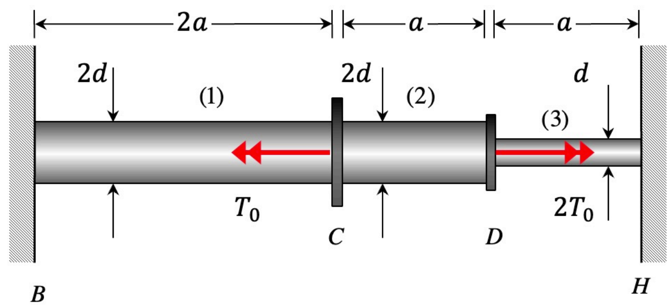
ME 323 Exam No. 2 **Name** *(Print)* SOLUTION
November 5, 2025 *(Last)* *(First)*

PROBLEM NO. 1 – 25 points max.

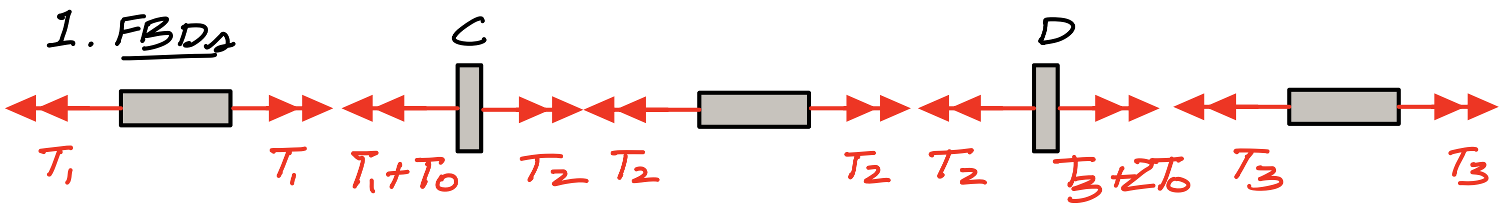
You are analyzing a stationary steel shaft in a transmission system. The shaft shown is made up of three segments having solid circular cross-sections, each having a shear modulus of G. The shaft has a total length $4a$. It is rigidly fixed against rotation at both ends (B and H). During operation, rigid gears mounted at points C and D apply concentrated torques to the shaft, as shown in the schematic below.



Follow the steps provided below and express your answers in terms of T_0, a, G and d.

1. Draw the free body diagram showing all relevant external and reactive torques. Write down the equation(s) of static equilibrium. Determine if the problem is statically determinate or indeterminate. Justify your answer.
2. Determine the polar area moment I_p for the cross-section of each segment of the shaft.
3. Write down the torque-angle of twist relationships for each of the segment of the shaft.
4. Write the compatibility equation for the shaft.
5. Determine the angle of twist at C (ϕ_C), and D (ϕ_D) relative to fixed end B.
6. Determine the maximum shear stress in each of the three segments of the shaft $(\tau_{max})_1$, $(\tau_{max})_2$, and $(\tau_{max})_3$.

1. FBDs



(1) $C: \Sigma M = -(T_1+T_0) + T_2 = 0 \Rightarrow T_1 = T_2 - T_0$

(2) $D: \Sigma M = T_3 + 2T_0 - T_2 = 0 \Rightarrow T_3 = T_2 - 2T_0$

INDETERMINATE

2. Polar area moments

$$I_{p1} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4 = I_{p2}$$

$$I_{p3} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$$

3. Torque/angle of twist

(3) $$\Delta\phi_1 = \frac{T_1(2a)}{(\pi d^4/2)G} = \frac{4}{\pi d^2}\frac{T_1 a}{G}$$

(4) $$\Delta\phi_2 = \frac{T_2 a}{(\pi d^4/2)G} = \frac{2}{\pi d^4}\frac{T_2 a}{G}$$

(5) $\Delta\phi_3 = \dfrac{T_3 a}{(\pi d^4/32)G} = \dfrac{32}{\pi d^4}\dfrac{T_3 a}{G}$

4. <u>Compatibility</u>

$$\phi_C = \cancelto{0}{\phi_B} + \Delta\phi_1 = \Delta\phi_1$$

$$\phi_D = \phi_C + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2$$

(6) $\phi_H = \phi_D + \Delta\phi_3 = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3 = 0$

5. <u>Solve for torques & angles of twist</u>

(7) (3) − (6) $\Rightarrow \dfrac{4\cancel{a}}{\cancel{\pi G d^4}} T_1 + \dfrac{2\cancel{a}}{\cancel{\pi G d^4}} T_2 + \dfrac{32\cancel{a}}{\cancel{\pi G d^4}} T_3 = 0$

(1), (2), (7) $\Rightarrow 4(T_2 - T_0) + 2T_2 + 32(T_2 - 2T_0) = 0$

$\hookrightarrow 38T_2 = 68T_0 \Rightarrow T_2 = \dfrac{34}{19} T_0$

(1) $\Rightarrow T_1 = \dfrac{34}{19} T_0 - T_0 = \dfrac{15}{19} T_0$

(2) $\Rightarrow T_3 = \dfrac{34}{19} T_0 - 2T_0 = -\dfrac{4}{19} T_0$

$\therefore \quad \phi_C = \Delta\phi_1 = \dfrac{4a}{\pi G d^4}\left(\dfrac{15}{19} T_0\right) = \dfrac{60}{19}\dfrac{T_0 a}{\pi G d^4}$ $\longleftarrow \phi_C$

