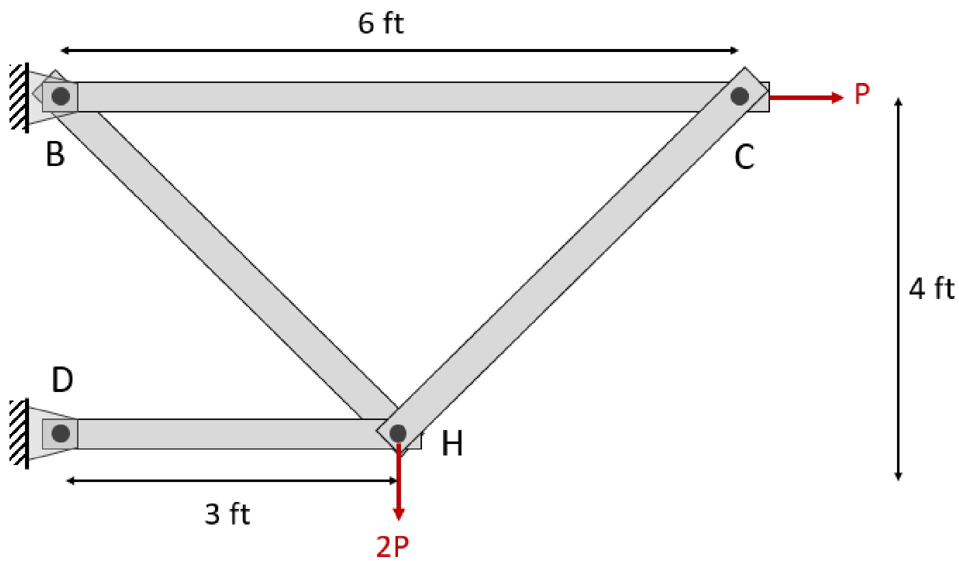
ME 323: Mechanics of Materials
Fall 2025

Homework Set 2
Due: Friday, September 12

Problem 2.1 (10 points)



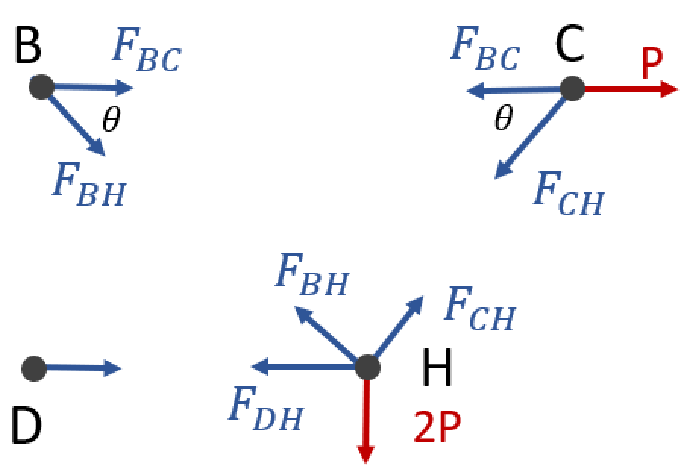
Each of the bars of truss have a cross-sectional area of 1.5 in^2 and an axial failure stress $\sigma_f = 30\ ksi$.

(a) Draw the free body diagrams.

(b) Determine the forces in each member as a function of the applied force of P.

(c) Determine the maximum magnitude P of the loads that can be applied to the truss that meets factor of safety FS = 2.

