

### ***Sample exam problems***

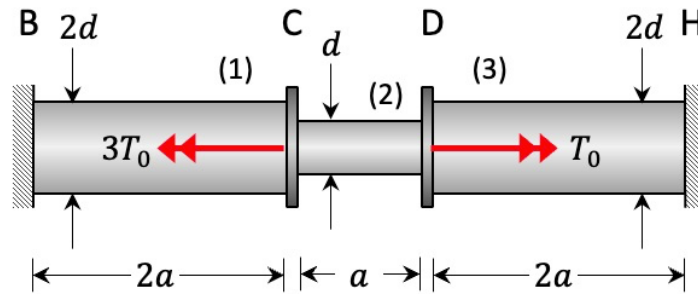
Attached here is a set of sample exam questions from past terms in the course. Please do not use these questions as an indication of which specific topics will appear on this term's midterm. These problems were chosen from past terms without regard to the specific questions that will appear on your exam.

Please use these questions to help you prepare for the midterm this summer. We will not be providing solutions for these questions, as we want you to use the questions to prepare under the exam-like situation of not knowing the answer, and you working through ways to check your work on your own, rather than looking at the answer to work backwards in checking your work. If you have questions regarding the solution of these problems, please check with your instructor and/or TA.

Please note that Weekly Joys has a good number of sample exam questions from past terms for which solutions are provided.

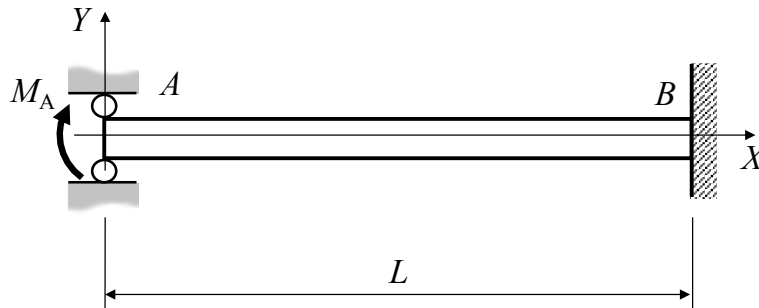
**Problem No. 2 – 20 pts**

The shaft shown is made up of three elements having solid circular cross-sections, each of a material having a shear modulus of  $G$ . Determine the torque carried by each element. *You are asked to clearly label the four steps in your solution.*



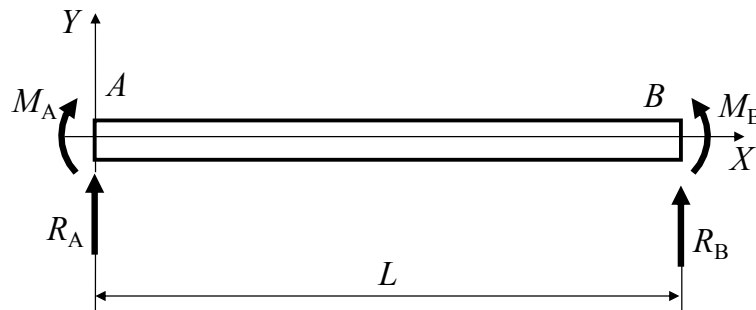
Name (Print) \_\_\_\_\_  
(Last) (First)

**PROBLEM # 2 (25 points).**



The linearly elastic beam shown in the figure supports a couple  $M_A$  at end A. The beam is homogeneous, with Young's modulus  $E$ , and has constant cross-section, with moment of inertia  $I$ .

- (a) Using the following free body diagram, write the equations of equilibrium and identify whether the structure is statically determinate or indeterminate.



Using the second-order integration method:

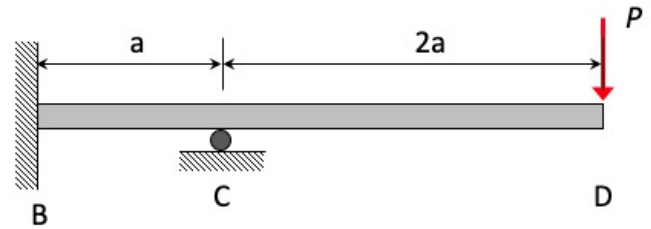
- (b) Determine the bending moment  $M(x)$  of the beam (as a function of the reactions at A, the external loads and the geometric parameters).
- (c) Determine the slope  $v'(x)$  and deflection  $v(x)$  of the beam.
- (d) Indicate the boundary conditions at supports A and B.
- (e) Solve for the reaction at A, i.e.,  $R_A$ .

