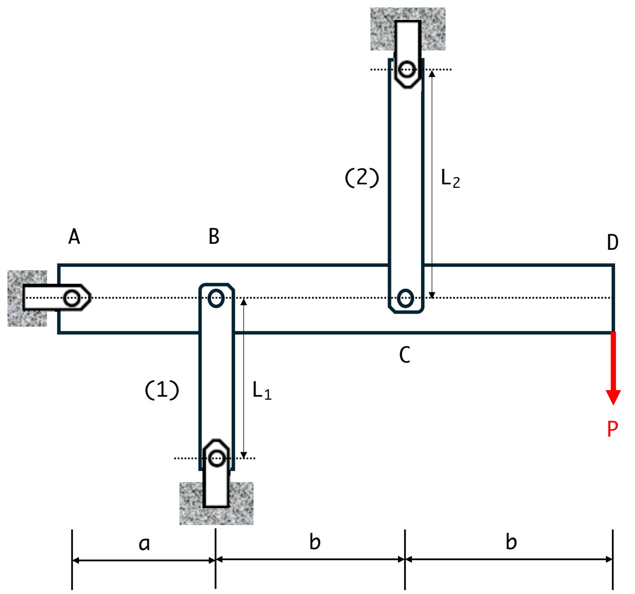
PROBLEM NO. 1 – 25 points max.

A rigid bar ABCD is supported by two rods (1) and (2) at B and C and has a pin support at A (two-sided pin with diameter d), as shown in the figure below. There is no strain in the vertical rods before load P is applied at point D.



The two rods are made of the same material with elastic modulus E and have the same cross-sectional area A. The length of rod (1), $L_1 = L$ and length of rod (2), $L_2 = 2L$. It is desired to determine the loads carried by rods (1) and (2). Also assume, $b = 2a$. Assuming a, d, E, L, A, and P are all known values, you are asked to:

(a) Draw relevant Free Body Diagram(s) and write corresponding equilibrium equations.

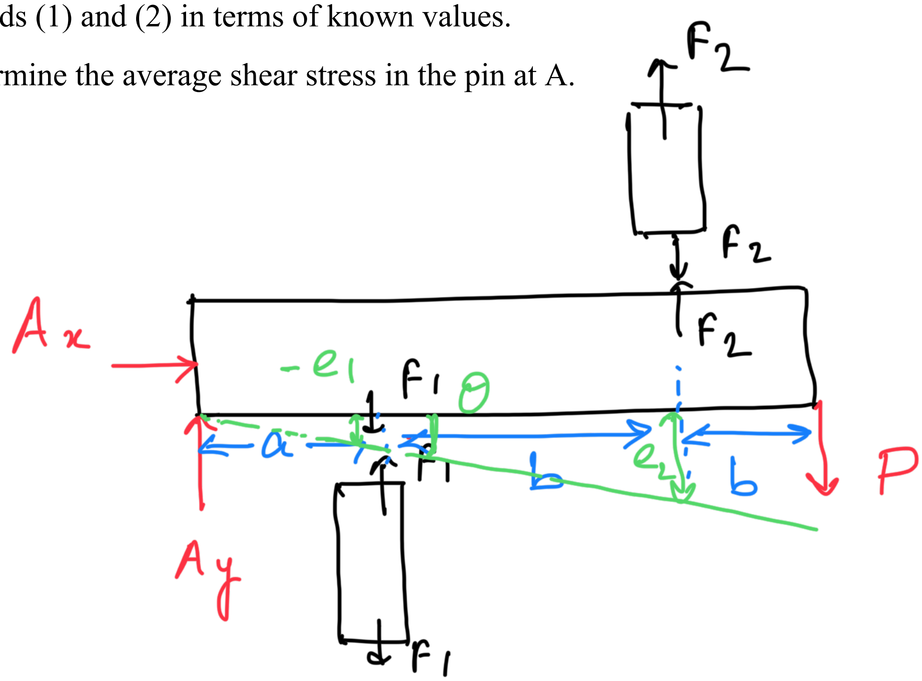
(b) Set up the force-elongation equations.

(c) Set up the compatibility equations.

(d) Verify that you have enough equations for the number of unknowns. Solve for the load carried by rods (1) and (2) in terms of known values.

(e) Determine the average shear stress in the pin at A.

