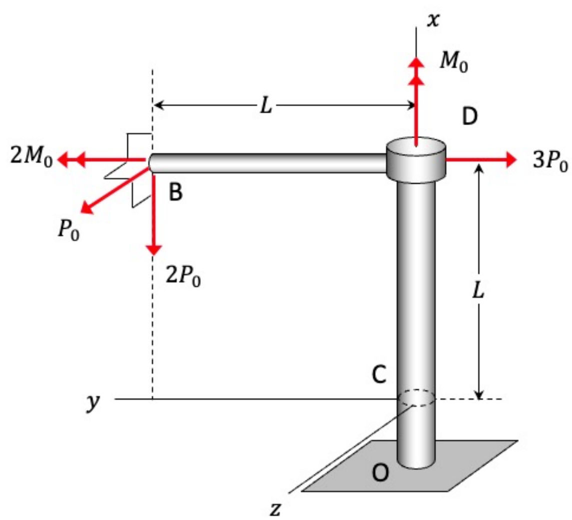
Problem 1.1 (10 points)

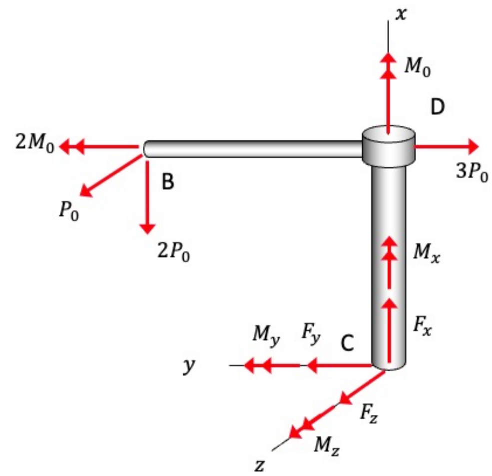


The angled structure shown is loaded with concentrated loads and couples at locations B and D. You are asked here to determine the internal resultants on the positive x-face of a mathematical cut on section OD at location C. Please show the following steps in your work. The grading of your work will be based on these steps.

1. Complete the free body diagram (FBD) of section BDC of the structure.
2. Using your FBD from above, write down the equilibrium equations for section BDC.
3. From your equilibrium equations, solve for the internal resultant components at location C.
4. In the figure of section OC, show the internal resultant components on the positive x-face of this cut. Show these resultant components as they actually exist with their correct directions. For this part of the problem, please use $M_0 = P_0 L$.
5. Write your final answers *as vectors* for the resultant internal force and internal couple acting on the positive x-face of the cut at C.

Leave your answers in terms of, at most: P_0, M_0 and L.

Part 1 – Shown to the right.



Part 2

$\sum F_x = F_x - 2P_0 = 0 \;\Rightarrow\; F_x = 2P_0$

$\sum F_y = F_y - 3P_0 = 0 \;\Rightarrow\; F_y = 3P_0$

$\sum F_z = F_z + P_0 = 0 \;\Rightarrow\; F_z = -P_0$

$$\begin{aligned}\sum \vec{M}_C = {} & (M_x + M_0)\hat{\imath} + (M_y + 2M_0)\hat{\jmath} + M_z\hat{k} + \\ & (L\hat{\imath} + L\hat{\jmath}) \times \left(-2P_0\hat{\imath} + P_0\hat{k}\right) + \\ & (L\hat{\imath}) \times (-3P_0\hat{\jmath})\end{aligned}$$

$$\vec{0} = (M_x + M_0 + LP_0)\hat{\imath} + (M_y + 2M_0 - LP_0)\hat{\jmath} + (M_z + 2LP_0 - 3LP_0)\hat{k}$$