(a)



(b) $\left(\Sigma F_y\right)_C = 0 = -F_{CH}\sin(\theta) \rightarrow F_{CH} = 0$

$(\Sigma F_x)_C = 0 = P - F_{BC} - F_{CH}\sin(\theta) \rightarrow F_{BC} = P$

$\left(\Sigma F_y\right)_H = 0 = F_{BH}\sin(\theta) + F_{CH}\sin(\theta) - 2P$

$\rightarrow F_{BH}\left(\frac{4}{5}\right) = 2P \rightarrow F_{BH} = \frac{5}{2}P$

$(\Sigma F_x)_H = 0 = F_{CH}\cos(\theta) - F_{BH}\cos(\theta) - F_{DH}$

$\rightarrow F_{DH} = -F_{BH}\left(\frac{3}{5}\right) = -\left(\frac{3}{5}\right)\left(\frac{5}{2}\right)P = -\frac{3}{2}P$

(c) $\sigma_{BC} = \frac{P}{A}$

$\sigma_{BH} = \frac{5P}{2A}$

$\sigma_{DH} = -\frac{3P}{2A}$

A is the same for all members → largest stress is in member BH.

$FS = \frac{\sigma_f}{\sigma_{BH}}$

$\sigma_{BH} = \frac{\sigma_f}{FS} = \frac{5P}{2A} \rightarrow P = \frac{2\sigma_f A}{5FS} = \frac{2(30{,}000\ psi)(1.5\ in^2)}{5*2} = 9000\ lbs = 9\ ksi$

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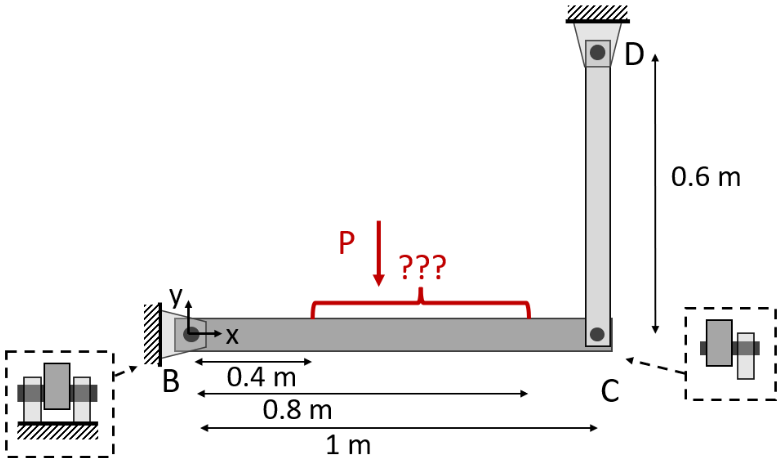
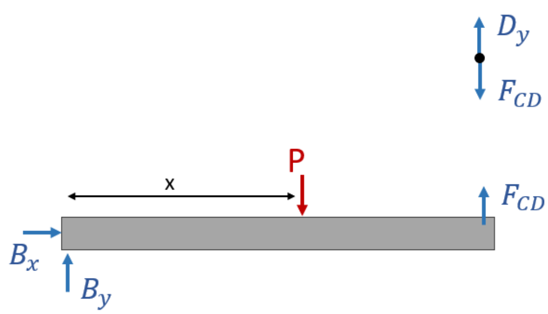
Problem 2.2 (10 points)



BCD is a shelf that has a double-sided pin at B and a single-sided pin at C. The ultimate stresses of the pins are both 60 MPa. A load of P = 1 kN will be placed on the BC section of the shelf somewhere between 0.4 m and 0.8 m from B.

(a) Determine the minimum diameter of the pin at B to achieve a factor of safety of at least 3.0 for all possible loading conditions.

(b) Determine the minimum diameter of the pin at C to achieve a factor of safety of at least 3.0 for all possible loading conditions.



$$\left(\Sigma F_y\right)_D = 0 = D_y - F_{CD} \rightarrow \; F_{CD} = D_y$$

$$(\Sigma M)_B = 0 = -Px + F_{CD}(1\,m) \rightarrow F_{CD} = Px \qquad 0.4 \le x \le 0.8$$

$$\left(\Sigma F_y\right)_{BC} = 0 = B_y - P + F_{CD} \rightarrow B_y = \; P - F_{CD} = P(1-x) \qquad 0.4 \le x \le 0.8$$

$$(\Sigma F_x)_{BC} = 0 = B_x$$

(a) Pin at B:

$$V_B = |B| = \sqrt{B_x^2 + B_y^2} = B_y = P(1 - x)$$

B_y is largest when x = 0.4.