$\phi_D = \Delta\phi_1 + \Delta\phi_2 = \dfrac{60}{19}\left(\dfrac{T_0 a}{\pi G d^4}\right) + \dfrac{2}{\pi d^4}\left(\dfrac{34}{19} T_0\right) a$ $\longleftarrow \phi_D$

$= \dfrac{128}{19}\dfrac{T_0 a}{\pi G d^4}$

6. <u>Maximum shear stresses</u>

$\tau_{1,max} = \dfrac{T_1(2d/2)}{I_{p1}} = \dfrac{\left(\frac{15}{19} T_0\right) d}{(\pi d^4/2)} = \dfrac{30}{19\pi}\dfrac{T_0}{G d^3}$ $\longleftarrow \tau_{1,max}$

$\tau_{2,max} = \dfrac{T_2(2d/2)}{I_{p2}} = \dfrac{\left(\frac{34}{19} T_0\right) d}{\pi d^4/2} = \dfrac{68}{19\pi}\dfrac{T_0}{G d^3}$ $\longleftarrow \tau_{2,max}$

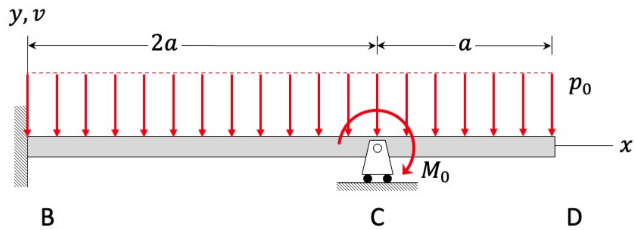
$\tau_{3,max} = \dfrac{T_3(d/2)}{I_{p3}} = -\dfrac{\left(\frac{4}{19} T_0\right)\frac{d}{2}}{(\pi d^4/32)} = -\dfrac{64}{19\pi}\dfrac{T_0}{G d^3}$ $\longleftarrow \tau_{3,max}$

ME 323 Exam No. 2 **Name** *(Print)* SOLUTION

November 5, 2025 *(Last)* *(First)*

PROBLEM NO. 2 – 25 points max.

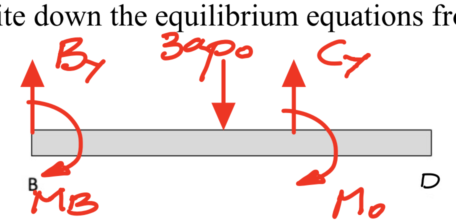
A propped-cantilever beam is acted upon by a constant line load p_0 along its length and by a couple $M_0 = p_0 a^2$ at the roller support C. The beam is made up of a material having a Young's modulus of E and whose cross-section has a second area moment of I. You are asked in this problem to determine the reactions on the beam at end B and the beam rotation at C using the 2nd-order integration method. You are asked to *follow the prescribed steps below*.



a) Write down the boundary conditions for this problem.

$v(0) = \theta(0) = v(2a) = 0$

b) Draw a free body diagram (FBD) of the beam and write down the equilibrium equations from this FBD.



$\Sigma M_C = -M_0 + (3ap_0)\left(\frac{a}{2}\right) - B_y(2a) - M_B = 0$

$\hookrightarrow M_B = -p_0 a^2 + \frac{3}{2} p_0 a^2 - 2B_y a$

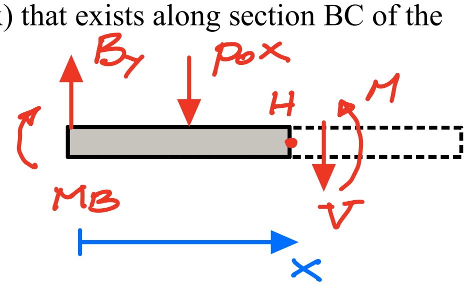
(1) $= \frac{1}{2} p_0 a^2 - 2B_y a$

(2) $\Sigma F_y = B_y + C_y - 3ap_0 = 0$

c) State whether the problem of determining the reactions on the beam is *determinate* or *indeterminate*.

Two equations/3 unknowns $\Rightarrow$ INDETERMINATE

d) Determine the expression for the bending moment M(x) that exists along section BC of the beam.



$\Sigma M_H = M + (p_0 x)\left(\frac{x}{2}\right) - B_y x - M_B = 0$

$\hookrightarrow M(x) = M_B + B_y x - \frac{1}{2} p_0 x^2$

e) Using the 2nd-order integration method to determine the general expression for the beam rotation angle $\theta(x)$ and the beam deflection $v(x)$ along section BC of the beam. These expressions should be in terms of, at most: the external reactions on the beam, x, E, I and p_0.