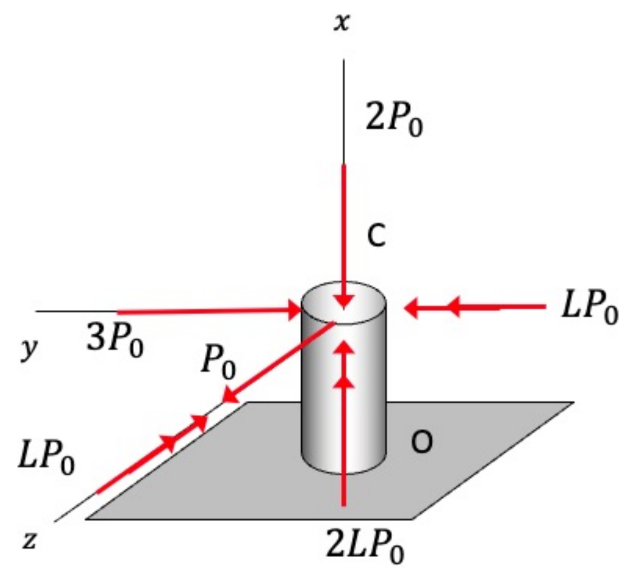
Part 3

$\hat{\imath}$: $M_x + M_0 + LP_0 = 0 \quad\Rightarrow\quad M_x = -M_0 - LP_0 = -2LP_0$

$\hat{\jmath}$: $M_y + 2M_0 - LP_0 = 0 \quad\Rightarrow\quad M_y = -2M_0 + LP_0 = -LP_0$

$\hat{k}$: $M_z - LP_0 = 0 \quad\Rightarrow\quad M_z = LP_0$

Part 4 – Shown to the right

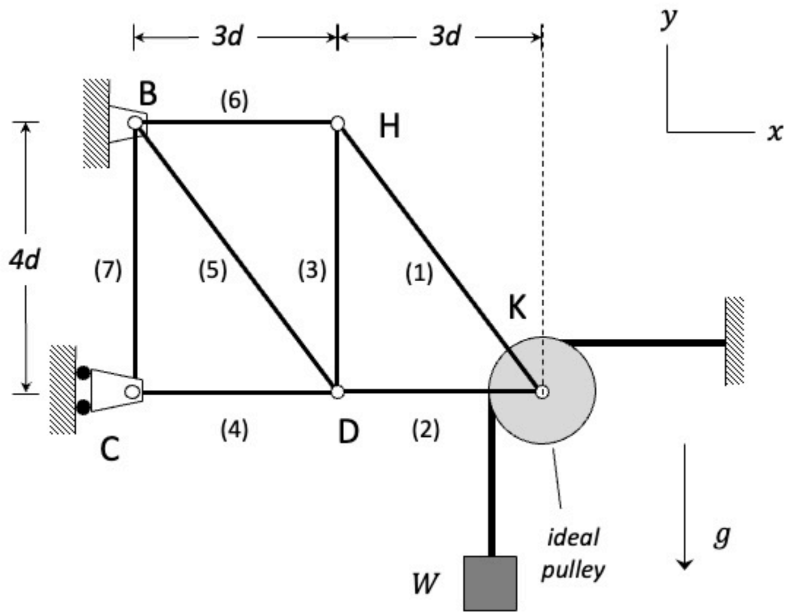


Part 5

$\vec{F} = \left(-2\hat{\imath} - 3\hat{\jmath} + \hat{k}\right)P_0$

$\vec{M} = \left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right)LP_0$

Problem 1.2 (10 points)

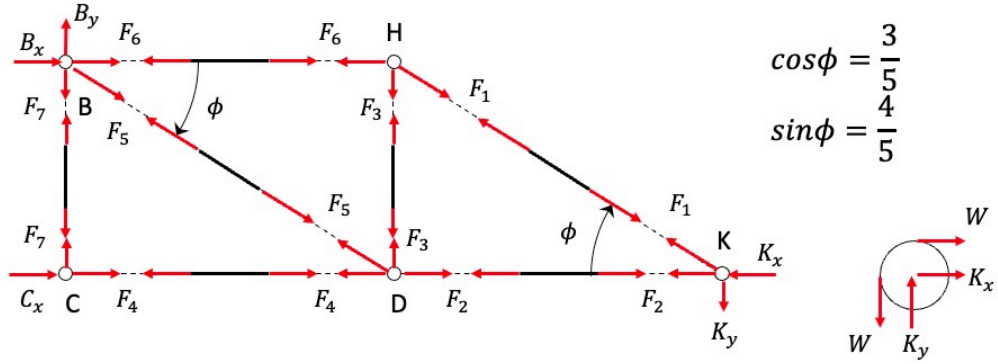


A truss being made up on members (1)-(7) supports a block of weight W through a simple cable-pulley system. Members (1)-(3) have cross-sectional areas of A, whereas members (4)-(7) have cross-sectional areas of $2A$. Consider the weights of the truss members and of the pulley to be negligible compared to the weight of the block. Assume a frictionless pulley and a massless, inextensible cable. You are asked here to determine the stress in each member of the truss. Please show the following steps in your work. The grading of your work will be based on these steps.

