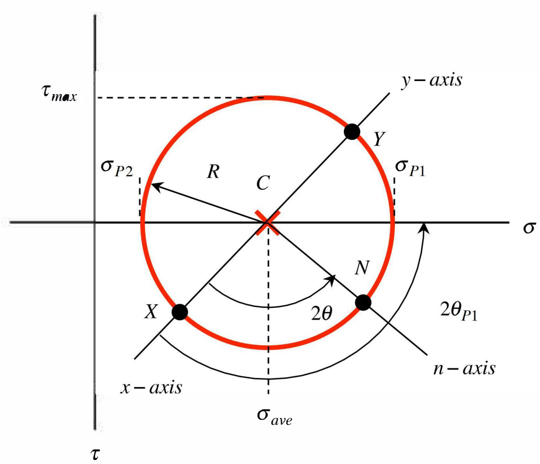
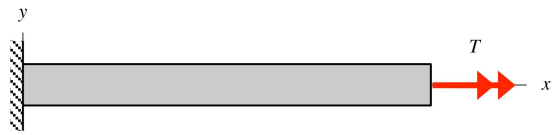
Construction of Mohr's circle for a general state of plane stress

For a given state of stress (σ_x, σ_y, τ_{xy}) for a point:

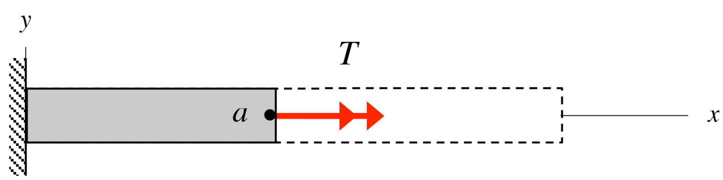
1) Establish a set of σ-τ axes (be sure to use the same scale on each axis)::
   - +σ points to *right*
   - +τ points *down*
2) Calculate the two parameters that define the location and size of Mohr's circle:
   - $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \textit{average normal stress}$
   - $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
3) Draw a circle in the $\sigma - \tau$ plane with its center C at $\left(\sigma, \tau\right) = \left(\sigma_{ave}, 0\right)$ and having a radius of R.
4) Show the point X given by the coordinates $\left(\sigma, \tau\right) = \left(\sigma_x, \tau_{xy}\right)$ on the Mohr's circle. Line OX is the x-axis. (Note that the y-axis is at a $180°$ from the x-axis in the $\sigma - \tau$ plane.)
5) The components of stress on the face of a stress element rotated through an angle of θ corresponds to a point N on Mohr's circle found through a rotation of 2θ on the circle.
6) The angle from the x-axis to the $\sigma -$axis in the Mohr's circle plane is $2\theta_{P1}$, where θ_{P1} the rotation angle for the stress element that produces the largest principal stress σ_{P1}. It is readily seen from the figure that the principal stresses are given by: $\sigma_{P1,P2} = \sigma_{ave} \pm R$.



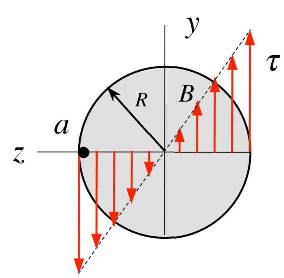
TORSIONAL LOADING



Internal loading:



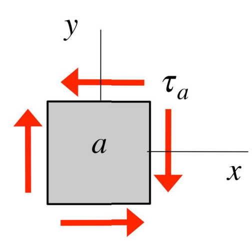
Stress distribution:



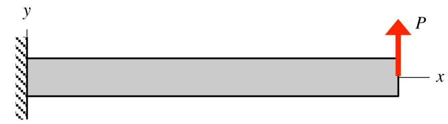
sign convention : T <u>*positive*</u> *OUTWARD on face (by right – hand rule)*

$$\tau_a = \frac{TR}{I_P} = \text{linear in radial position} \quad ; \quad I_P = \text{polar area moment}$$

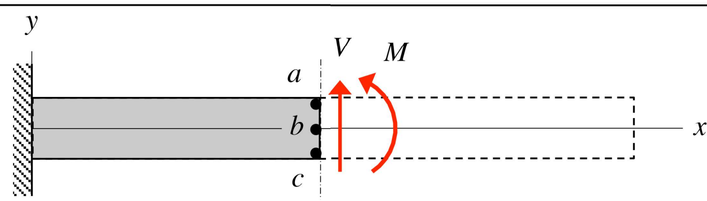
Stress element:



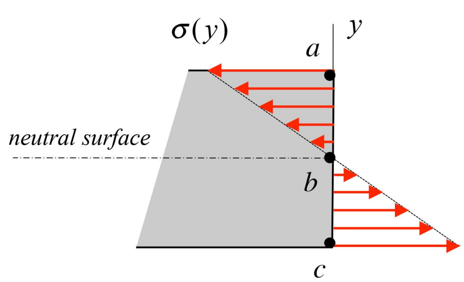
TRANSVERSE LOADING (e.g., rectangular cross section)



Internal loading:

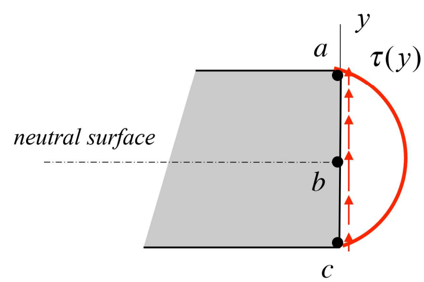


Normal stress distribution:



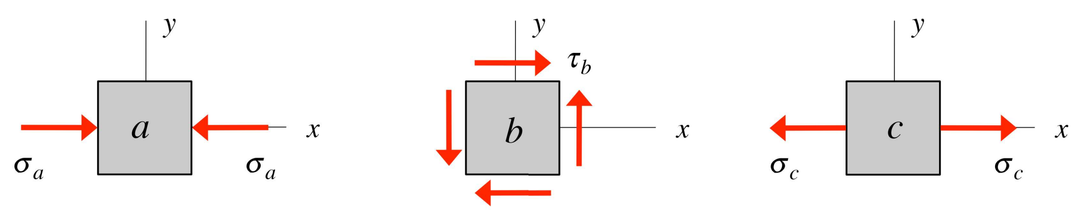
$$|\sigma_a| = \frac{M|y_a|}{I} \qquad \sigma_b = 0 \qquad |\sigma_c| = \frac{M|y_c|}{I}$$

Shear stress distribution:

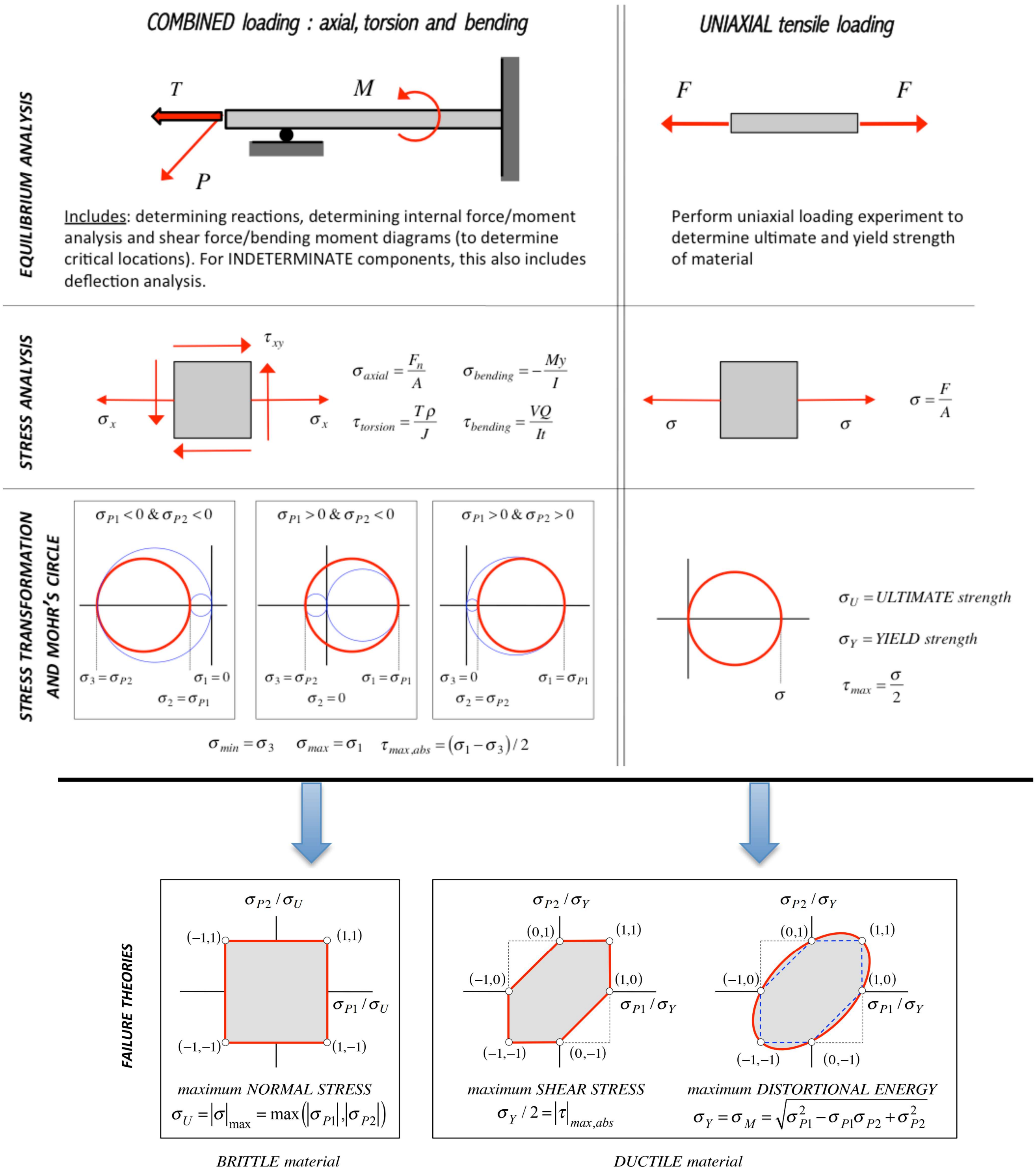


$$\tau_a = 0 \qquad |\tau_b| = \frac{3}{2}\frac{|V|}{A} \qquad \tau_c = 0$$

Stress element:



Summary: failure analysis *(what the whole course of ME 323 leads up to…)*

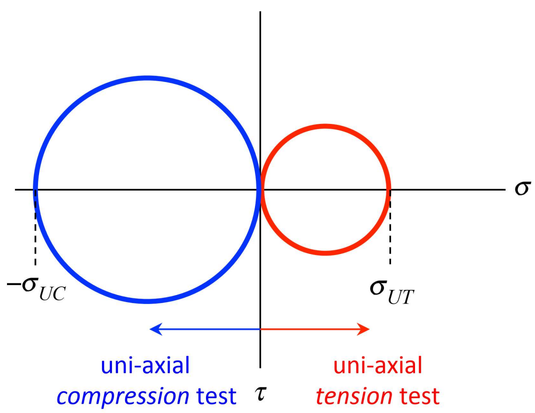


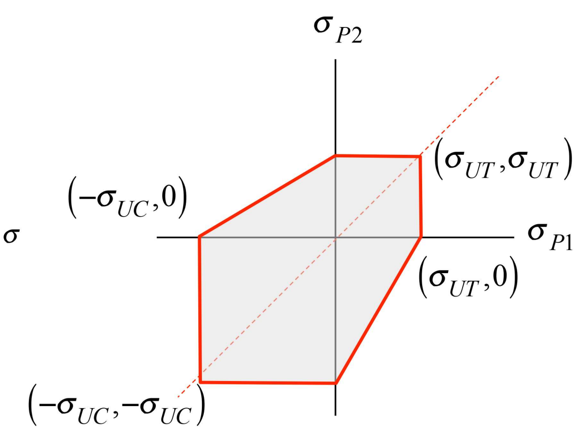
d) Failure theory for brittle materials – Mohr's criterion

Many brittle materials have a lower ultimate strength in tension σ_{UT} than in compression σ_{UC}; i.e., $\sigma_{UT} < \sigma_{UC}$. In this case, the maximum normal stress criterion cannot be used. An alternative theory, Mohr's criterion, states that:

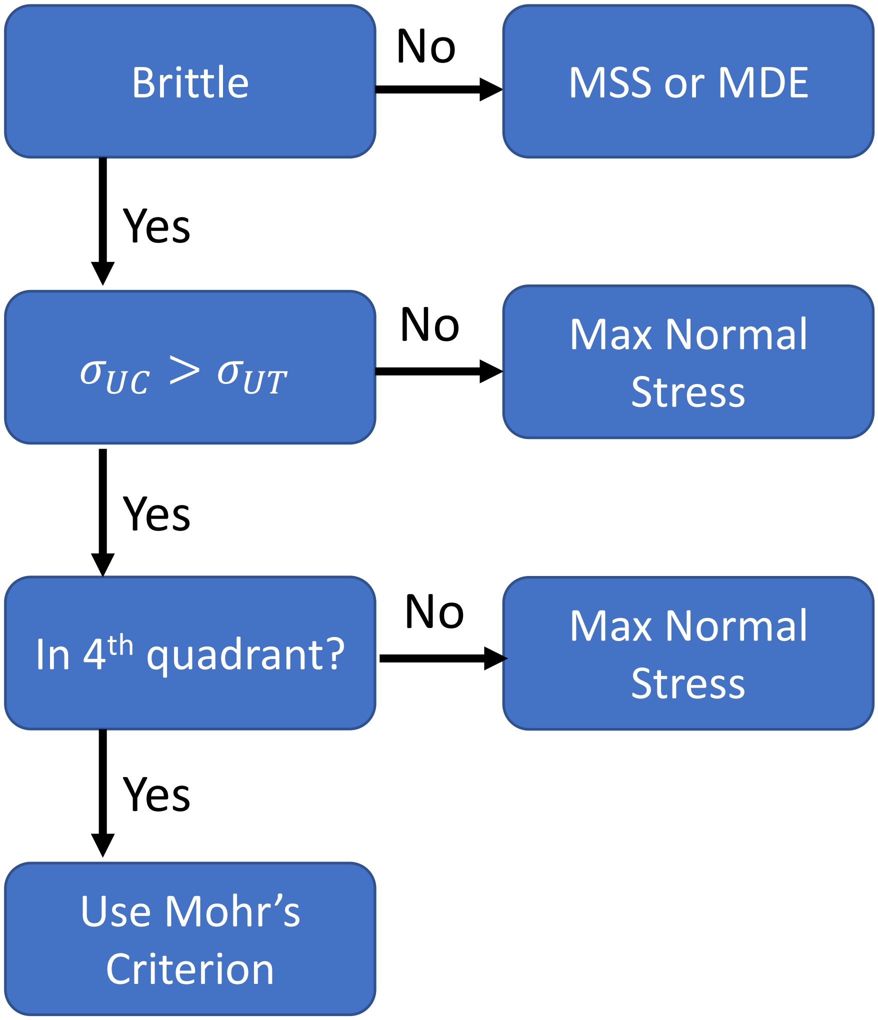
- when σ_{P1}, and σ_{P2} have the same sign, failure occurs if either of the following stress limits is reached: $\sigma_{\max} = \sigma_{TU} \quad OR \quad \sigma_{\min} = -\sigma_{CU}$.
- when σ_{P1} and σ_{P2} have opposite signs, Mohr's failure criterion states that failure occurs when: $\dfrac{\sigma_{P1}}{\sigma_{UT}} = \dfrac{\sigma_{P2}}{\sigma_{UC}} + 1$.

The above combination constitutes Mohr's failure criterion.





general state of plane stress


Brittle
No
MSS or MDE
Yes
$\sigma_{UC} > \sigma_{UT}$
No
Max Normal Stress
Yes
In 4th quadrant?
No
Max Normal Stress
Yes
Use Mohr's Criterion

Exam 3

May 5, 2023 Name (Print)_______________________________________

PROBLEM # 2 (25 points)

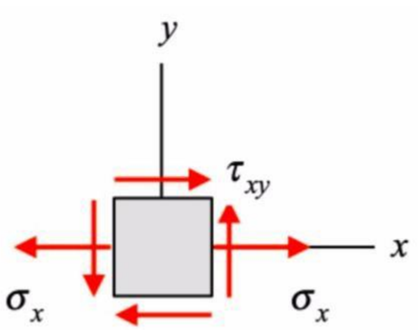
Consider a state of plane stress in a structural component represented by the stress element provided below where σ_x is yet unknown and τ_{xy} = 50 ksi.

a) Suppose that the structural component is made of a *ductile* material with a yield strength σ_Y = 200 ksi.

   a. Using the *maximum shear stress theory*, determine the maximum value of σ_x for which the ductile material in the structural component does not fail.

   b. For the maximum value of σ_x found above, does the *maximum distortional energy theory* predict failure of the material?

b) Suppose now that the structural component is instead made of a *brittle* material with equal ultimate strengths in tension and compression of $\sigma_{UT} = \sigma_{UC}$ = 200 ksi. Using *Mohr's failure criteria*, determine the maximum value of σ_x for which the brittle material in the structural component does not fail.



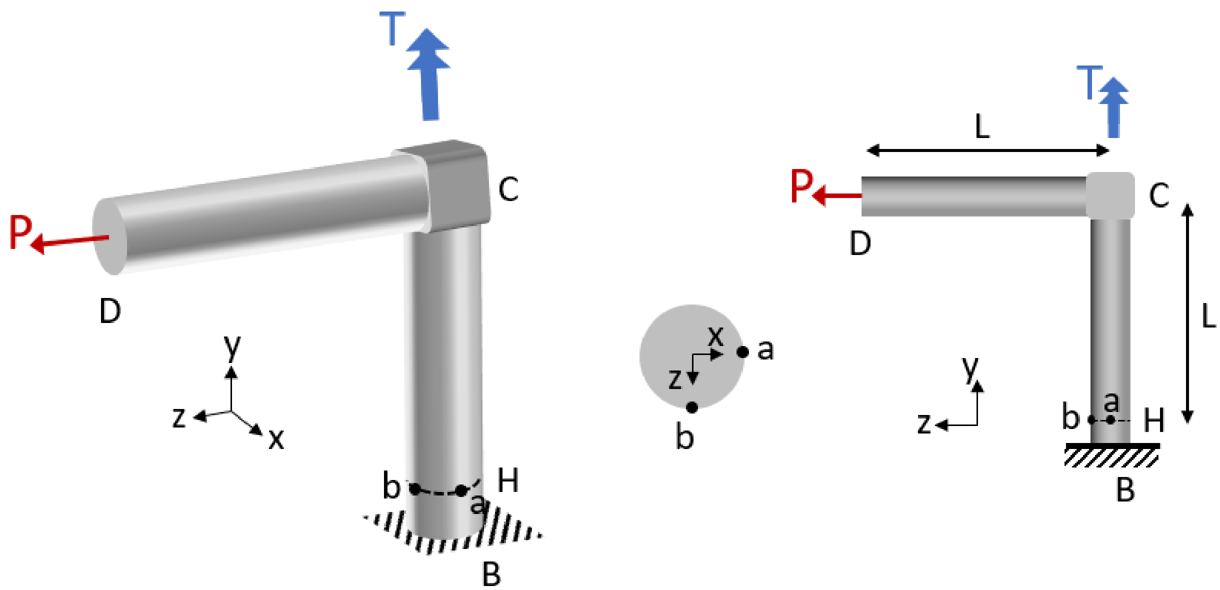
a) Suppose now that the structural component is instead made of a *brittle* material with equal ultimate strengths in tension and compression of σ_{UT} = 200 ksi and σ_{UC} = 600 ksi. Using *Mohr's failure criteria*, determine the maximum value of σ_x for which the brittle material in the structural component does not fail.

Exam 3

May 5, 2023 Name (Print)____

PROBLEM #3 (25 points)

BCD is an "L-shaped" structure made of a solid cylindrical rod with a diameter of d that is fixed to a wall at node B. A force of P is applied in the z-direction at node D and a torque of T is applied in the y-direction at node C. The structure is made of a material with a Young's modulus of E and a shear modulus of G.



(a) Determine the internal reactions at cross-section H.
(b) Draw the relevant stress distributions on the cross-sections on the next page.
(c) Determine the stresses at point a and b (which are located on the cross section at H) and write them in Table 1.
(d) Draw the stress element at points a and b.