## Examination 2 - Summer 2023

## Problem No. 2 – 20 pts

A point load  $P$  is applied to end D of a propped cantilevered beam. The beam is made up of a material having a Young's modulus of  $E$ , and the cross-section of the beam has a centroidal 2<sup>nd</sup> area moment of  $I$ .

You are asked to use the 2<sup>nd</sup>-order integration method to determine the reactions on the beam at end B. In order to receive full credit for your work, you will need to show the following in your solution:



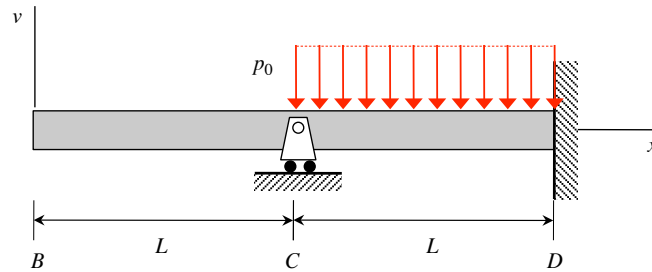
**Step 1 – Equilibrium analysis.** Draw a free body diagram (FBD) of the beam. Clearly label all forces and couples in your FBD. Write down the appropriate equilibrium equation(s) from this FBD. State whether the structure is statically determinate.

**Step 2 – Force/deformation.** Use equilibrium analyses to determine the bending moment in the beam over section BC. Use this result in integrations to determine the deflection  $v(x)$  along that section of the beam.

**Step 3 – Compatibility.** Enforce the boundary condition at C to produce an additional equation relating the reactions at B.

**Step 4 – Solve.** Solve your equations above for the reactions on the beam at B.

August 3, 2023

**PROBLEM NO. 1 – 20 points max.**

A beam having a constant cross-section with a second-order area moment of  $I$  and made up of a material with a Young's modulus of  $E$  is supported by a roller at  $C$  and a fixed support at end  $D$ . A constant line load of  $p_0$  acts on the beam between  $C$  and  $D$ . The goal of this problem is to determine the reaction force at the roller support  $C$  of the beam. Here, you are allowed to ignore shear effects in your work.

In order to receive full credit for your work, you will need to show the following in your solution:

**Step 1 – Equilibrium analysis.** Draw a free body diagram (FBD) of the entire beam. Clearly label all forces. Write down the appropriate equilibrium equations from this FBD. Choose your set of redundant load(s) for the problem.

**Step 2 – Strain energy function.** Write the strain energy function for the beam in terms of the redundant and applied loads. Be sure to include the appropriate free body diagrams on which these functions are based.

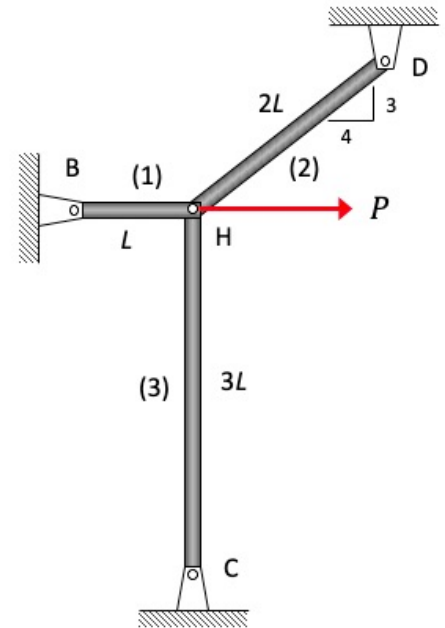
**Step 3 – Castigliano's 2<sup>nd</sup> theorem.** Apply Castigliano's 2<sup>nd</sup> theorem to set up the equation(s) needed to solve.