(a)



given, $b = 2a$

PROBLEM NO. 1 – *continued*

$$\Sigma M_A = -F_1(a) + F_2(a+b) - P(a+2b) = 0$$

$$= -F_1 + 3F_2 - 5P = 0 \Rightarrow 3F_2 - F_1 = 5P \quad \text{—— (1)}$$

2 unknowns, 1 equation $\Rightarrow$ indeterminate

(b) force - elongation equations -

$$e_1 = \frac{F_1 L_1}{E_1 A_1} = \frac{(F_1)(L)}{EA} = \frac{F_1 L}{EA} \quad \text{—— (2)}$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} = \frac{(F_2)(2L)}{EA} = \frac{2F_2 L}{EA} \quad \text{—— (3)}$$

(c) compatibility -

$$\tan\theta = \frac{-e_1}{a} = \frac{e_2}{3a} \Rightarrow -3e_1 = e_2 \quad \text{—— (4)}$$

from (2), (3), (4) -

$$-\frac{3F_1 L}{EA} = \frac{2F_2 L}{EA}$$

$$\Rightarrow F_2 = -\frac{3}{2} F_1 \quad \text{—— (5)}$$

from (1) and (5) -

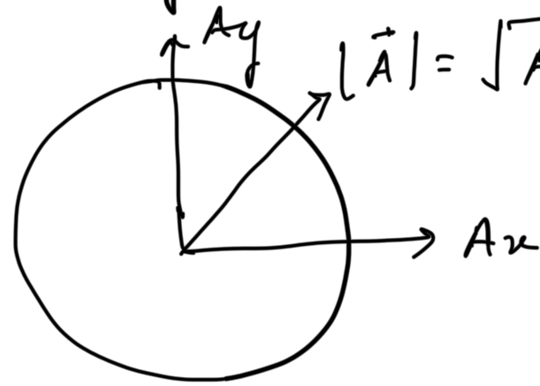
$$-\frac{9}{2}F_1 - F_1 = 5P \Rightarrow F_1 = \frac{-10P}{11}$$

from (5) -

$$F_2 = -\frac{3}{2}\left(-\frac{10P}{11}\right) = \frac{15P}{11}$$

$$\boxed{F_1 = \frac{-10P}{11}} \qquad \boxed{F_2 = \frac{15P}{11}}$$

(e) need to find A_x and A_y



$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$$\Sigma F_y = A_y - F_1 + F_2 - P = 0$$

$$A_y = F_1 - F_2 + P$$

$$= \frac{-10P}{11} - \frac{15P}{11} + P = \frac{-14P}{11}$$

$$\Sigma F_x = A_x = 0$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \frac{14P}{11}$$

$$\tau_{avg} = \frac{|\vec{A}|/2}{\pi \left(\frac{d}{2}\right)^2} = \frac{\frac{1}{2}\left(\frac{14P}{11}\right)}{\pi\left(\frac{d}{2}\right)^2}$$

$$\tau_{avg} = \frac{7P}{11}\left(\frac{4}{\pi d^2}\right)$$

$$\boxed{\tau_{avg} = \frac{28P}{11\pi d^2}}$$

PROBLEM NO. 2 – 25 points max.

Rods (1) and (2) in Fig. 2(a) are connected by a rigid connector at A, and they are fixed to the rigid walls at the ends B and C. Note that Rod (2) passes through a hole in the wall at B. Rod (1) has length L, and rod (2) has length $L/2$. The Young's moduli of rods (1) and (2) are $E_1 = E$ and $E_2 = E/10$, respectively. Rod (1) has a solid cross section of diameter d, and rod (2) has a hollow cross section of outer diameter $3d$ and inner diameter $2d$, as shown in Figs. 2(b). An external force F_A is applied at connector A, and rod (1) experiences an increment in temperature equal to $\Delta T_1 > 0$, with $\Delta T_2 = 0$. The coefficient of thermal expansion of rod (1) is α.

1. Assuming all members under tension, draw the free body diagram (FBD) of the rigid connector A.
2. Write down the equilibrium equation(s) of the rigid connector A.
3. Indicate if the structure is statically determinate or indeterminate.
4. Write down the force/elongation equations relating the elongations of rods (1) and (2) to the mechanical and thermal loads in rods (1) and (2).
5. Write down the compatibility equation relating the elongations in rods (1) and (2).
6. Calculate the axial load in rods (1) and (2). Indicate if each rod (2) is in tension (T) or compression (C). Express your results in terms of F_A, ΔT_1, α, d, L, E, and π.
7. Determine the force F_A that results in zero stress in rod (1). Express your results in terms of ΔT_1, α, d, L, E, and π.