Exam 3

May 5, 2023 Name (Print)______________________________________

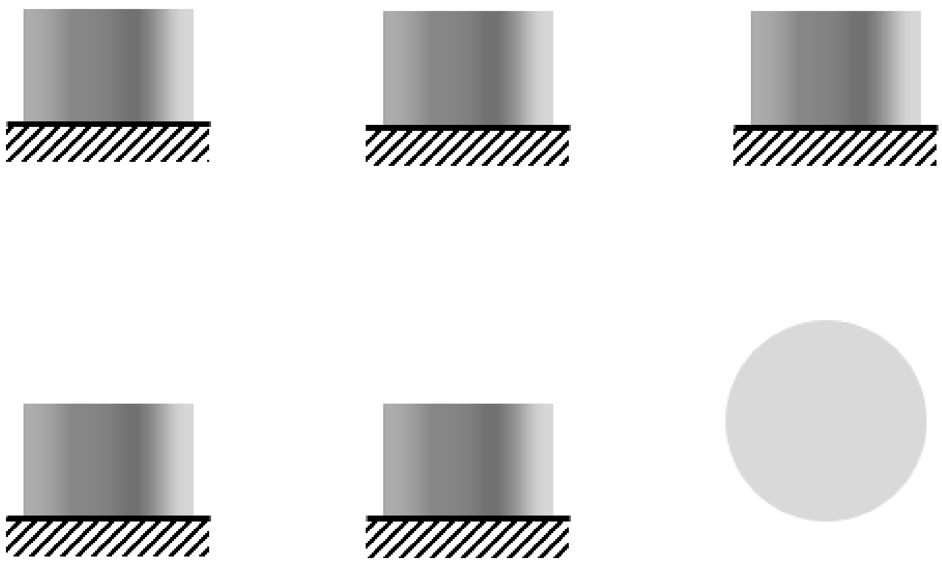
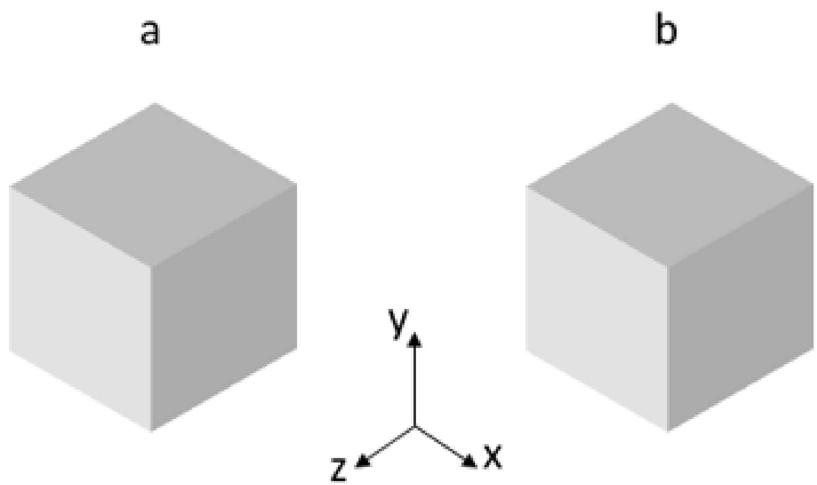


Table 1

Force Component	a	b

Exam 3

May 5, 2023 Name (Print)____________________________________



ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: ______________________________
(Last) (First)

PROBLEM #1 (25 pts)

The compressed-air tank is fabricated of steel plate that is welded along a helix that makes an angle of β=60° with respect to the longtigudinal axis of the tank. The inside diameter of the cylinder is 48 inch, the wall thickness is 0.5 inch, and the internal pressure is 200 psi.

(a) Determine the axial stress σ_a and hoop stress σ_h.

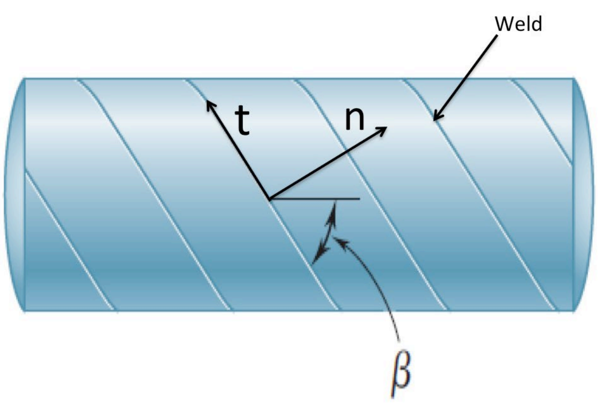
(b) Draw the Mohr's circle on the given graph.

(c) Use Mohr's circle to determine

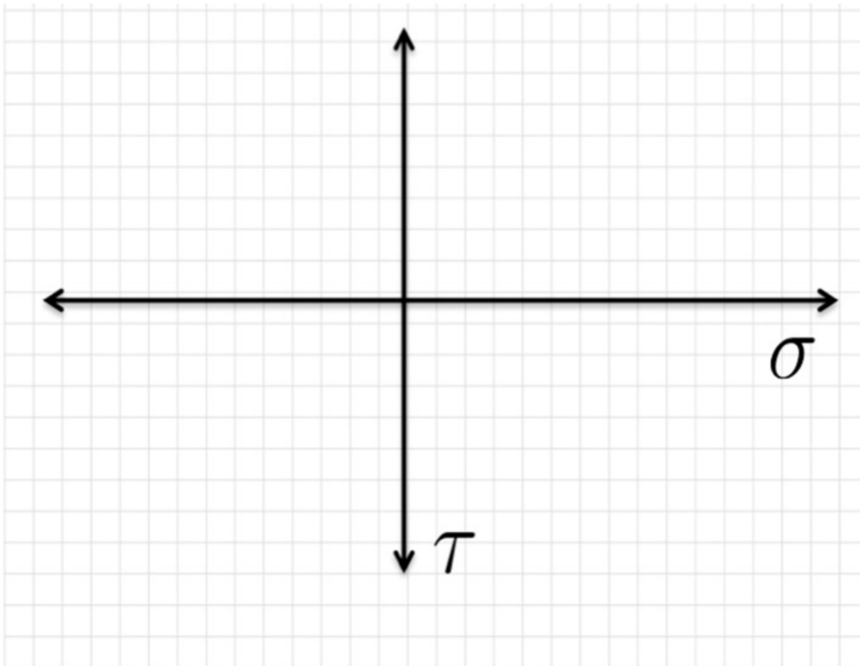
(i) the principal stresses ($\sigma_1, \sigma_2, \sigma_3$), and the absolute maximum shear stress $\tau_{\max}^{abs}$.

(ii) the stresses in the directions perpendicular (n) and tangent (t) to the weld. Mark the two stress states on Mohr's circle. And show the stresses in a properly oriented stress element.

(d) Knowing that the Young's modulus of steel is E=29×10^6 psi and the Poisson's ratio is ν=0.3, determine the strain ε_t tangent to the weld.