(3) $\therefore \ \theta(x) = \cancel{\theta(0)}^{\,0} + \frac{1}{EI}\int_0^x \left(M_B + B_y x - \frac{1}{2}p_0 x^2\right)dx$

$= \frac{1}{EI}\left[M_B x + \frac{1}{2}B_y x^2 - \frac{1}{6}p_0 x^3\right]$

(4) $v(x) = \cancel{v(0)}^{\,0} + \int_0^x \frac{1}{EI}\left[M_B x + \frac{1}{2}B_y x^2 - \frac{1}{6}p_0 x^3\right]dx$

$= \frac{1}{EI}\left[\frac{1}{2}M_B x^2 + \frac{1}{6}B_y x^3 - \frac{1}{24}p_0 x^4\right]$

f) Using your results above in e), determine the reactions on the beam at B and the beam rotation at C. Your answers should be in terms of, at most: E, I, a and p_0.

$v(2a) = 0 = \cancel{\frac{1}{EI}}\left[\frac{1}{2}M_B(2a)^2 + \frac{1}{6}B_y(2a)^3 - \frac{1}{24}p_0(2a)^4\right]$

$= 2M_B\cancel{a^2} + \frac{4}{3}B_y a^{\cancel{3}} - \frac{2}{3}p_0 a^{\cancel{4}\,2}$

(5) $\therefore \ M_B = -\frac{2}{3}B_y a + \frac{1}{3}p_0 a^2$

• (1) and (5):

$\frac{1}{2}p_0 a^2 - 2B_y a = -\frac{2}{3}B_y a + \frac{1}{3}p_0 a^2$

↳ $B_y = \frac{1}{8}p_0 a$ ← B_y

• (5): $M_B = -\frac{2}{3}\left(\frac{1}{8}p_0 a\right)a + \frac{1}{3}p_0 a^2 = \frac{1}{4}p_0 a^2$ ← M_B

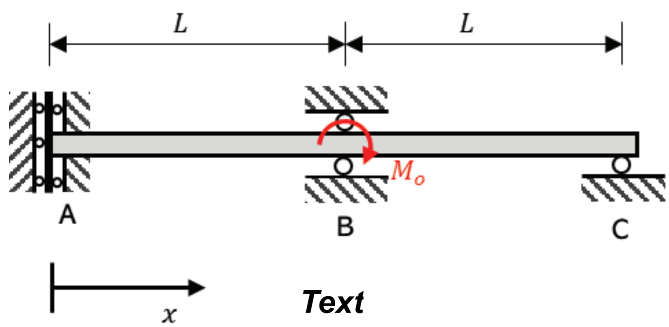
• (3): $\theta_C = \theta(2a) = \frac{1}{EI}\left[\frac{1}{4}p_0 a^2(2a) + \frac{1}{2}\left(\frac{1}{8}p_0 a\right)(2a)^2 - \frac{1}{6}p_0(2a)^3\right]$

$= -\frac{7}{12}\frac{p_0 a^3}{EI}$ ← θ_C

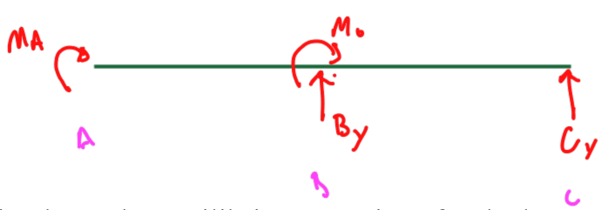
Please note that there are THREE solutions provided for Problem 3: one for each possible redundant load.

PROBLEM NO. 3 – 25 points max.

The beam ABC has an elastic modulus E and a second area moment I. The beam is supported by a vertical slider at A and by rollers at B and C. A bending moment M_o is applied at B. Assume that deformation due to shear is negligible. Use the coordinate system provided with its origin at A.



a) Draw the free body diagram for the beam ABC. 3



b) Write down the equilibrium equations for the beam. 2

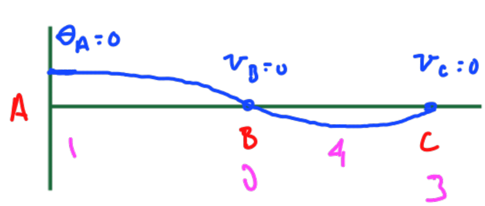
$\Sigma F_y = 0: \quad B_y + C_y = 0 \quad \rightarrow \quad C_y = -B_y$

$\Sigma M_C^{\circlearrowleft +} = 0: \quad -M_A - M_o - B_y L = 0 \quad \rightarrow \quad B_y = -\dfrac{M_A}{L} - \dfrac{M_o}{L}$

c) State if the beam is statically determinate or indeterminate (justify your answer). 2

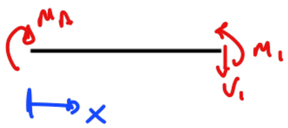
3 Unknowns > 2 Equations ∴ Indeterminate

d) Make a sketch of the expected shape of the beam deflection resulting from this applied load. 4



e) Write down the strain energy equations for beam segments AB and BC in terms of the redundant support of your choice. Neglect the contribution of shear to the strain energy.