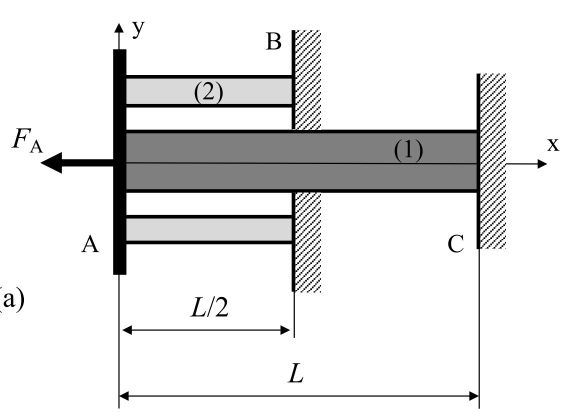


Fig. 2(a)

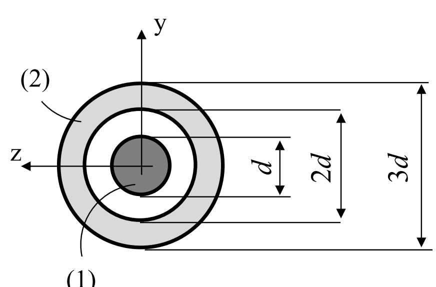
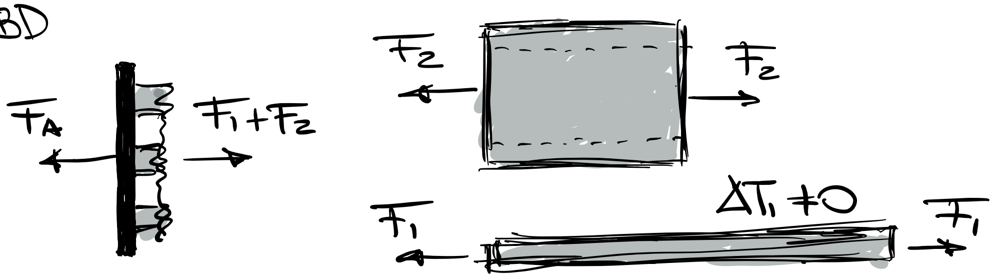


Fig. 2(b)

- Assume all members under tension

- FBD



- Equilibrium $\oplus \rightarrow \quad \Sigma F = 0 = F_1 + F_2 - F_A \Rightarrow$ Statically Indeterminate

- Force-elongation equation

$$e_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha_1 \Delta T_1 L_1 \; ; \quad e_2 = \frac{F_2 L_2}{E_2 A_2} \quad \text{with} \quad A_1 = \frac{\pi d^2}{4}$$

$$A_2 = \frac{\pi d^2}{4}\left[9-4\right] = \frac{5}{4}\pi d^2$$

- Compatibility conditions

$$e_1 = u_A - u_C \; ; \quad e_2 = u_A - u_B \; ; \quad u_C = u_B = 0 \Rightarrow e_1 = e_2$$

- Solve $\{F_1 \; F_2\}$ if $E_1 = 10 E_2 = E$

$$\left.\begin{aligned} & F_1 + F_2 - F_A = 0 \\ & \frac{F_1 L}{E_1 \pi d^2/4} + \alpha_1 \Delta T_1 L = \frac{F_2 L/2}{E_2 \, 5\pi d^2/4} \end{aligned}\right\} \quad \text{2 equations \& 2 unknowns}$$

$$F_1 + \alpha_1 \Delta T_1 E \frac{\pi d^2}{4} = F_2 = F_A - F_1$$

$$2F_1 + \alpha_1 \Delta T_1 E \frac{\pi d^2}{4} = F_A$$

$$F_1 = \frac{F_A}{2} - \alpha_1 \Delta T_1 E \frac{\pi d^2}{8}$$

$$F_2 = F_A - F_1 = \frac{F_A}{2} + \alpha_1 \Delta T_1 E \frac{\pi d^2}{8}$$ (T) if $\Delta T_1 > 0$

