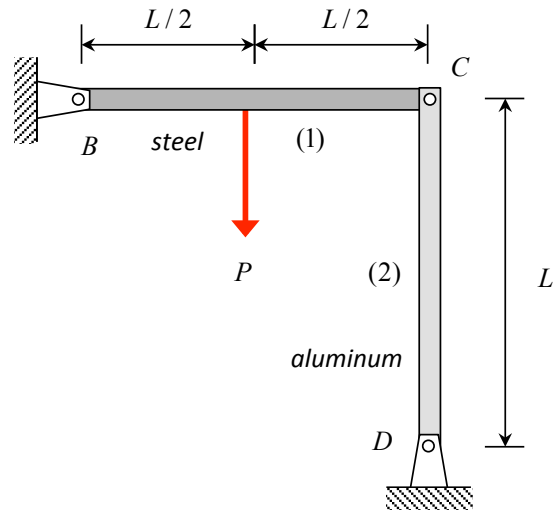
A truss is made up of members (1) and (2) as shown above, with each member being composed of a material with a Young's modulus of E and thermal expansion coefficient of α , and having a cross-sectional area of A . With the members being initially unstressed and unstrained, the temperature of (1) is increased by an amount of ΔT and the temperature of (2) is decreased by an amount of $2\Delta T$.

- Provide an argument supporting the claim that members (1) and (2) remain *unstressed* after the temperature change described above. *HINT*: Consider the free body diagram of joint C.
- TRUE* or *FALSE*: Members (1) and (2) have zero strain.

PROBLEM NO. 5 - PART E – 4 points max.



Structure (a)



Structure (b)

- Structure (a) is an L-shaped frame made up of elements (1) and (2), with all connections in the frame being pin joints. Elements (1) and (2) are both made up of *steel*. Let $(\sigma_2)_a$ be the axial stress in element (2) of structure (a).
- Structure (b) is identical to structure (a), except here element (2) is made up of *aluminum*. Let $(\sigma_2)_b$ be the axial stress in element (2) of structure (b).

Circle the answer below that most accurately describes the states of stress in element (2) for these two frames:

- a) $|(\sigma_2)_a| < |(\sigma_2)_b|$
- b) $|(\sigma_2)_a| = |(\sigma_2)_b|$
- c) $|(\sigma_2)_a| > |(\sigma_2)_b|$