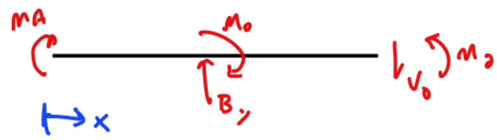
[A,B]



$\sum M_C^{\curvearrowleft +} : 0 = M_1 - M_A = 0$

$M_1 = M_A$

$$U_{AB} = \frac{1}{2EI}\int_0^L M_A^2\,dx$$



$\sum M_C^{\curvearrowleft +} = 0 = -M_A - M_o - B_y(x-L) + M_2 = 0$

$$M_2 = M_A + M_o + \underbrace{B_y}_{\left(-\frac{M_A}{L} - \frac{M_o}{L}\right)}(x-L)$$

$$U_{BC} = \frac{1}{2EI}\int_L^{2L}\left(-\frac{M_A x}{L} - \frac{M_o}{L}x + 2M_A + 2M_o\right)^2 dx$$

$$U_T = U_{AB} + U_{BC}$$

f) Determine reaction forces and moments using Castigliano's second theorem. Your answer should be in terms of, at most: M_o, E, I, L.

$$U_T = \frac{1}{2EI}\int_0^L M_A^2\,dx + \frac{1}{2EI}\int_L^{2L}\left(-\frac{M_A x}{L} - \frac{M_o}{L}x + 2M_A + 2M_o\right)^2 dx$$

$$\theta_A = 0 = \frac{\partial U_T}{\partial M_A} = \int_0^L M_A\,dx + \int_L^{2L}\left(M_A\left(2-\frac{x}{L}\right) + M_o\left(2-\frac{x}{L}\right)\right)\left(2-\frac{x}{L}\right)dx$$

$$0 = M_A x\Big|_0^L + \left(2M_A\left(2x - \frac{x^2}{2L}\right) + 2M_o\left(2x - \frac{x^2}{2L}\right) - M_A\left(\frac{x^2}{L} - \frac{x^3}{3L^2}\right) - M_o\left(\frac{x^2}{L} - \frac{x^3}{3L^2}\right)\right)\Big|_L^{2L}$$

$$0 = M_A x\Big|_0^L + (M_A + M_o)\left(4x - \frac{2x^2}{L} + \frac{x^3}{3L^2}\right)\Big|_L^{2L}$$

$$0 = M_A L + (M_A + M_o)\left[\cancel{8L} - \cancel{8L} + \frac{8L}{3} - 4L + 2L - \frac{L}{3}\right]$$

$$0 = M_A L + \frac{L}{3}M_A + \frac{L}{3}M_o$$

$$\frac{4}{3}LM_A = -\frac{L}{3}M_o \rightarrow \boxed{M_A = -\frac{M_o}{4}}$$

$$\boxed{B_y = +\frac{M_o}{4L} - \frac{M_o}{L} = -\frac{3M_o}{4L}}$$

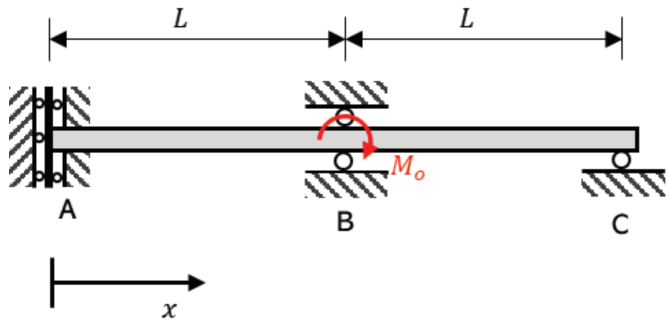
$$\boxed{C_y = -B_y = \frac{3M_o}{4L}}$$

Please note that there are THREE solutions provided for Problem 3: one for each possible redundant load.

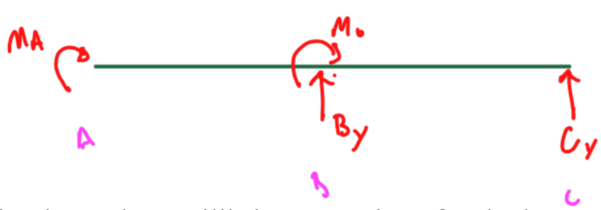
ME 323 Exam No. 2 **Name** *(Print)* Solution B
November 5, 2025 *(Last)* *(First)*

PROBLEM NO. 3 – 25 points max.

The beam ABC has an elastic modulus E and a second area moment I. The beam is supported by a vertical slider at A and by rollers at B and C. A bending moment M_o is applied at B. Assume that deformation due to shear is negligible. Use the coordinate system provided with its origin at A.



a) Draw the free body diagram for the beam ABC. 3



b) Write down the equilibrium equations for the beam. 2

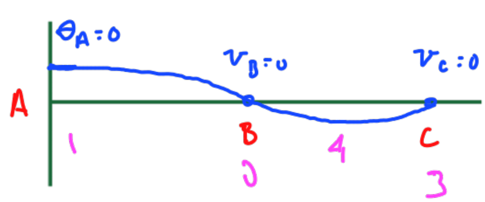
$\Sigma F_y = 0: \; B_y + C_y = 0 \;\rightarrow\; C_y = -B_y$

$\Sigma M_C = 0: \; -M_A - M_o - B_y L = 0 \;\rightarrow\; M_A = -M_o - B_y L$

c) State if the beam is statically determinate or indeterminate (justify your answer). 2

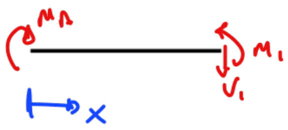
3 Unknowns > 2 Equations $\therefore$ Indeterminate

d) Make a sketch of the expected shape of the beam deflection resulting from this applied load. 4



e) Write down the strain energy equations for beam segments AB and BC in terms of the redundant support of your choice. Neglect the contribution of shear to the strain energy.