- Stress free rod (1) $\Rightarrow \sigma_1 = 0 = F_1 / A_1 \Rightarrow F_1 = 0$

if $F_1 = 0 = \frac{F_A}{2} - \alpha_1 \Delta T_1 E \frac{\pi d^2}{8} \Rightarrow F_A = \alpha_1 \Delta T_1 \frac{E \pi d^2}{4}$

ME 323 Exam No. 1 **Name** <u>***SOLUTION***</u>
September 24, 2025

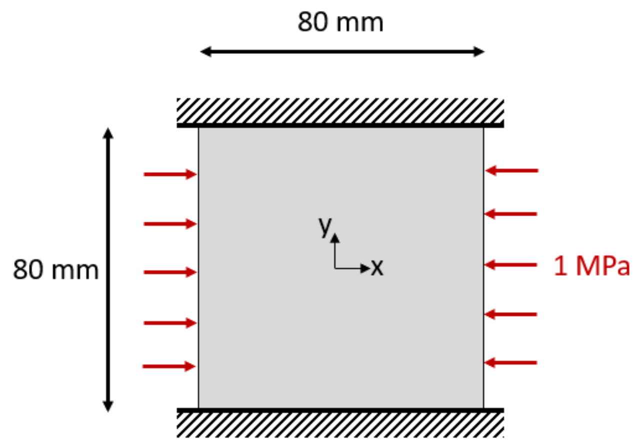
Problem 3

A square sheet of polymer is 80 x 80 mm in the x and y directions and is 20 mm thick in the z-direction. The Young's modulus of the material is 280 MPa, the Poisson's ratio is 0.4, and the thermal expansion coefficient is 80×10^{-6} °C^{-1}.

(a) The sample is fixed between two frictionless plates in the y-direction. A compressive stress is of 1 MPa is applied in the x-direction and the temperature of the block is increased by 20 °C. The sample has no stresses or constraints in the z-direction.

(i) Calculate the stress on the plates in the y-direction.

(ii) Calculate the change in thickness in the z direction.

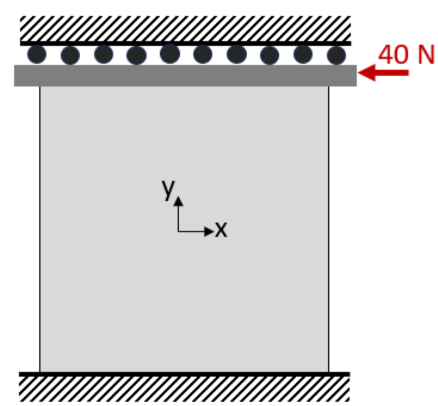


(b) The sample is next fixed in a shear apparatus in which the positive y face can displace, and a force of 40 N is applied in the x-direction.

(i) Determine the magnitude and sign of the shear stress with the correct indices.

(ii) Determine the shear strain.

(iii) Determine the x-displacement of the connector at the positive y edge of the sample.



ME 323 Exam No. 1 **_Name_ ________ *SOLUTION* ________**
September 24, 2025

Problem 3

$\epsilon_x = ?$ $\qquad \sigma_x = 1\ MPa$

$\epsilon_y = 0$ $\qquad \sigma_y = ?$

$\epsilon_z = ?$ $\qquad \sigma_z = 0$

$$\epsilon_x = \left(\frac{1}{E}\right)\left(\sigma_x - \nu\left(\sigma_y + \cancelto{0}{\sigma_z}\right)\right) + \alpha\Delta T$$

$$\cancelto{0}{\epsilon_y} = \left(\frac{1}{E}\right)\left(\sigma_y - \nu\left(\sigma_x + \cancelto{0}{\sigma_z}\right)\right) + \alpha\Delta T$$

$$\epsilon_z = \left(\frac{1}{E}\right)\left(\cancelto{0}{\sigma_z} - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T$$

(a)(i) Use equation for strain in y:

$$0 = \left(\frac{1}{E}\right)\left(\sigma_y - \nu\sigma_x\right) + \alpha\Delta T$$

$$\sigma_x = \nu\sigma_x - \alpha\Delta T E = 0.4 * (-1 * 10^6) - (80 * 10^{-6}) * (20) * 280 * 10^6 = -848\ kPa$$