**Step 4 – Solve.** Solve your above equations for the reaction on the beam at  $C$ . Leave your answer in terms of  $p_0$  and  $L$ .

## Examination 2 - Summer 2023

## Problem No. 1 – 20 pts

The truss shown is made up of members (1), (2) and (3), with all members made up the same material (having a Young's modulus of  $E$ ) and having the same cross-sectional area  $A$ . The lengths of members (1), (2) and (3) are known to be  $L$ ,  $2L$  and  $3L$ , respectively. A force  $P$  acts to the right at joint H, as shown in the figure. The weights of the members can be considered to be negligible.



You are asked to use Castigliano's method to determine the load carried by member (2) in the truss. In order to receive full credit for your work, you will need to show the following in your solution:

**Step 1 – Equilibrium analysis.** Draw a free body diagram (FBD) of joint H. Clearly label all forces. Write down the appropriate equilibrium equations from this FBD. Choose your set of redundant load(s) for the problem.

**Step 2 – Strain energy function.** Write the strain energy function for the truss in terms of the redundant and applied load(s).

**Step 3 – Castigliano's 2<sup>nd</sup> theorem.** Apply Castigliano's 2<sup>nd</sup> theorem to set up the equation(s) needed to solve.

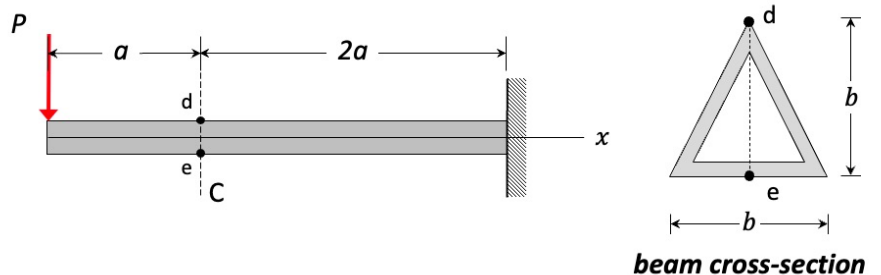
**Step 4 – Solve.** Solve your above equations for the load carried by member (2). Leave your answer in terms of  $P$ . State whether the member is in tension or compression.

## Problem No. 5 (continued)

**PART 5.C – 4 points**

Consider the end-loaded cantilevered beam shown that has a triangular-shaped tubular cross-section of uniform wall thickness.

Let  $\sigma_d$  and  $\sigma_e$  be the *normal stresses* at points “d” and “e” on the cross-section at C, respectively. Choose the correct response below. Provide a short justification.



- a)  $|\sigma_d| > |\sigma_e|$
- b)  $|\sigma_d| = |\sigma_e|$
- c)  $|\sigma_d| < |\sigma_e|$
- d) More information is needed in order to answer this question.

**PART 5.D – 4 points**

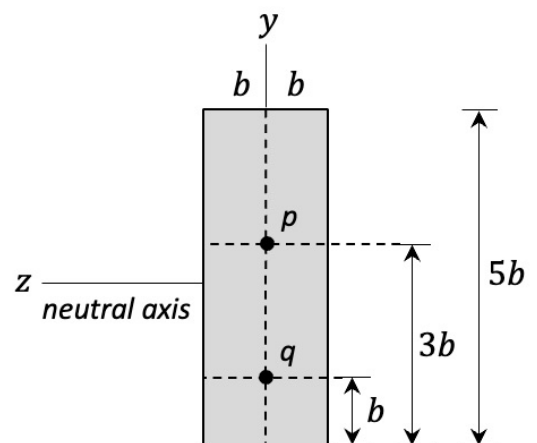
Same cantilevered beam as Part 5.C, except here the shape of the beam cross-section is unknown. Choose the correct TRUE/FALSE response below. Provide a short justification.

TRUE FALSE: Shear stress at “d” and “e” is zero, for any cross-section shape.

HINT: Consider the stress elements at “d” and “e”.