Name: ___________________________________
(Last) (First)



PROBLEM #4 (25 Points):

PART A – 5 points

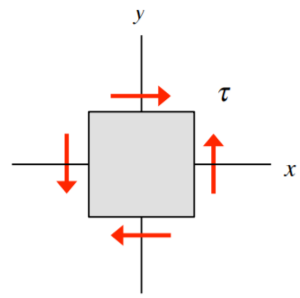
(i) For the state of stress shown below, $\boldsymbol{\sigma_x = \sigma_y = 0}$ and $\boldsymbol{\tau_{xy} = \tau}$. Let $\boldsymbol{\tau_{MSS}}$ and $\boldsymbol{\tau_{MDE}}$ be the values of $\boldsymbol{\tau}$ required to cause yielding based on the maximum shear stress and maximum distortional energy theories, respectively. The yield strength of the ductile material is $\boldsymbol{\sigma_Y}$.

Circle the answer that best describes the relative sizes of $\boldsymbol{\tau_{MSS}}$ and $\boldsymbol{\tau_{MDE}}$.

(a) $\boldsymbol{\tau_{MSS} > \tau_{MDE}}$

(b) $\boldsymbol{\tau_{MSS} = \tau_{MDE}}$

(c) $\boldsymbol{\tau_{MSS} < \tau_{MDE}}$



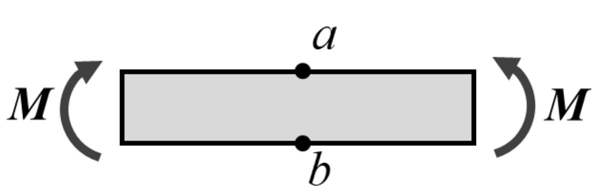
(ii) The beam shown below has a square cross section and is made of a brittle material where the ultimate compressive strength is ***larger*** than the ultimate tensile strength. The beam is subjected to a bending moment $\boldsymbol{M} > \boldsymbol{0}$ as shown below. Let $\boldsymbol{M_a}$ and $\boldsymbol{M_b}$ be the values of the bending moment required to cause brittle failure at points *a* and *b*, respectively, based on **Mohr's failure criterion**.

Circle the answer that best describes the relative sizes of $\boldsymbol{M_a}$ and $\boldsymbol{M_b}$.

(a) $\boldsymbol{M_a > M_b}$

(b) $\boldsymbol{M_a = M_b}$

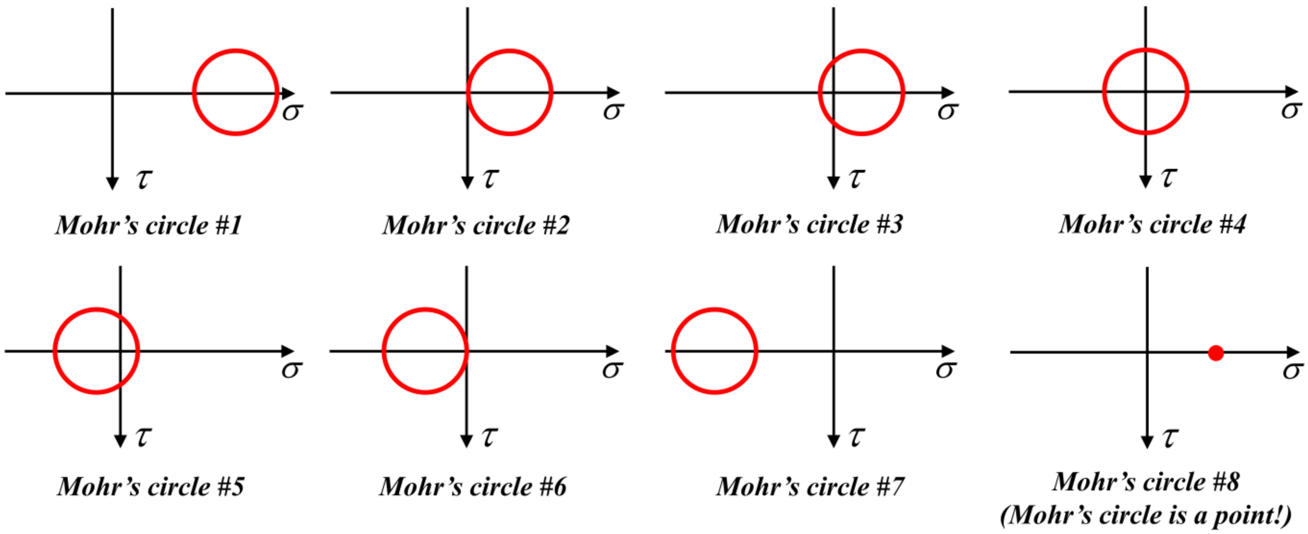
(c) $\boldsymbol{M_a < M_b}$



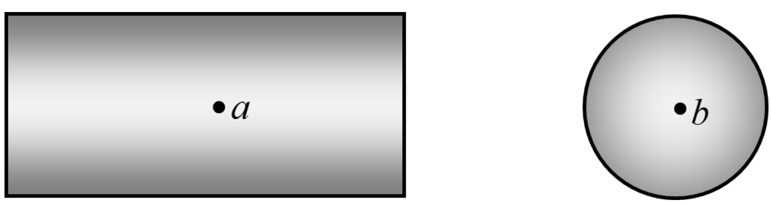
PROBLEM #4 (cont.):

PART B – 10 points

Questions (i) and (ii) in Part B refer to the eight Mohr's circles shown below.



(i) The cylindrical and spherical thin-walled pressure vessels are each subjected to an internal pressure. Point *a* is on the surface of the cylindrical pressure vessel. Point *b* is on the surface of the spherical pressure vessel.



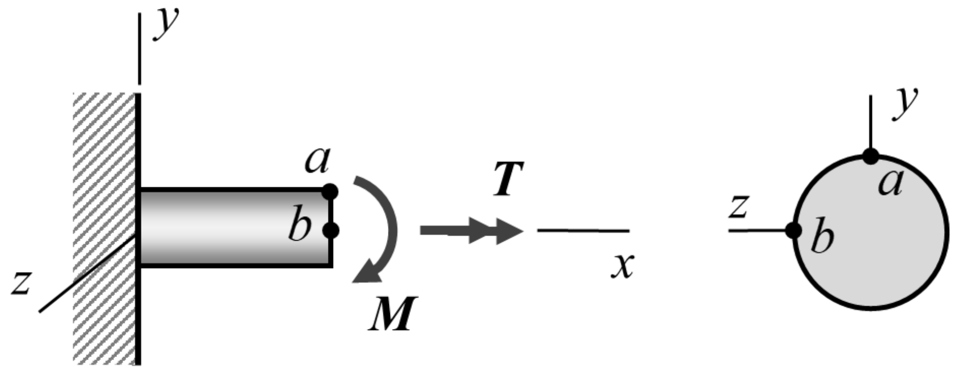
Circle the number of the **correct in-plane Mohr's circle** for the state of stress at:

- Point *a*: #1 #2 #3 #4 #5 #6 #7 #8
- Point *b*: #1 #2 #3 #4 #5 #6 #7 #8

PROBLEM #4 (cont.):

PART B (cont.)