$$|B| = P(1 - 0.4) = 0.6P$$

Double shear:

$$\tau_B = \frac{|B|}{2A} = \frac{0.6P}{2\left(\pi\left(\frac{d_B}{2}\right)^2\right)}$$

$$FS = \frac{\tau_U}{\tau_B} \rightarrow \tau_B = \frac{\tau_U}{FS}$$

$$\frac{\tau_U}{FS} = \frac{0.6P}{2\left(\pi\left(\frac{d_B}{2}\right)^2\right)}$$

$$d_B = \sqrt{\frac{0.6P(2)FS}{\tau_U \pi}} = \sqrt{\frac{0.6(1000\ N)(2)(3)}{\pi(60*10^6)}} = 0.0044\ m = 4.4\ mm$$

(b) Pin at C:

$$V_C = F_{CD} = Px$$

F_{CD} is largest when x = 0.8 m

$$F_{CD} = 0.8P$$

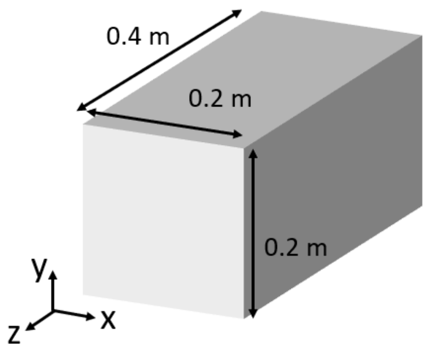
$$\tau_C = \frac{F_{CD}}{A} = \frac{0.8P}{\pi\left(\frac{d_C}{2}\right)^2}$$

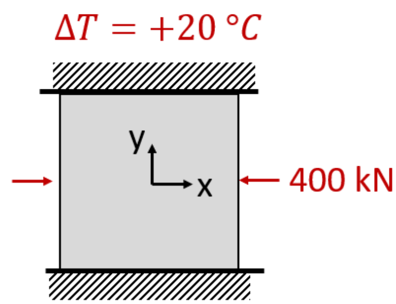
$$FS = \frac{\tau_U}{\tau_C} \rightarrow \tau_C = \frac{\tau_U}{FS}$$

$$\frac{\tau_U}{FS} = \frac{0.8P}{\pi\left(\frac{d_C}{2}\right)^2}$$

$$d_C = \sqrt{\frac{0.8P(4)FS}{\tau_U \pi}} = \sqrt{\frac{0.8(1000\ N)(4)(3)}{\pi(60*10^6)}} = 0.0071\ m = 7.1\ mm$$

Problem 2.3 (10 points)





A block of materials has initial dimensions of 0.2, 0.2, and 0.4 m in the x, y, and z directions when no stress is applied. The block is constrained in the y-direction and has a force of 400 kN applied in the x-direction and has no constraint in the z-direction. The temperature of the block is increased by 20 °C ($\Delta T = 20\ °C$). The block is made of epoxy with a Young's modulus of 4 GPa, a Poisson's ratio of 0.45, and a thermal expansion coefficient of 40 x10^{-6} °C^{-1}.

(a) Determine the stress applied on the fixed surfaces in the y-direction when the load is applied and the temperature is increased.

(b) Determine the length of the block in the z direction when the load is applied and the temperature is increased.

$\epsilon_x = ?$ $\quad\quad$ $\sigma_x = -\frac{400{,}000N}{0.2*0.4} = 5*10^6 Pa = 5\ MPa$

$\epsilon_y = 0$ $\quad\quad$ $\sigma_y = ?$

$\epsilon_z = ?$ $\quad\quad$ $\sigma_z = 0$

→3 known and 3 unknown

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) + \alpha\Delta T$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) + \alpha\Delta T$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T$$

(a) Use equation for ϵ_y because we know all values in that equation except σ_y.

$$0 = \left(\sigma_y - \nu\sigma_x\right) + E\alpha\Delta T$$

$\sigma_y = \nu\sigma_x - E\alpha\Delta T = 0.45(-5 * 10^6) - 4 * 10^9(40 * 10^{-6}) * 20$

$\sigma_y = -2.25\ MPa - 3.2\ MPa = -5.45\ MPa$

(b) Use equation for ϵ_z because the question is asking about z deformation.