[A, B]

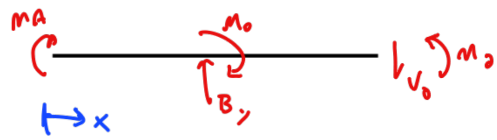


$\Sigma M_C = 0: M_1 - M_A = 0$

$M_1 = M_A$

$M_1 = -M_0 - B_y L$

$U_{AB} = \frac{1}{2EI}\int_0^L (-M_0 - B_y L)^2 \, dx$



$\Sigma M_C = 0 = -M_A - M_0 - B_y(x-L) + M_2 = 0$

$M_2 = \underbrace{M_A}_{-M_0 - B_y L} + M_0 + B_y(x-L)$

$U_{BC} = \frac{1}{2EI}\int_L^{2L} (B_y x - 2B_y L)^2 \, dx$

$U_T = U_{AB} + U_{BC}$

f) Determine reaction forces and moments using Castigliano's second theorem. Your answer should be in terms of, at most: M_o, E, I, L.

$$U_T = \frac{1}{2EI}\int_0^L (-M_0 - B_y L)^2 \, dx + \frac{1}{2EI}\int_L^{2L} (B_y x - 2B_y L)^2 \, dx$$

$$v_B = 0 = \frac{\partial U_T}{\partial B_y} = \int_0^L (-M_0 - B_y L)(-L)\, dx + \int_L^{2L} (B_y x - 2B_y L)(x - 2L)\, dx$$

$$0 = M_0 L x + B_y L^2 x \Big|_0^L + \frac{B_y x^3}{3} - \underbrace{\frac{\cancel{2} B_y L x^2}{\cancel{2}} - \frac{\cancel{2} B_y L x^2}{\cancel{2}}}_{-2B_y L x^2} + 4 B_y L^2 x \Big|_L^{2L}$$

$$0 = M_0 L^2 + B_y L^3 + \frac{8 B_y L^3}{3} - \cancel{8 B_y} L^3 + \cancel{2B_y} L^3 - \frac{B_y L^3}{3} + 2B_y L^3 - 4 B_y L^3$$

$$\frac{4 B_y L^3}{3} + M_0 L^2 = 0$$

$\boxed{B_y = -\frac{3M_0}{4L}}$ $\boxed{C_y = -B_y = \frac{3M_0}{4L}}$

$M_A = -M_0 - B_y L$

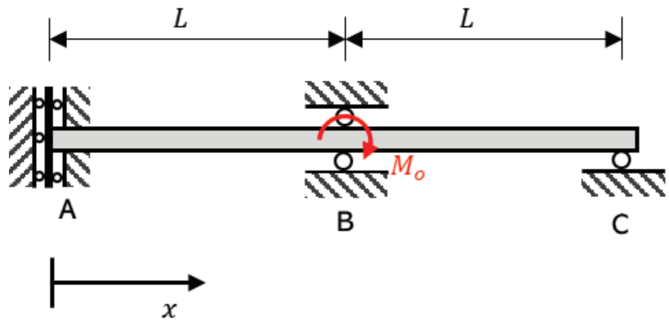
$\boxed{M_A = -M_0 + \frac{3M_0}{4} = -\frac{M_0}{4}}$

Please note that there are THREE solutions provided for Problem 3: one for each possible redundant load.

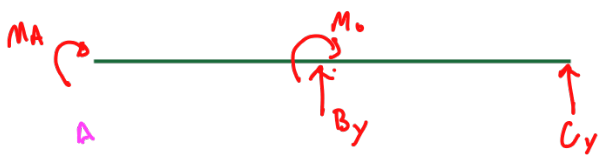
ME 323 Exam No. 2 **Name** *(Print)* Solution C
November 5, 2025 *(Last)* *(First)*

PROBLEM NO. 3 – 25 points max.

The beam ABC has an elastic modulus E and a second area moment I. The beam is supported by a vertical slider at A and by rollers at B and C. A bending moment M_o is applied at B. Assume that deformation due to shear is negligible. Use the coordinate system provided with its origin at A.



a) Draw the free body diagram for the beam ABC. 3



b) Write down the equilibrium equations for the beam. 2

$\Sigma F_y = 0$

$B_y + C_y = 0$

$B_y = -C_y$

$\Sigma M_A = 0$ (counterclockwise +)

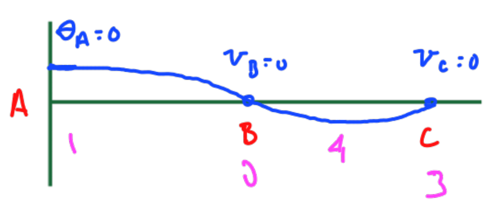
$-M_A + B_y L + 2 C_y L - M_o = 0$

$\rightarrow M_A = B_y L + 2 C_y L - M_o = C_y L - M_o$

c) State if the beam is statically determinate or indeterminate (justify your answer). 2

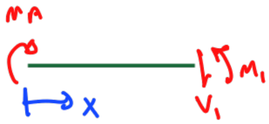
3 Unknowns > 2 Equations $\therefore$ Indeterminate

d) Make a sketch of the expected shape of the beam deflection resulting from this applied load. 4



e) Write down the strain energy equations for beam segments AB and BC in terms of the redundant support of your choice. Neglect the contribution of shear to the strain energy.