(a)(ii) Use the equation for strain in z:

$$\epsilon_z = \left(\frac{1}{E}\right)\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta \mathrm{T}$$

$$\epsilon_z = \left(\frac{1}{E}\right)\left(-\nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta \mathrm{T}$$

$$= \left(\frac{1}{280 * 10^6}\right)(-0.4(-1 * 10^6 - 0.848 * 10^6)) + 20 * \ 80 * 10^{-6}$$

$$\epsilon_z = \ 0.00424$$

$$\Delta z = z_o\epsilon_z = 0.0848\ mm$$

ME 323 Exam No. 1 *Name* **_SOLUTION_**
September 24, 2025

Problem 3

(b)(i) $\tau_{xy} = \frac{V}{A} = \frac{40\ N}{0.08*0.02} = 25\ kPa$

(b)(ii) $G = \frac{E}{2(1+\nu)} = \frac{280*10^6}{2.8} = 100\ MPa$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-25\ kPa}{100\ MPa} = -0.00025$

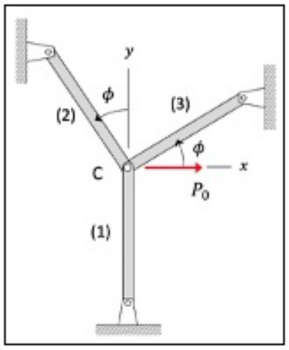
(b)(iii) $\delta x = \gamma_{xy} x_o = -0.00025 * (80\ mm) = -0.02\ mm = -20\ \mu m$

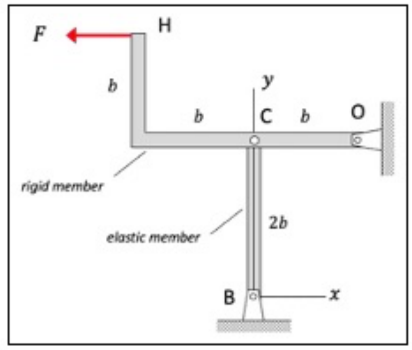
ME 323 Exam No. 1
September 24, 2025

Name SOLUTION

PROBLEM NO. 4 - PART A – 3 points max.

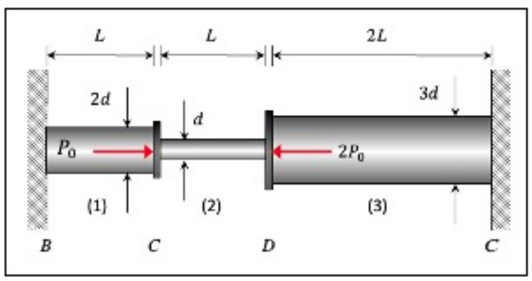
Answer the following TRUE/FALSE questions regarding whether the following structures are statically determinate.





4A.1 $\boxed{\text{TRUE}}$/FALSE: This structure is statically indeterminate

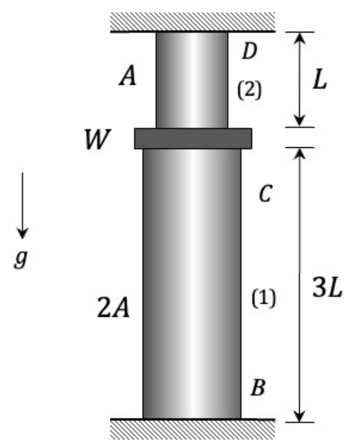
4A.2 TRUE/$\boxed{\text{FALSE}}$: This structure is statically indeterminate



4A.3 $\boxed{\text{TRUE}}$/FALSE: This structure is statically indeterminate

PROBLEM NO. 4 - PART B – 4 points max.