**PART 5.E – 4 points**

Again, the same cantilevered beam as Part 5.C, except here the beam has the rectangular cross-section shown. Let  $\tau_p$  and  $\tau_q$  represent the shear stress on the beam cross-section at points “p” and “q”. Choose the correct response below. Provide a short justification.



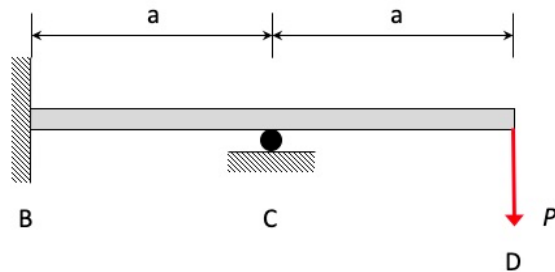
- a)  $|\tau_p| > |\tau_q|$
- b)  $|\tau_p| = |\tau_q|$
- c)  $|\tau_p| < |\tau_q|$
- d) More information is needed in order to answer this question.

## Examination 2 - Summer 2023

## Problem No. 4

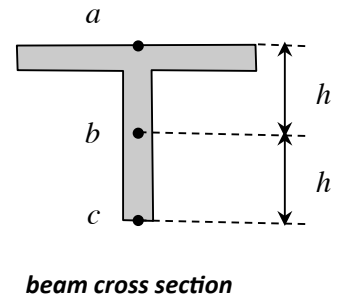
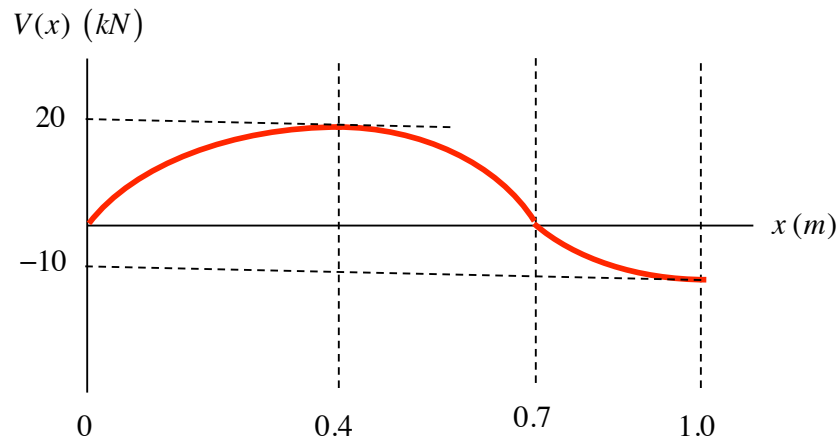
**PART C – 5 points**

A propped-cantilevered beam is acted upon by a downwardly-acting point load  $P$  at end D. The beam is made up of a material having a Young's modulus of  $E$  and has a centroidal 2<sup>nd</sup> area moment of  $I$ . Using the superposition approach, determine the reaction force on the beam at the roller support C. Leave your answer in terms of  $P$ .