[A, B]



$\sum \overset{\curvearrowleft +}{M_c} = 0 = M_1 - M_A = 0$

(2) $M_1 = M_A$

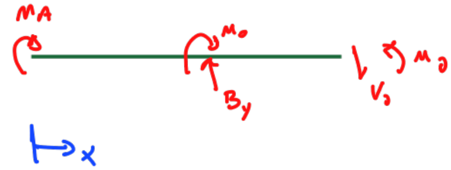
$U_{AB} = \frac{1}{2EI}\int_0^L M_A^2 \, dx$

$U_{AB} = \frac{1}{2EI}\int_0^L (C_yL - M_o)^2 \, dx$

(2)

$U_T = U_{AB} + U_{BC}$

[B, C]



$\sum \overset{\curvearrowleft +}{M_c} = M_2 - M_A - M_0 - B_y(x-L) = 0$

$M_2 = M_A + M_o + B_y(x-L)$ (2)

$U_{BC} = \frac{1}{2EI}\int_L^{2L}\Big[\underset{C_yL - M_o}{\underbrace{M_A + M_o}} + \underset{-C_y}{\underbrace{B_y}}(x-L)\Big]^2 dx$

$U_{BC} = \frac{1}{2EI}\int_L^{2L}(2C_yL - C_yx)^2 \, dx$ (2)

f) Determine reaction forces and moments using Castigliano's second theorem. Your answer should be in terms of, at most: M_o, E, I, L.

$U_T = \frac{1}{2EI}\int_0^L (C_yL - M_o)^2 dx + \frac{1}{2EI}\int_L^{2L}(2C_yL - C_yx)^2 dx$

A $v_c = 0 = \frac{\partial U_T}{\partial C_y} = \int_0^L (C_yL - M_o)(L)\,dx + \int_L^{2L}(2C_yL - C_yx)(2L - x)\,dx$

$0 = C_yL^2x - M_oLx\Big|_0^L + 4C_yL^2x \underbrace{- \frac{\cancel{2}C_yLx^2}{\cancel{2}} - \frac{\cancel{2}C_yLx^2}{2}}_{-2C_yLx^2} + \frac{C_yx^3}{3}\Big|_L^{2L}$

$0 = C_yL^3 - M_oL^2 + \cancel{8C_yL^3} - \cancel{8C_yL^3} + \frac{8}{3}C_yL^3 - 4C_yL^3 + 2C_yL^3 - \frac{C_yL^3}{3}$

$0 = \frac{4}{3}C_yL^3 - M_oL^2 \rightarrow \boxed{C_y = \frac{3M_o}{4L}}\uparrow \quad \boxed{B_y = -C_y = -\frac{3M_o}{4L}}\downarrow$

$\boxed{M_A = \frac{3}{4}M_o - M_o = -\frac{M_o}{4}}$ $\circlearrowleft$ CCW

2

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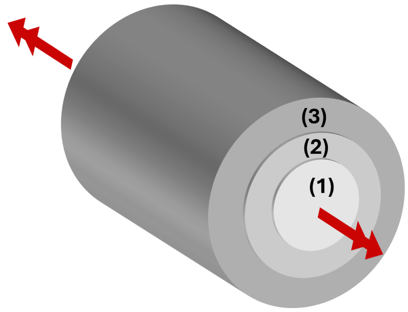
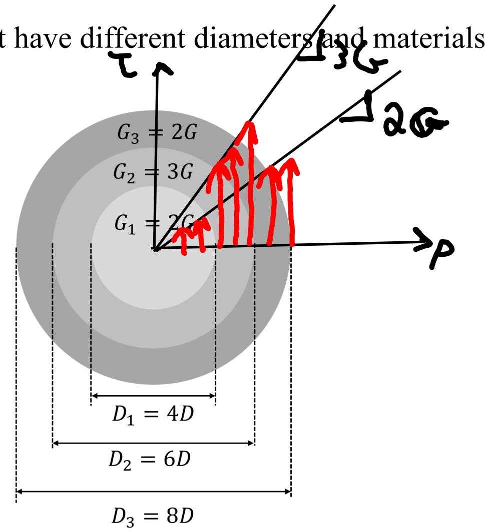
PROBLEM NO. 4 - PART A – 3 points max.

A torsion member consists of three concentric sections that have different diameters and materials properties as labeled in the image.





In what material is the maximum shear stress in the torsion member (circle one)?

