ME 323 Final Exam **Name** *(Print)* ______________________________
November 5, 2025 *(Last)* *(First)*

Equation sheets

Generalized Hooke's Law

$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$

$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$ $\quad \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$

$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$ $\quad G = \frac{E}{2(1+\nu)}$

Uni-axial loadings

$\sigma_{ave} = \frac{F_N}{A} \quad FS = \frac{\sigma_{fail}}{\sigma_{allow,member}}$

$e_{AB} = u_B - u_A \quad e = \int_0^L \frac{F(x)}{E(x)A(x)}dx + \int_0^L \alpha\Delta T dx \quad e = \frac{FL}{EA} + \alpha\Delta TL$

$e = u cos(\theta) + v sin(\theta)$

Direct shear loadings

$\tau_{ave} = \frac{V}{A} \quad FS = \frac{\tau_{fail}}{\tau_{allow,member}}$

Torsional loadings

$\phi_{AB} = \phi_B - \phi_A \quad \Delta\phi = \int_0^L \frac{T(x)}{G(x)I_p(x)}dx \quad \Delta\phi = TL/GI_p$

$\gamma = \rho\frac{d\phi}{dx} \quad \tau = G\rho\frac{d\phi}{dx} \quad \gamma = \frac{\rho T}{GI_p} \quad \tau = \frac{\rho T}{I_p}$

$I_p = \int_A \rho^2 dA \quad I_p = \frac{\pi}{2}r^4 \ (solid) \quad I_p = \frac{\pi}{2}(r_o^4 - r_i^4)$

Loadings resulting in bending

$\frac{dV}{dx} = p(x); \quad \frac{dM}{dx} = V(x); \quad \frac{d\theta}{dx} = \frac{1}{EI}M(x); \quad \frac{dv}{dx} = \theta(x); \quad \Delta V = P; \quad \Delta M = -M_o$

$\sigma(x,y) = -\frac{Ey}{\rho} = -\frac{M_z y}{I_{zz}}$

$\tau(x,y) = \frac{VQ}{I_{zz}t} = \frac{VA^*(y)\bar{y}^*(y)}{I_{zz}t(y)}$

$\tau_{max} = \frac{3V}{2A}\ (rectangle); \quad \tau_{max} = \frac{4V}{3A}\ (circle); \quad \tau_{max} = \frac{2V}{A}\ (hollow\ circle)$

$I_z = \frac{bh^3}{12}\ (rectangle), \quad I_z = \frac{\pi}{4}r^4\ (circle), \quad I_z = \frac{bh^3}{36}\ (triangle)$

Finite Element method

$k = \frac{(EA)_{ave}}{L}$

Stress transformations

$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(2\theta) + \tau_{xy}\sin(2\theta) \quad \tau_{nt} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin(2\theta) + \tau_{xy}\cos(2\theta)$

$\sigma_{ave} = \frac{\sigma_x+\sigma_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_{P1} = \sigma_{ave} + R, \quad \sigma_{P2} = \sigma_{ave} - R$

Energy methods

$$\bar{u} = \frac{1}{2}\left[\sigma_x(\varepsilon_x - \alpha\Delta T) + \sigma_y(\varepsilon_y - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right]$$

$$U = \frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx \qquad U = \frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx \qquad U = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}dx$$

$f_s = 6/5 \ \ (rectangular\ cross\ section), \quad f_s = 10/9 \ \ (circular\ cross\ section)$

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \qquad \theta_{M_i} = \frac{\partial U}{\partial M_i} \qquad \phi_{T_i} = \frac{\partial U}{\partial T_i} \qquad \frac{\partial U}{\partial R_i} = 0 \ \ (for\ redundant\ loads)$$

Pressure vessels

Cylindrical: $\sigma_h = \frac{pR}{t} \qquad \sigma_a = \frac{pR}{2t}$

Spherical: $\sigma_s = \frac{pR}{2t}$

Failure theories

Maximum shear stress (MSS) failure theory:

$\tau_{max,abs} = \frac{\sigma_Y}{2}$

Maximum distortional energy (MDE) failure theory:

$\sigma_M = \sigma_Y$

$\sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}$

Maximum normal stress (MNS) failure theory:

$|\sigma_{P1}| = \sigma_U \quad or \quad |\sigma_{P2}| = \sigma_U$

Mohr's failure theory:

If σ_{P1} and σ_{P2} are of the same sign: $\sigma_{P1} = \sigma_{UT} \quad or \quad \sigma_{P2} = -\sigma_{UC}$

If σ_{P1} and σ_{P2} are of different signs: $\frac{\sigma_{P1}}{\sigma_{UT}} - \frac{\sigma_{P2}}{\sigma_{UC}} = 1$

Column buckling

$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2}$