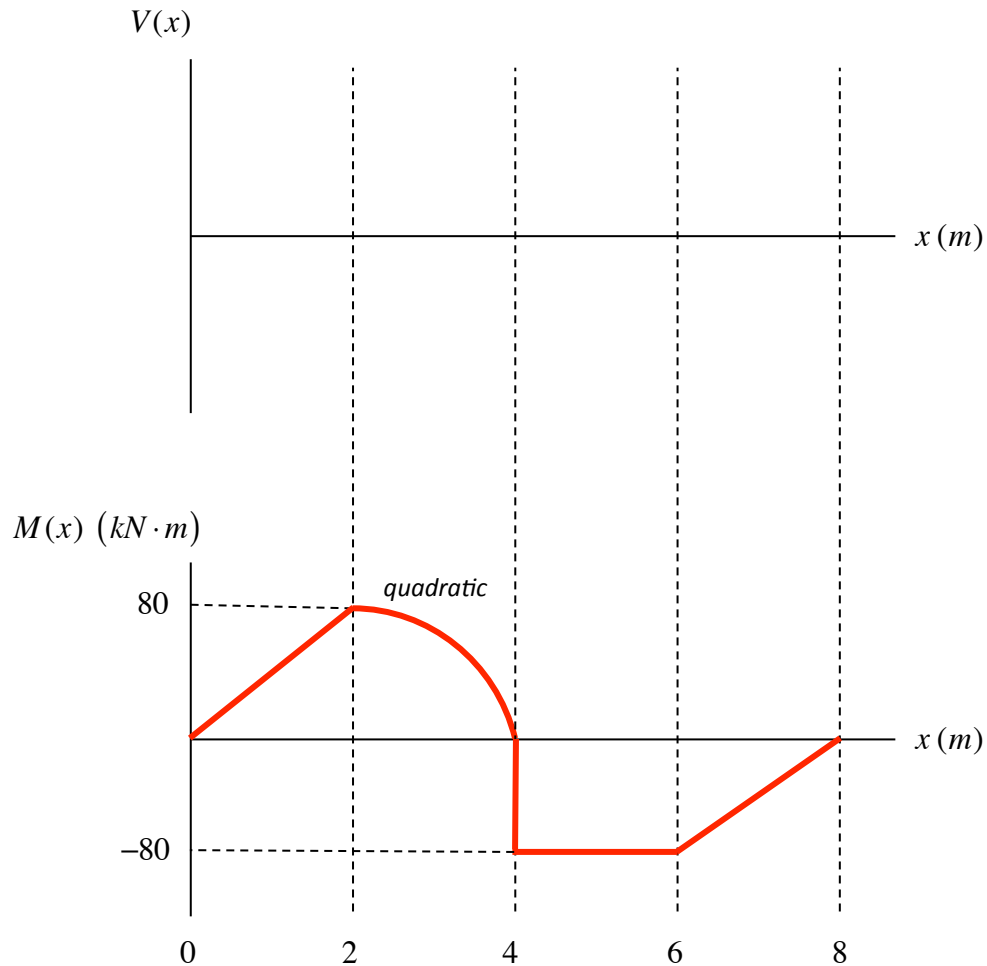
July 20, 2018

**PROBLEM NO. 4 - PART A – 4 points max.**

Consider a beam with a T-shaped cross section along a longitudinal  $x$ -axis. The distribution of shear force as a function of  $x$  is shown above. It is known that the bending moment in the beam at  $x = 0$  is given by  $M(0) = 0$ .

- At what location along the  $x$ -axis is the largest magnitude shear stress in the beam?
- For the location of maximum shear stress in a) above, which point on the cross section “a”, “b” or “c” has the largest magnitude shear stress?
- At what location along the  $x$ -axis is the largest magnitude normal stress in the beam?
- For the location of maximum normal stress in c) above, which point on the cross section “a”, “b” or “c” has the largest magnitude normal stress?

July 20, 2018

**PROBLEM NO. 4 - PART F – 5 points max.**

The loading of a beam produces the moment diagram shown above. All segments of  $M(x)$  are linear except for between  $2\text{m} < x < 4\text{m}$  where the relationship is quadratic.

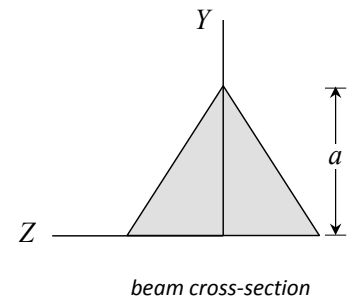
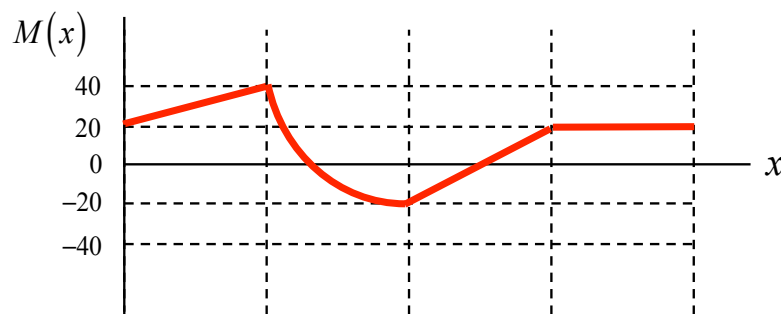
Draw the shear force  $V(x)$  vs.  $x$  for this beam.

