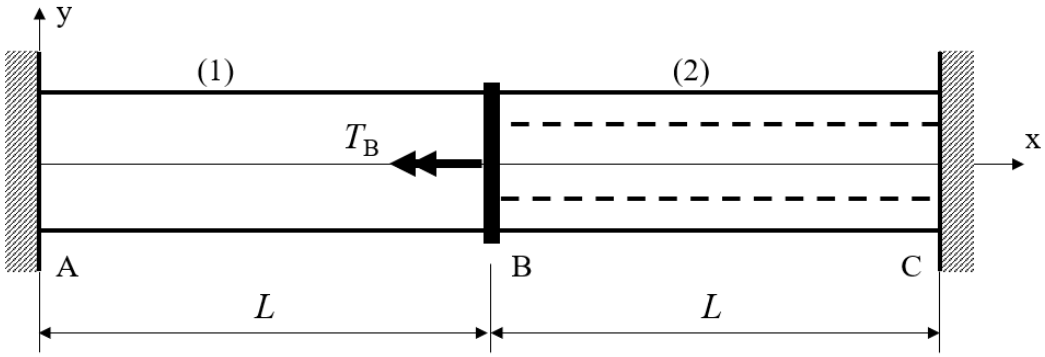
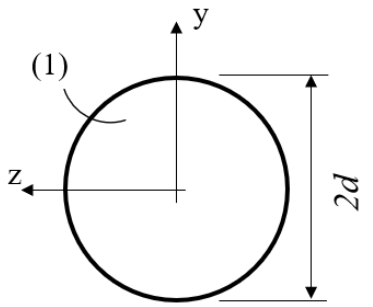
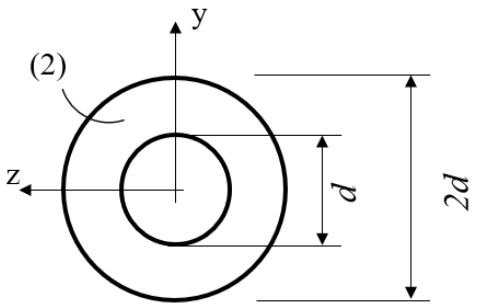
Problem 8.1 (10 points)
Shaft ABC is made up of members 1 and 2. The shear modulus of both members is G. Member 1 is a solid shaft, with diameter $2d$. Member 2 is a hollow shaft, with outer diameter $2d$ and inner diameter d. Both members are attached to a fixed wall and to rigid connector B. A torque T_B is applied to connector B.





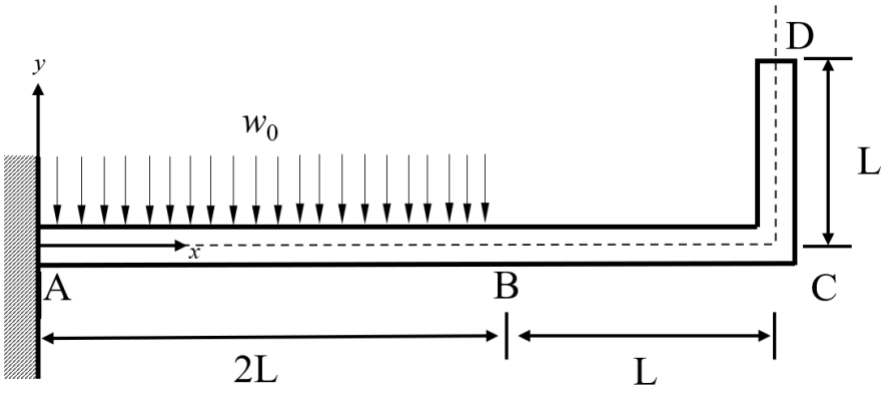


Follow the steps provided below to determine the torque in each member and the angle of twist at connector B:

(a) Draw the necessary free body diagram(s).
(b) Write down the corresponding equilibrium equations.
(c) Determine whether the structure is statically determinate or indeterminate.
(d) State your choice for redundant load(s), if needed.
(e) Determine the elastic strain energy of the shaft ABC, in terms of T_B, G, L, d, and the redundant load(s).
(f) Use Castigliano's second theorem to find the torque in each member.
(g) Use Castigliano's second theorem to find the angle of twist at connector B.

Express your results in terms of T_B, G, L, and d.

Problem 8.2 (10 points)
An L-shaped beam ABCD has a Young's modulus of E and a square cross section with side length b. The beam is connected to a wall at A, and it is loaded with a distributed load per unit of length w_0 over the segment AB.