A: (1)

B: (2)

C: (3)

$$\Delta\phi = \frac{TL}{GI_p} \Rightarrow T = GI_p\left(\frac{\Delta\phi}{L}\right)$$

$$\tau = \frac{T\rho}{I_p} = G\rho\left(\frac{\Delta\phi}{L}\right)$$

$$\tau_{max,1} = (2G)(2D)\left(\frac{\Delta\phi}{L}\right)$$

$$\tau_{max,2} = (3G)(3D)\left(\frac{\Delta\phi}{L}\right) \leftarrow \text{largest.}$$

$$\tau_{max,3} = (2G)(4D)\left(\frac{\Delta\phi}{L}\right)$$

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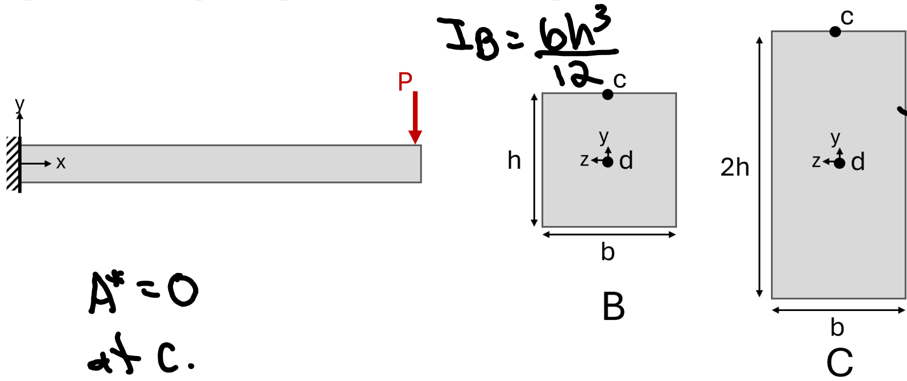
PROBLEM NO. 4 - PART B

A point load is applied at the end of a simple cantilever. We want to compare the stresses and deflections of two beams with different cross-sections: a square (B) and a rectangle (C). The stresses are compared at point c (the top) and point d (at the neutral plane).



$I_B = \frac{bh^3}{12}$ $I_C = \frac{b(2h)^3}{12}$ $I_C = 8I_B$

$\tau = \frac{VA^*\bar{y}^*}{It}$ $A^* = 0$ at c. $\tau_{max} = \frac{3V}{2A}$

B(i) (1 point) How does the ***shear stress at point c*** compare between the two beams (circle one)?

$\tau_C(c) = \tau_B(c)/4$
$\tau_C(c) = \tau_B(c)/2$
$\boxed{\tau_C(c) = \tau_B(c)}$
$\tau_C(c) = 2\tau_B(c)$
$\tau_C(c) = 4\tau_B(c)$

B(ii) (1 point) How does the ***shear stress at point d*** compare between the two beams (circle one)?

$A_C = 2A_B$

$\tau_C(d) = \tau_B(d)/4$
$\boxed{\tau_C(d) = \tau_B(d)/2}$
$\tau_C(d) = \tau_B(d)$
$\tau_C(d) = 2\tau_B(d)$
$\tau_C(d) = 4\tau_B(d)$

$\tau_{max,C} = \frac{3V}{2(2A_B)} = \frac{\tau_{max,B}}{2}$

B(iii) (1 point) How does the ***flexural stress at point c*** compare between the two beams (circle one)?

$\sigma_C(c) = \sigma_B(c)/4$ Typo
$\sigma_C(c) = \sigma_B(c)/2$
$\sigma_C(c) = \sigma_B(c)$ ⇒ everyone
$\sigma_C(c) = 2\sigma_B(c)$ gets this
$\sigma_C(c) = 4\sigma_B(c)$ point.

B(iv) (1 point) How does the ***flexural stress at point d*** compare between the two beams (circle one)?

$\sigma_C(d) = \sigma_B(d)/8$
$\sigma_C(d) = \sigma_B(d)/2$
$\boxed{\sigma_C(d) = \sigma_B(d)}$
$\sigma_C(d) = 2\sigma_B(d)$
$\sigma_C(d) = 8\sigma_B(d)$

$\sigma = -\frac{My}{I}$ $y = 0$

⇒ both zero.

B(v) (2 points) How does the ***maximum beam deflection*** compare between the two cross-sections (circle one)?

$\boxed{v_{max,C} = v_{max,B}/8}$
$v_{max,C} = v_{max,B}/2$
$v_{max,C} = v_{max,B}$
$v_{max,C} = 2v_{max,B}$
$v_{max,C} = 8v_{max,B}$

$v = \frac{1}{EI}\int_0^L\int_0^L M(x)\,dx\,dx.$

$v_C = \frac{1}{E(8I_B)}\int_0^L\int_0^L M(x)\,dx\,dx = \frac{v_B}{8}$

B(vi) (2 points) How does the ***strain energy due to flexural stresses (U_M)*** compare between the two beams (circle one)?