**PROBLEM #4** (*continued*)

**Part C – 6 points**

The bending moment diagram for a loaded beam is shown below. The beam is known to have the triangular cross section shown below. Provide a justification for each answer.

- At what location(s) on the beam does the maximum *tensile* normal stress exist? Provide both  $x$  and  $Y$  components of the location of this point(s). You are not asked to solve for this value of stress.
- At what location(s) on the beam does the maximum *compressive* normal stress exist? Provide both  $x$  and  $Y$  components of the location of this point(s). You are not asked to solve for this value of stress.

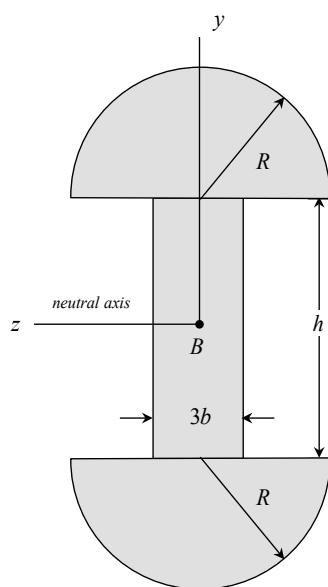


**PROBLEM #4** (*continued*)

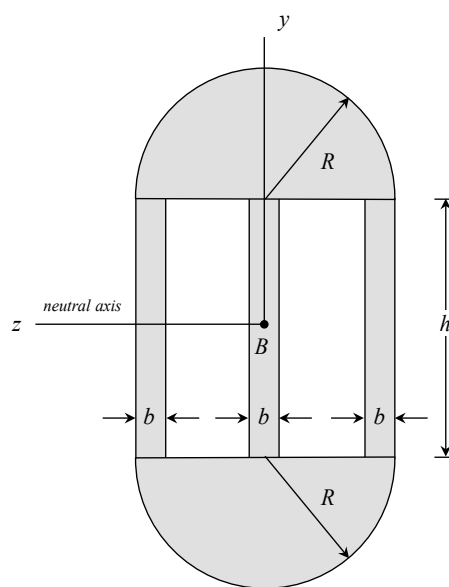
**Part D – 6 points**

The cross-sections for Beams 1 and 2 are shown below. Let  $I_1$  and  $I_2$  represent the centroidal second area moment (about the z-axis) for beams 1 and 2, respectively. Each beam is experiencing the same shear force of  $V$  at the cross section. Let  $\tau_{1B}$  and  $\tau_{2B}$  be the shear stress at points B on Beams 1 and 2, respectively.

- a) Circle the correct answer below in regard to the relative sizes of  $I_1$  and  $I_2$ . You are not asked to provide numerical values for these second area moments, or justification for your answers.
- $I_1 > I_2$
  - $I_1 = I_2$
  - $I_1 < I_2$
- b) Circle the correct answer below in regard to the relative sizes of  $|\tau_{1B}|$  and  $|\tau_{2B}|$ . You are not asked to provide numerical values for these stresses, or justification for your answers.
- $|\tau_{1B}| > |\tau_{2B}|$
  - $|\tau_{1B}| = |\tau_{2B}|$
  - $|\tau_{1B}| < |\tau_{2B}|$



**Beam 1**



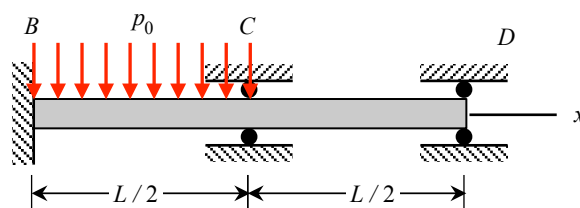
**Beam 2**

Name (Print) \_\_\_\_\_  
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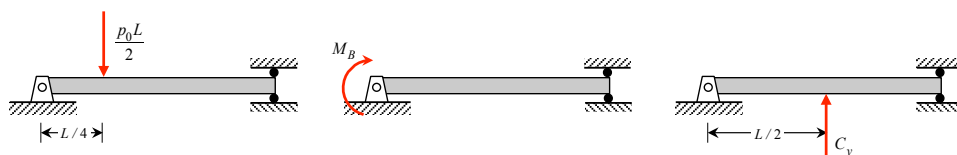
**PROBLEM #4 (25 Points)**