A rod is made up of two solid cross-section members, (1) and (2), having cross-sectional areas of $2A$ and A, respectively, and each member made up of a material with a Young's modulus of E. The connector C joining the two segments has a weight of W, with the weights of the members being negligible compared to W. Let F_1 and F_2 be the axial forces carried by members (1) and (2), respectively. Choose the correct response below regarding the sizes of these member loads.



a) $|F_1| > |F_2|$

b) $|F_1| = |F_2|$

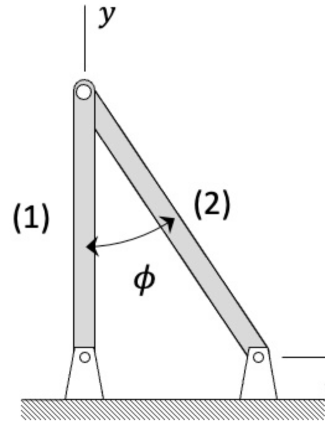
c) $\boxed{|F_1| < |F_2|}$

d) More information is needed in order to answer this question.

$$|e_1| = |e_2| \Rightarrow \frac{|F_1|(3\cancel{L})}{\cancel{E}(2\cancel{A})} = \frac{|F_2|\cancel{L}}{\cancel{E}\cancel{A}} \Rightarrow |F_2| = \frac{3}{2}|F_1|$$

PROBLEM NO. 4 - PART C

A truss is made up of members (1) and (2) with the material makeup and cross-sectional areas of the members *unknown*. The temperature of member (1) is increased by ΔT and the temperature of (2) is held constant. Let F_1 and F_2 be the axial loads carried by the members, and ϵ_1 and ϵ_2 be the strains in the members. Choose the correct responses below regarding the sizes of these member loads and strains.



4C.1 – 3 points max.

$$\sum F_x = F_2 \sin\phi = 0 \Rightarrow F_2 = 0$$
$$\sum F_y = -F_1 - \cancel{F_2}^{0}\cos\phi = 0$$
$$\hookrightarrow F_1 = 0$$

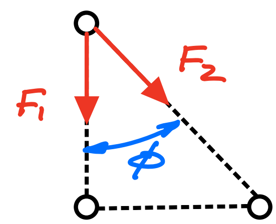
a) $|F_1| > |F_2|$

b) $|F_1| = |F_2| > 0$

$\boxed{\text{c) } |F_1| = |F_2| = 0}$

d) $|F_1| < |F_2|$

e) More information is needed in order to answer this question.



4C.2 – 3 points max.

$$\varepsilon_1 = \frac{\sigma_1}{E_1} + \alpha\Delta T = \frac{\cancel{F_1}^{0}}{E_1A_1} + \alpha\Delta T$$
$$\varepsilon_2 = \frac{\sigma_2}{E_2} = \frac{\cancel{F_2}^{0}}{E_2A_2} = 0$$

$\boxed{\text{a) } |\epsilon_1| > |\epsilon_2|}$

b) $|\epsilon_1| = |\epsilon_2| > 0$

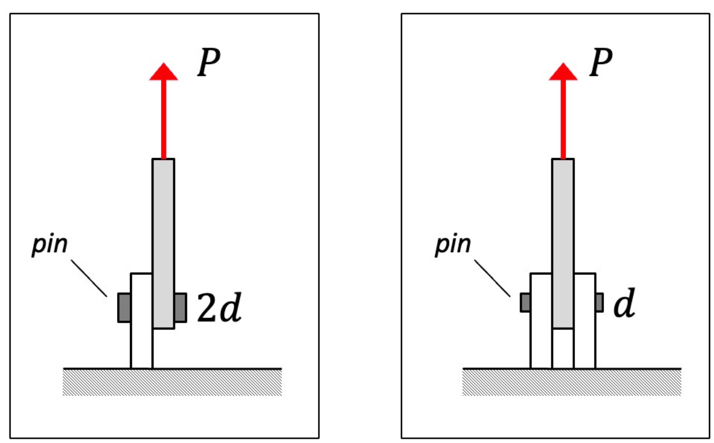
c) $|\epsilon_1| = |\epsilon_2| = 0$

d) $|\epsilon_1| < |\epsilon_2|$

e) More information is needed in order to answer this question.

PROBLEM NO. 4 - PART D – 4 points max.

Consider the single-sided and double-sided pin joints shown with each having a member force of P applied. The pin diameters for pin joints 1 and 2 are $2d$ and d, respectively. The maximum allowable shear stresses for the material of pins 1 and 2 are τ_Y and $2\tau_Y$, respectively. Let $P_{1,max}$ and $P_{2,max}$ represent the maximum member load P that can be applied without the pin in joint 1 and in joint 2 failing. Choose the correct response below regarding the sizes of these maximum member loads.