$\boxed{U_{M,C} = U_{M,B}/8}$
$U_{M,C} = U_{M,B}/2$
$U_{M,C} = U_{M,B}$
$U_{M,C} = 2U_{M,B}$
$U_{M,C} = 8U_{M,B}$

$U = \frac{1}{2EI}\int_0^L M^2\,dx.$

$U_C = \frac{1}{2E(8I_B)}\int_0^L M^2\,dx$

$U_C = \frac{U_B}{8}.$

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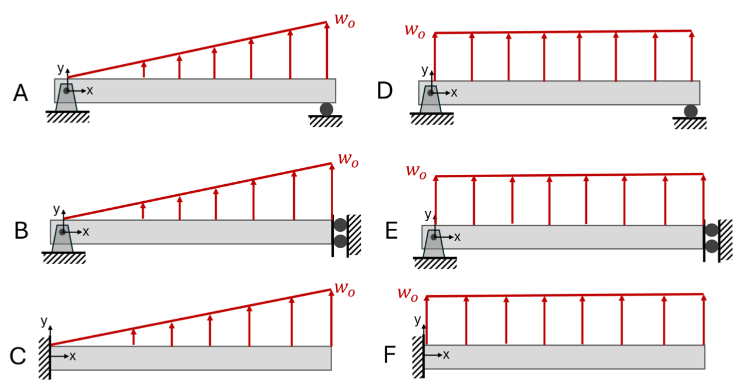
PROBLEM NO. 4 - PART C – 4 points max.

The equation for the moment along the length (L) of a beam is given by:

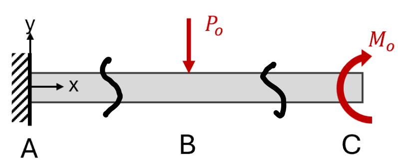
$$M(x) = \left(\frac{1}{2}\right) w_0 x^2 + w_o L x + \left(\frac{1}{2}\right) w_o L^2$$

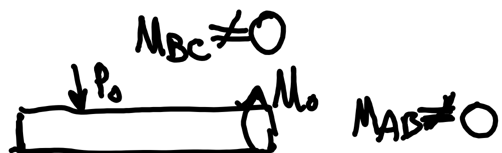
Typos ⇒ everyone gets these points.

Choose the loading condition that corresponds to this moment equation (circle one):



PROBLEM NO. 4 - PART D





$V_{AB} \neq 0$

D(i) (2 points) How many non-zero terms would be in the total equation for strain energy (each section of the structure can have 4 possible terms – axial, torsion, moment, and shear) (circle one)?

A: 2 B: 3 C: 4

D: 5 E: 6 F: 8

D(ii) (2 points) (separate from part (i)) To find the deflection of the beam in the y direction at B, you would need to take the partial derivative of the energy (U) with respect to:

A: P_o

B: M_o

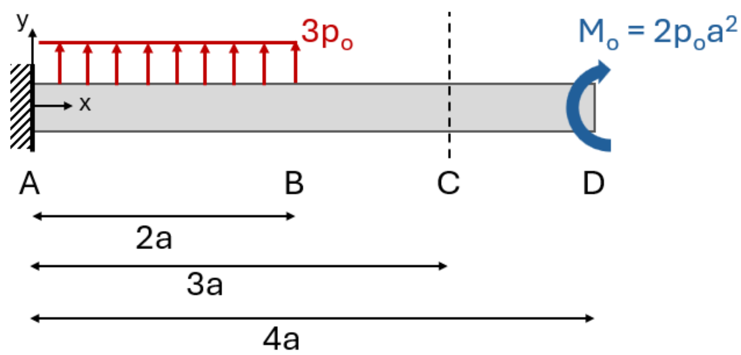
C: A dummy force at C

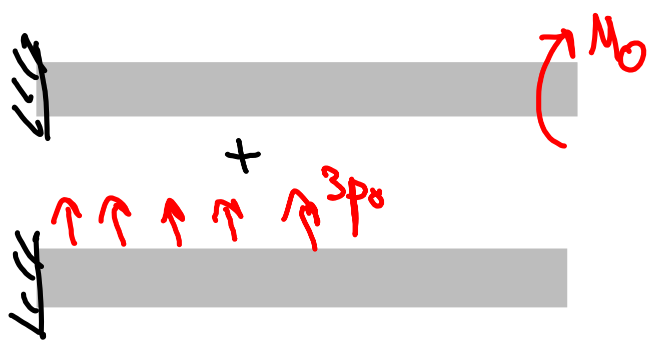
D: A dummy moment at B

E: A dummy torque at C

PROBLEM NO. 4 - PART E – 6 points max.

A simple cantilever has a Young's modulus of E and a second area moment of I. The beam has a couple M_0 applied at the end and a constant line load $3p_0$ applied on section AB of the beam. Use the superposition tables to find the displacement of point C in terms of p_o, a, E, and I.





$$v = v_{M_0} + v_{p_0}$$

$$v_{p_0} = \frac{x^2}{24}\left[6(2a)^2 - 4x(2a) + x^2\right]\frac{3p_0}{EI} \qquad 0 < x < 2a$$

$$\frac{(2a)^3}{24}\left[4x - 2a\right]\frac{3p_0}{EI} \qquad 2a < x < 4a$$

$$v_{M_0} = -\frac{1}{2}\left[x^2\right]\frac{(2p_0a^2)}{EI}$$

$$v(3a) = \frac{8a^3}{24}\left[12a - 2a\right]\frac{3p_0}{EI} - \frac{p_0a^2}{EI}\left[3a\right]^2$$

$$v(3a) = \frac{p_0a^4}{EI}\left[10 - 9\right] = \frac{p_0a^4}{EI}$$