**Part A – 5 points**

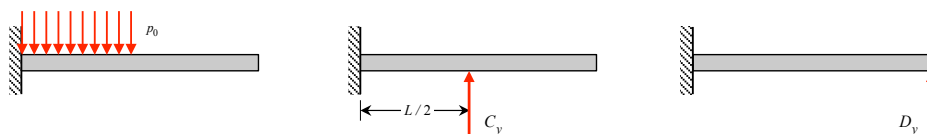
Consider the beam shown to the right. The superposition method is to be used to determine the reactions on the beam at locations C and D. Consider the following *True/False* questions regarding whether the loadings provided can be used in this analysis. No justification is needed for your answers.



a) *TRUE FALSE*



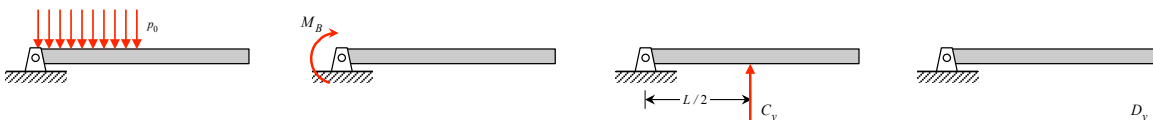
b) *TRUE FALSE*



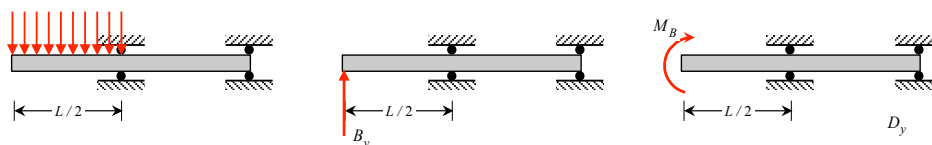
c) *TRUE FALSE*



d) *TRUE FALSE*



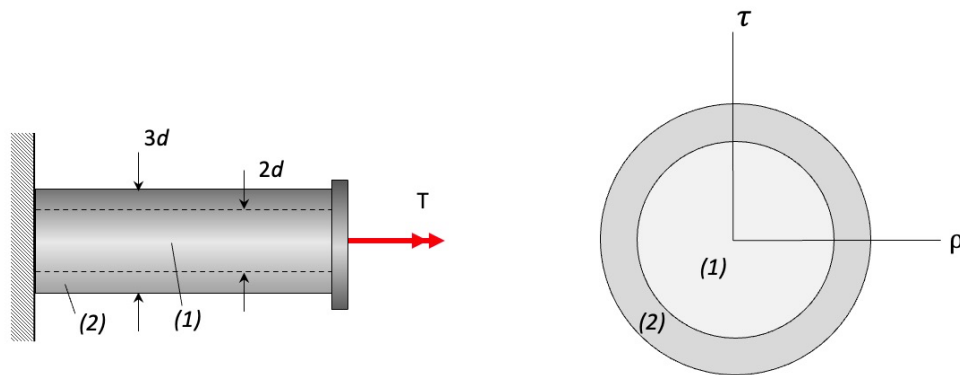
e) *TRUE FALSE*



## Problem No. 4 (continued)

**PART 4.C – 4 points**

A composite shaft is made up of an outer shell (2) and an inner core (1). The shear moduli for (1) and (2) are known to be  $2G$  and  $G$ , respectively. On the figure of the cross-section below, make a sketch of the distribution of shear stress  $\tau$  vs. the radial position  $\rho$  on the cross-section of the composite shaft. Determine the *maximum shear stress* on the cross-section and indicate its location.

**PART 4.E – 6 points**

Draw the *shear force/bending moment diagrams* for the simply-supported beam shown below.