$\epsilon_z = \frac{1}{E}\left(-\nu(\sigma_x + \sigma_y)\right) + \alpha\Delta T$

$\epsilon_z = \left(\frac{1}{4*10^9}\right)\left(-0.45 * (-5 * 10^6 - 5.45 * 10^6)\right) + (40 * 10^{-6}) * 20 = 0.001976$

$\epsilon_z = \frac{\Delta z}{z_o} \rightarrow \Delta z = \epsilon_z * z_o = 0.001176 * 0.4\ m = 0.00079\ m = 0.79\ mm$

$Length = z_o + \Delta z = 0.4\ m + 0.00079\ m = 0.40079\ m$

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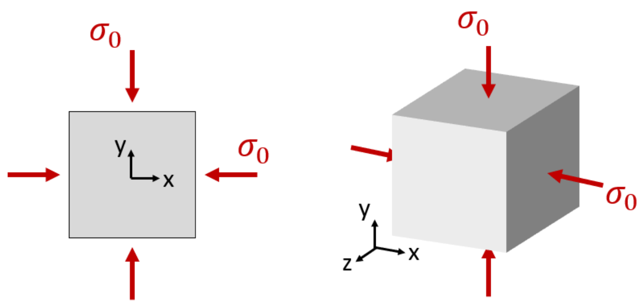
Problem 2.4 (5 points)

A cube of material is subjected to compressive stresses of σ_0 in the x and y directions and is free to deform in the z direction (no stresses or constraints in the z direction).



Assume small strains and linear materials properties. For different values of the Poisson's ratio, indicate whether the change in volume (ΔV) and the strain in the z direction (ϵ_z) are >0, =0, or <0.

ν	ΔV	ϵ_z
0.5	0	>0
0.2	<0	>0
0	<0	=0
-0.2	<0	<0

Conceptual explanation: the volume change is zero when the Poisson's ratio is 0.5. If the Poisson's ratio is anything other than 0.5, the volume change will be reflective of the stresses: compressive stress results in negative volume change and tensile stress results in positive volume change.

Mathematical explanation:

$$V_o = x_o y_o z_o$$

$$V = x_o(1+\epsilon_x)y_o(1+\epsilon_y)z_o(1+\epsilon_z)$$

$$V = x_o y_o z_o(1+\epsilon_x+\epsilon_y+\epsilon_x\epsilon_y)(1+\epsilon_z)$$

$$V = x_o y_o z_o(1+\epsilon_x+\epsilon_y+\epsilon_x\epsilon_y+\epsilon_z+\epsilon_x\epsilon_z+\epsilon_y\epsilon_z+\epsilon_x\epsilon_y\epsilon_z)$$

When strains are small, $\epsilon_x\epsilon_y, \epsilon_x\epsilon_z, \epsilon_y\epsilon_z \ll \epsilon_x, \epsilon_y, \epsilon_z$

$$V \sim x_o y_o z_o\left(1-2\left(\frac{\sigma_o}{E}\right)(1-\nu)+\left(\frac{\sigma_o}{E}\right)(2\nu)\right) = x_o y_o z_o\left(1+\left(\frac{\sigma_o}{E}\right)(4\nu-2)\right)$$

$$\Delta V = V - V_o = x_o y_o z_o \left(\left(\frac{\sigma_o}{E} \right) (4\nu - 2) \right)$$

Which is 0 when $\nu = 0.5$ and negative for all other values of the Poisson's ratio.

$$\epsilon_z = \left(\frac{1}{E} \right) \left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right) \right) = \left(\frac{1}{E} \right) 2\nu|\sigma_o|$$