1: single-sided pin joint ***2: double-sided pin joint***

a) $P_{1,max} > P_{2,max}$

$\boxed{\text{b) } P_{1,max} = P_{2,max}}$

c) $P_{1,max} < P_{2,max}$

d) More information is needed in order to answer this question.

$$\tau_1 = \frac{P}{\pi(2d/2)^2} = \frac{P}{\pi d^2} = \tau_Y \Rightarrow P_{1,max} = \pi\tau_Y d^2$$

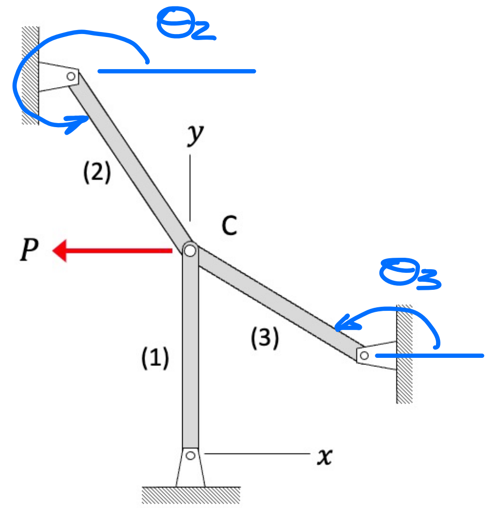
$$\tau_2 = \frac{P/2}{\pi(d/2)^2} = \frac{2P}{\pi d^2} = 2\tau_Y \Rightarrow P_{2,max} = \pi\tau_Y d^2$$

PROBLEM NO. 4 - <u>PART E</u>

Consider the three-member truss shown. Let e_1, e_2 and e_3 represent the elongations in members (1), (2) and (3), respectively. These member elongations are related to the x- and y-components of displacement of joint C (u_C and v_C, respectively) through the standard compatibility equations:

$$e_i = u_C cos\theta_i + v_C sin\theta_i$$

for i = 1, 2, 3. Choose the correct response below regarding the member angles.



4E.1 – 2 points max.

a) $0 < \theta_2 < 90°$

b) $90° < \theta_2 < 180°$

c) $180° < \theta_2 < 270°$

d) $\boxed{270° < \theta_2 < 360°}$

4E.2 – 2 points max.

a) $0 < \theta_3 < 90°$

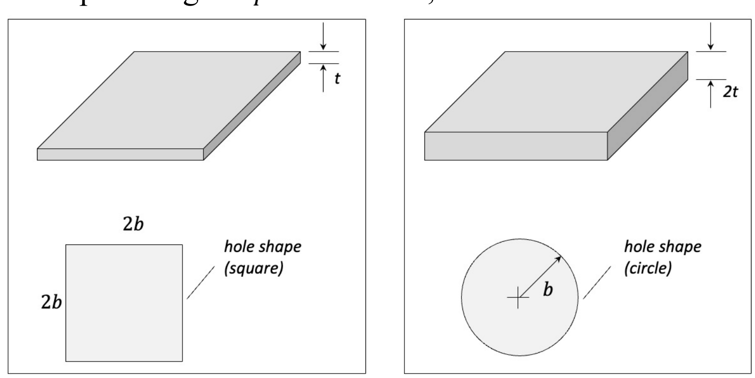
b) $\boxed{90° < \theta_3 < 180°}$

c) $180° < \theta_3 < 270°$

d) $270° < \theta_3 < 360°$

PROBLEM NO. 4 - <u>PART F</u> – 4 points max.

Consider two punching tasks. For Task 1, a square hole is to be punched into a plate of thickness t and of a material for which the shear stress required for punching is τ_P. For Task 2, a circular hole is to be punched into a plate of thickness $2t$ and of a material for which the shear stress required for punching is $2\tau_P$. Let P_1 and P_2 be the punching forces required for Tasks 1 and 2, respectively. Choose the correct response below regarding the sizes of these punching forces.



Punching task 1 ***Punching task 2***

a) $P_1 > P_2$

b) $P_1 = P_2$

c) $\boxed{P_1 < P_2}$

d) More information is needed in order to answer this question.

$$\tau_1 = \frac{P_1}{8bt} = \tau_P \Rightarrow P_1 = 8bt\tau_P$$

$$\tau_2 = \frac{P_2}{(2\pi b)(2t)} = 2\tau_P \Rightarrow P_2 = 8\pi bt\tau_P$$