(ii) At a cut in the circular rod shown below, the internal resultant loads are determined to be a bending moment ***M*** about the negative *z*-axis and a torque ***T*** about the positive *x*-axis.



Circle the number of the **correct in-plane Mohr's circle** for the state of stress at:

- Point *a*: #1 #2 #3 #4 #5 #6 #7 #8
- Point *b*: #1 #2 #3 #4 #5 #6 #7 #8

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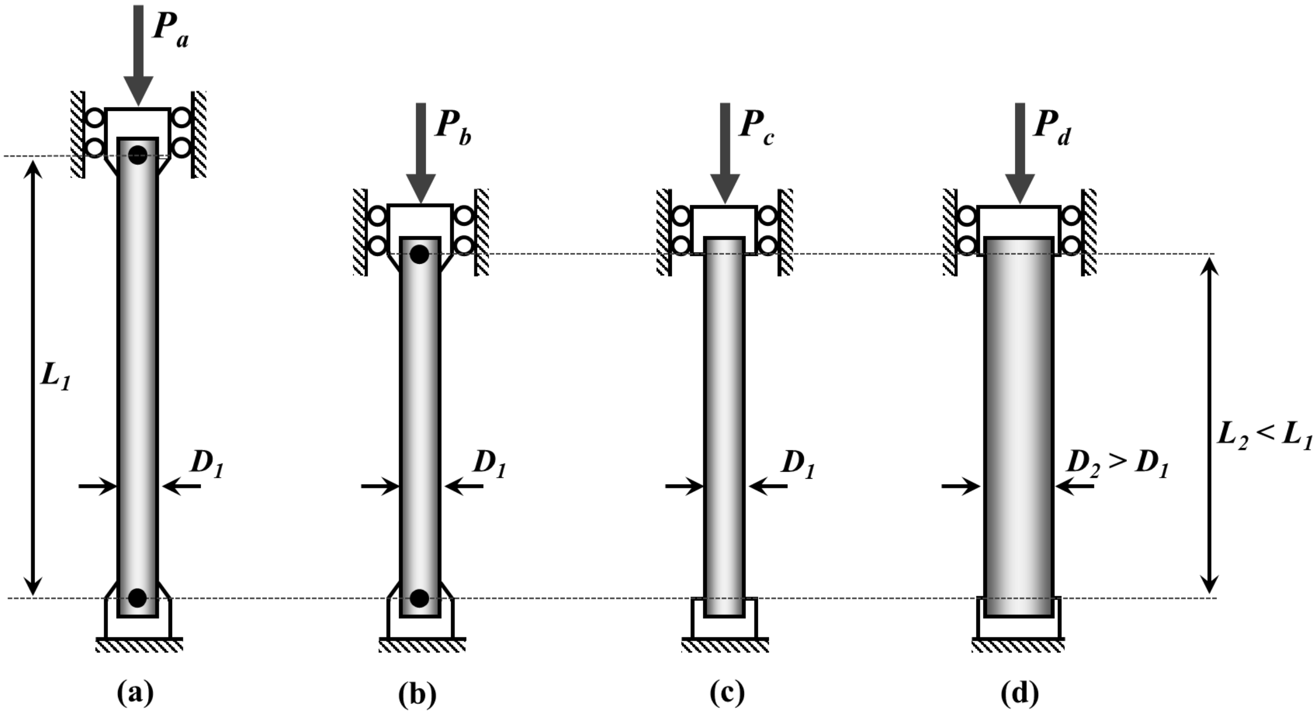
(Print) (Last) (First)

PART C – 6 points



Solid cylindrical columns (a), (b), (c), and (d) are made of the same material with Young's modulus $\boldsymbol{E}$. A compressive axial load is applied to each column. Let $\boldsymbol{P_{a,cr}}$, $\boldsymbol{P_{b,cr}}$, $\boldsymbol{P_{c,cr}}$, and $\boldsymbol{P_{d,cr}}$ represent the critical buckling loads for columns (a), (b), (c), and (d), respectively, according to Euler's buckling theory.

<u>Rank order</u> the critical buckling loads for each column from 1 to 4, where 1 represents the *largest* critical buckling load, and 4 represents the *smallest* critical buckling load, on the lines below.

$\boldsymbol{P_{a,cr}}$ ________

$\boldsymbol{P_{b,cr}}$ ________

$\boldsymbol{P_{c,cr}}$ ________

$\boldsymbol{P_{d,cr}}$ ________

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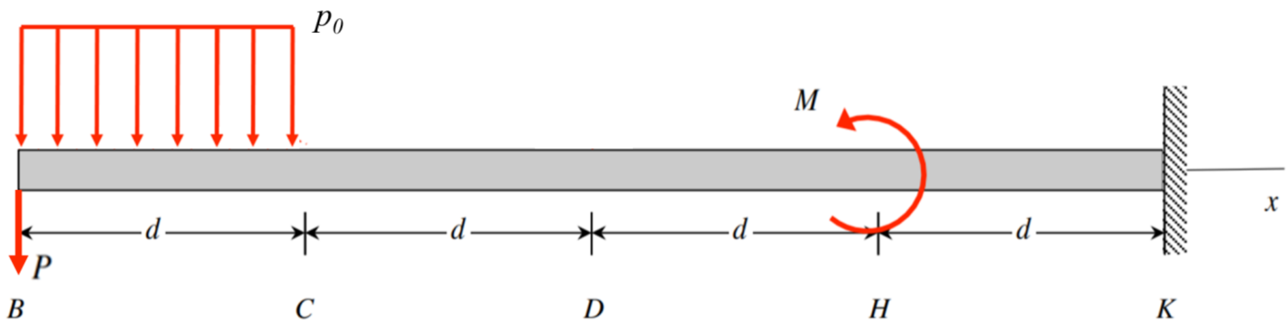
(Print) (Last) (First)

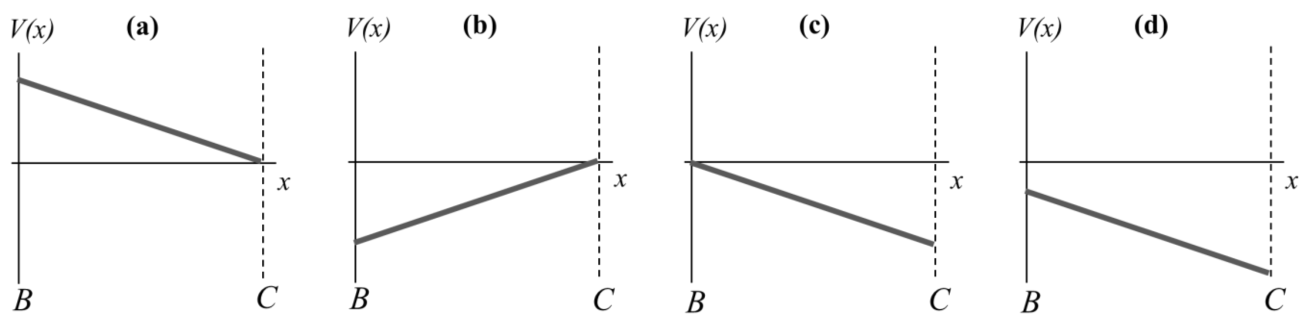
<u>PROBLEM #4 (cont.)</u>:

PART D – 4 points

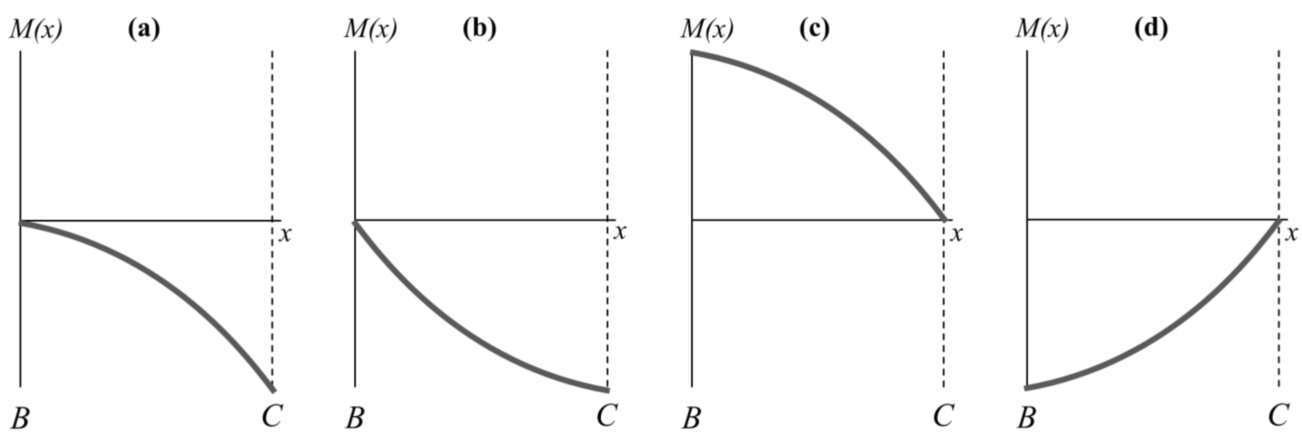
A cantilevered beam is loaded with a force ***P***, a distributed load $\boldsymbol{p_0}$, and a moment ***M***.



(i) Circle the answer that most accurately describes the internal **shear force** between points *B* and *C*.



(ii) Circle the answer that most accurately describes the internal **bending moment** between points B and C.



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(Print) **(Last)** **(First)**

PROBLEM #1 (25 Points):

Two elastic elements (1) and (2) are connected to the ends A and B of a rigid inverted T-shaped bar ABCD. Each elastic member has a Young's modulus E, circular cross section of radius R, length L, and yield stress σ_Y. The rigid bar is pinned at C and a load P is applied at end D, as shown in Figure 1.

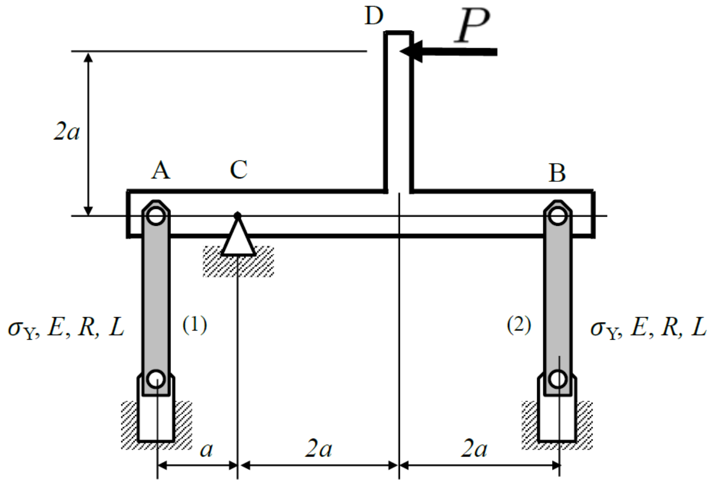

Figure 1

(a) Assuming both elastic elements are under tension, draw a free body diagram of the rigid T-shaped bar.

(b) Write the equilibrium equations for the rigid T-shaped bar and the compatibility condition(s) that relate the elongation of the elastic elements (1) and (2).

(c) Determine the axial force on elastic elements (1) and (2) and, for $P > 0$, indicate whether the element is under compression or tension.

(d) Determine the smallest force $P > 0$ ((i.e., the smallest force P in the direction shown in the figure)) that will induce *Euler buckling* on the assembly and indicate whether it will be on element (1) or (2). Express your result in terms of material properties and geometric parameters.

(e) If the slenderness of elements (1) and (2) is significantly reduced to avoid buckling, then determine the smallest force $P > 0$ that will induce *ductile failure* on the assembly and indicate whether it will be on element (1) or (2). Express your result in terms of material properties and geometric parameters.
Note: Use the maximum-distortional-energy theory.

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(Print) **(Last)** **(First)**

PROBLEM #2 (25 points)

An angled wrench ABC is fixed to the ground at A. The wrench is aligned along the *xz* plane as shown in the Figure 2A, such that the AB is along the *x* axis and BC is along the *z* axis. Two point-forces of the same magnitude *P*, **one in the negative *x* direction and the other in the positive *y* direction**, are applied at the end C. The segment AB has a circular cross section of the radius *r* as shown in Figure 2B. The length of AB and BC are *a* and *b*, respectively.

a) Determine the resultant load (forces and moments) on the cross section at the ground due to the applied forces at C.

b) Determine the state of stress at point M located on the cross section at the ground. Show the non-zero stresses on the given stress element.

c) Determine the state of stress at point N located on the cross section at the ground. Show the non-zero stresses on the given stress element.