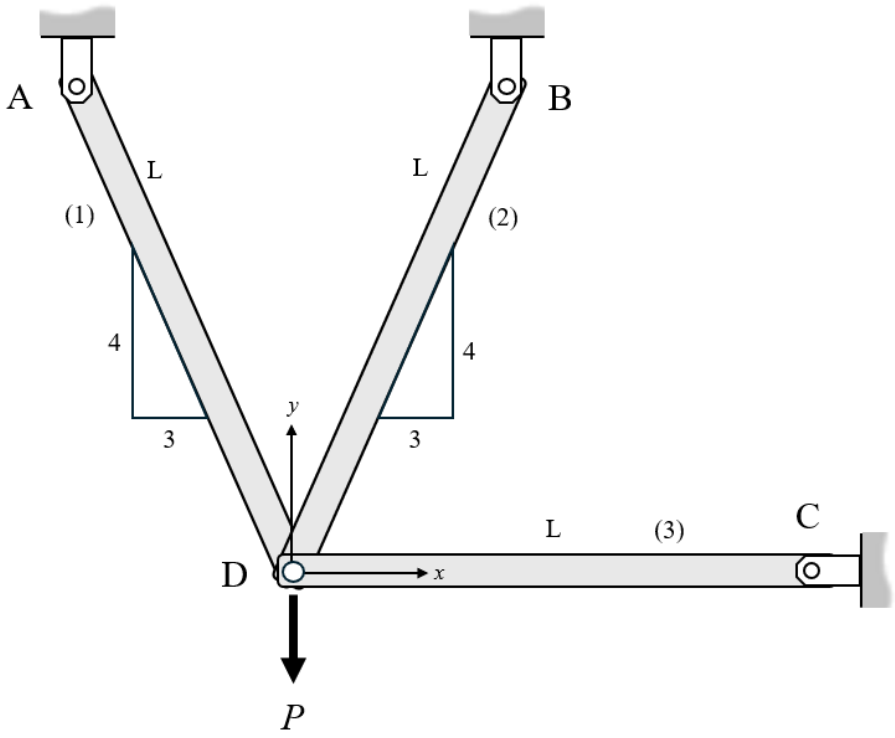


Ignoring shear effects, follow the steps provided below to determine the vertical deflection at D:

(a) Draw the necessary free body diagram(s).
(b) Write down the corresponding equilibrium equations.
(c) Determine whether the structure is statically determinate or indeterminate.
(d) State your choice for redundant load(s), if needed.
(e) State your choice for dummy load(s), if needed.
(f) Determine the elastic strain energy of the beam ABCD—set up, but do not evaluate, the integral for the strain energy. Ignore shear effects.
(g) Use Castigliano's second theorem to determine the vertical deflection at D.

Express your results in terms of w_0, E, L, and b.

Problem 8.3 (10 points)
Consider the truss depicted below. All members have a cross-sectional area A, length L, and Young's modulus E. A load P is applied at D as shown.



Follow the steps provided below to determine the vertical displacement of pin D:

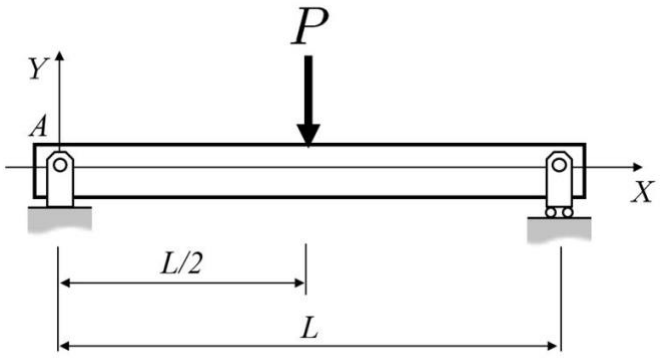
(a) Draw the necessary free body diagram(s).
(b) Write down the corresponding equilibrium equations.
(c) Determine whether the structure is determinate or indeterminate.
(d) State your choice for redundant load(s), if needed.
(e) State your choice for dummy load(s), if needed.
(f) Determine the elastic strain energy of the beam ABCD, in terms of P, E, L, A, and the redundant load(s).
(g) Determine the axial loads in the three elements.
(h) Use Castigliano's second theorem to determine the vertical displacement of pin D.

Express your results in terms of P, E, L, and A.

Problem 8.4.a (5 points)
A large value of $\partial U/\partial P$ for a given external load P implies:

A. The structure is very stiff in the direction of P.
B. The structure has low stiffness or high flexibility in the direction of P.
C. The structure is under pure torsion.
D. The structure is statically determinate.

Problem 8.4.b (5 points)
A simply supported beam of length L carries a concentrated load P at its midspan. You are asked to determine the slope at the left support A using Castigliano's Second Theorem.
Which of the following is the correct conceptual procedure?



A. Introduce a dummy moment M' at the left support, express total strain energy U as a function of M', and compute $\theta_A = \partial U/\partial M'|_{M'=0}$
B. Introduce a dummy force P' at midspan and compute $\delta_A = \partial U/\partial P'|_{P'=0}$.
C. Differentiate U directly with respect to P to obtain slope at support.
D. Compute slope using 2nd-order integration method because Castigliano's theorem is not applicable to slopes.