Using Castigliano's 2nd Theorem

Notation for generalized displacements and generalized loads

- Displacements and forces: Δ and P
- Shaft rotations and torques: ϕ and T
- Beam rotations and bending moments: θ and M
- Redundant loads: R (forces, torques, and/or bending moments)
- Dummy loads: F_d (forces, torques, and/or bending moments)

Finding reactions for indeterminate structures

Draw FBD(s) and write down the equilibrium equations.

Determine the number of redundant loads N_R, and choose your redundant loads:

$$R_i;\ \ i = 1, 2, \dots, N_R$$

Using the equilibrium equations, write the non-redundant loads in terms of the redundant loads.

Write strain energy function U *in terms of only the redundant and applied loads*.

Apply Castigliano's theorem for indeterminate structures:

$$\frac{\partial U}{\partial R_i} = 0\ ;\ \ i = 1, 2, \dots, N_R$$

Solve Castigliano equations, along with equilibrium equations, to find the external reactions.

Finding displacements

Draw FBD(s) and write down the equilibrium equations. If no load at point or direction of desired displacement, then add dummy load F_d.

Is structure indeterminate? If so, use *Castigliano's theorem for indeterminate structures* (shown to the left) to find external reactions FIRST.

Write the strain energy function U for the structure. Substitute in the external reactions found either from equilibrium (determinate) or from Castigliano and equilibrium (indeterminate). *At this point, U should be in terms of only applied loads, and possibly dummy loads*.

Apply Castigliano's theorem for determining the generalized displacements:

$$\Delta_i = \frac{\partial U}{\partial P_i}$$

$$\phi_i = \frac{\partial U}{\partial T_i}$$

$$\theta_i = \frac{\partial U}{\partial M_i}$$

Set any dummy load $F_d = 0$ at this point.