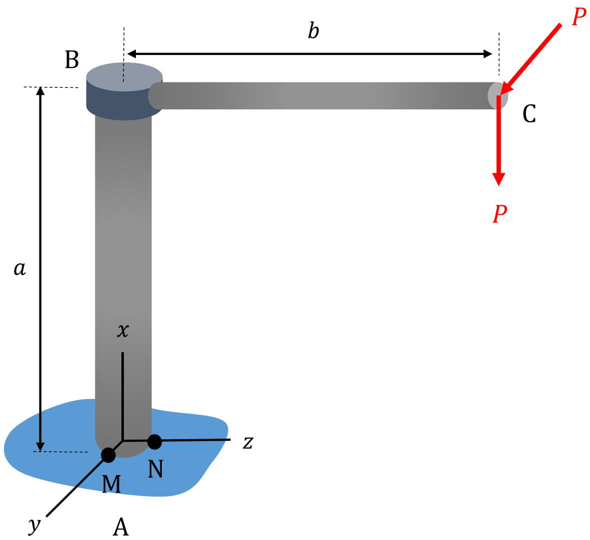


Figure 2A

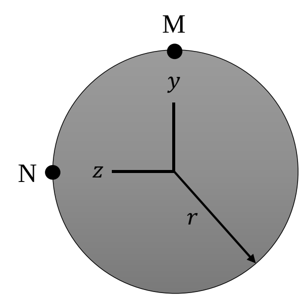


Figure 2B

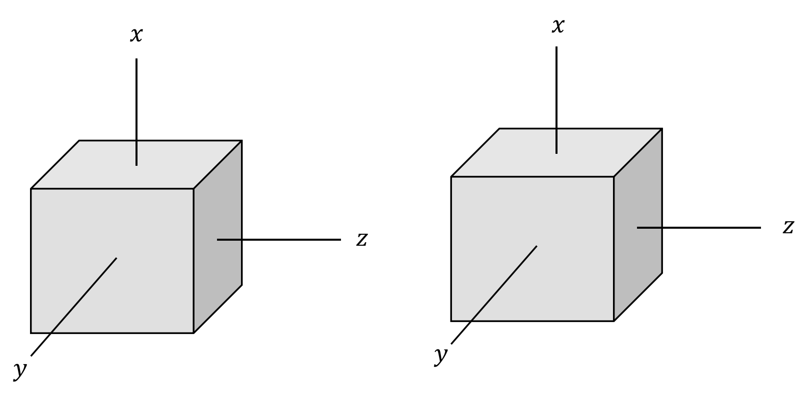


Figure 2C

PROBLEM #3 (25 points)

A point A on the structure in Figure 3A is subjected to in-plane stresses as shown in Figure 3B.

(a) Use the stress element in Figure 3B to draw the **Mohr's circle** on the attached graph paper.

(b) Use the **Mohr's circle** to calculate:

i. The principal stresses in the X-Y plane.
ii. The maximum in-plane shear stress.
iii. The absolute maximum shear stress.
iv. The angle of rotation from the X-axis to the direction of the in-plane principal stress σ_{p1}.
v. Draw a stress element to show the in-plane principal stresses correctly oriented with respect to the X axis

(c) Determine the normal and shear stresses in the X'-Y' directions, draw a stress element to show the calculated stresses, and mark the state of stress in the X'-Y' directions on the Mohr's circle. Note: The X' axis is oriented at 45° from the X axis as shown in Figure 3A.

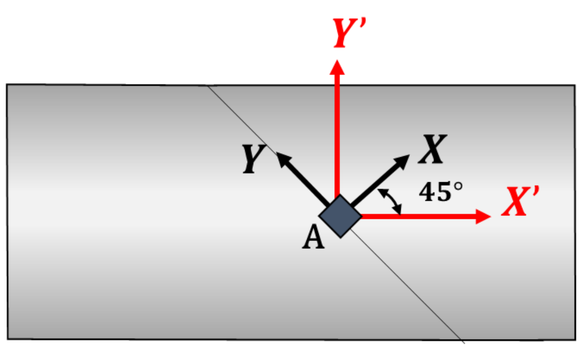


Figure 3A: Structure with element A

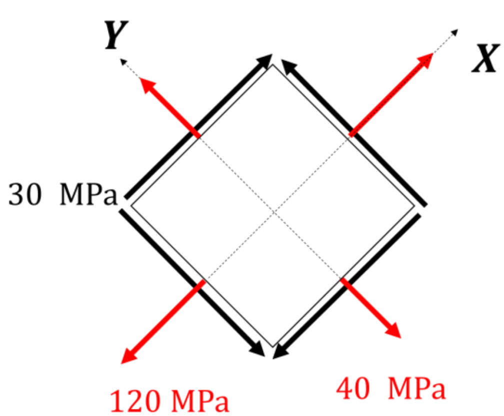
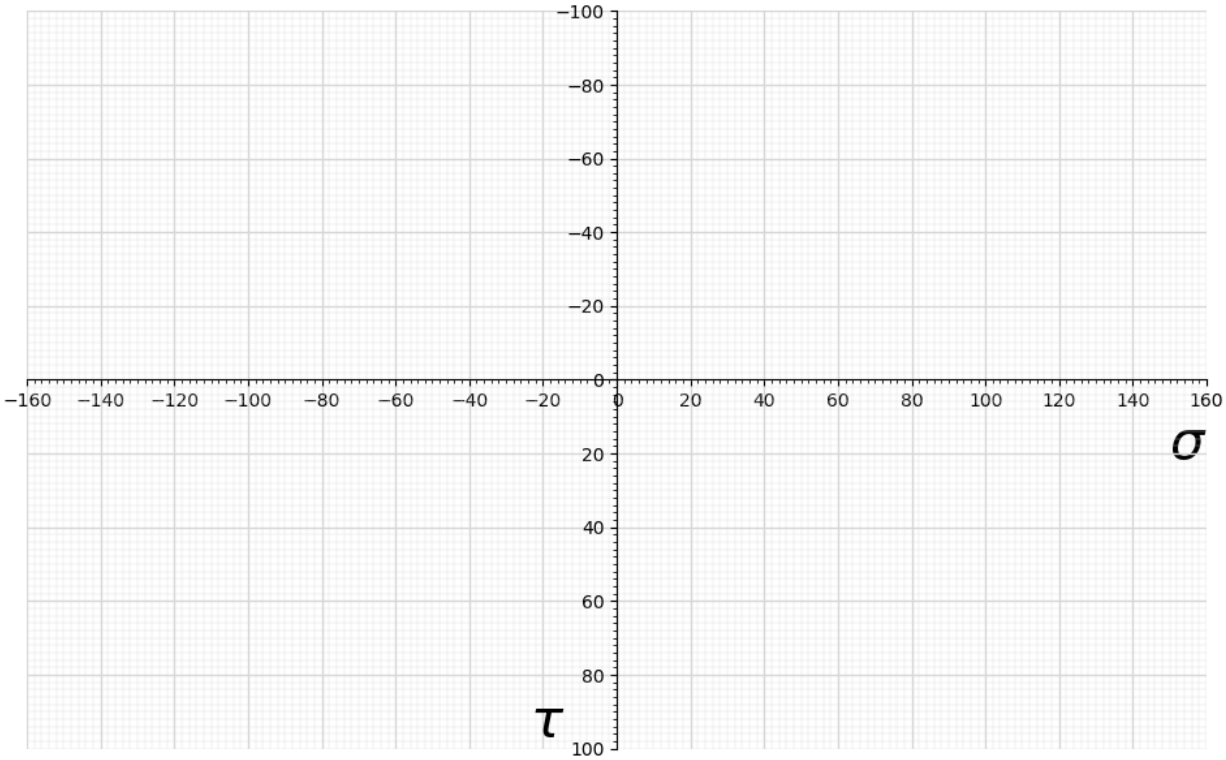


Figure 3B: Stress element A


−100
−80
−60
−40
−20
0
−160 −140 −120 −100 −80 −60 −40 −20 0 20 40 60 80 100 120 140 160
σ
20
40
60
80
τ
100