1. Draw individual free body diagrams of joints C, D, H and K, as well of the pulley.
2. From your FBD's determine the load carried by each member in the truss. Identify each member as being either in tension, or compression, or carrying zero load.
3. Determine the stress in each member.
4. Identify the member with the largest *tensile* stress. Identify the member with the largest *compressive* stress.

Leave your answers in terms of, at most: W and A.

Part 1



Part 2

<u>Pulley</u>:

$\sum F_x = K_x + W = 0 \quad \Rightarrow \quad K_x = -W$
$\sum F_y = K_y - W = 0 \quad \Rightarrow \quad K_y = W$

<u>Joint K</u>:

$\sum F_y = -K_y + F_1 sin\phi = 0 \quad \Rightarrow \quad F_1 = K_y/sin\phi = \frac{5}{4}W \quad (T)$

$\sum F_x = -F_2 - K_x - F_1 cos\phi = 0 \quad \Rightarrow \quad F_2 = -K_x - F_1 cos\phi = W - \left(\frac{5}{4}W\right)\frac{3}{5} = \frac{W}{4} \quad (T)$

<u>Joint H</u>:
$\sum F_y = -F_3 - F_1 sin\phi = 0 \quad \Rightarrow \quad F_3 = -F_1 sin\phi = -W \quad (C)$
$\sum F_x = -F_6 + F_1 cos\phi = 0 \quad \Rightarrow \quad F_6 = F_1 cos\phi = \left(\frac{5}{4}W\right)\frac{3}{5} = \frac{3}{4}W \quad (T)$

<u>Joint D</u>:

$\sum F_y = F_3 + F_5 sin\phi = 0 \quad \Rightarrow \quad F_5 = -F_3 \,/\, sin\phi = \frac{5}{4}W \quad (C)$

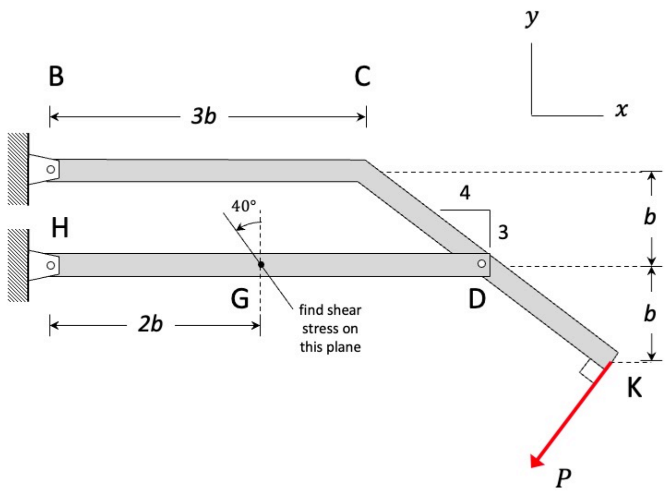
$\sum F_x = -F_4 + F_2 - F_5 cos\phi = 0 \quad \Rightarrow \quad F_4 = F_2 - F_5 cos\phi = \frac{W}{4} - \left(\frac{5}{4}W\right)\frac{3}{5} = -\frac{1}{2}W \quad (C)$

<u>Joint C</u>:
$\sum F_y = F_7 = 0$

Part 3
$\sigma_1 = \frac{F_1}{A} = \frac{5}{4}\frac{W}{A}$ (T)
$\sigma_2 = \frac{F_2}{A} = \frac{1}{4}\frac{W}{A}$ (T)
$\sigma_3 = \frac{F_3}{A} = -\frac{W}{A}$ (C)
$\sigma_4 = \frac{F_4}{2A} = -\frac{1}{2}\frac{W}{A}$ (C)
$\sigma_5 = \frac{F_5}{2A} = \frac{5}{8}\frac{W}{A}$ (T)
$\sigma_6 = \frac{F_6}{2A} = \frac{3}{8}\frac{W}{A}$ (T)
$\sigma_7 = \frac{F_7}{2A} = 0$

Part 4
Member 1 has the largest tensile stress.
Member 3 has the largest compressive stress.

Problem 1.3 (10 points)

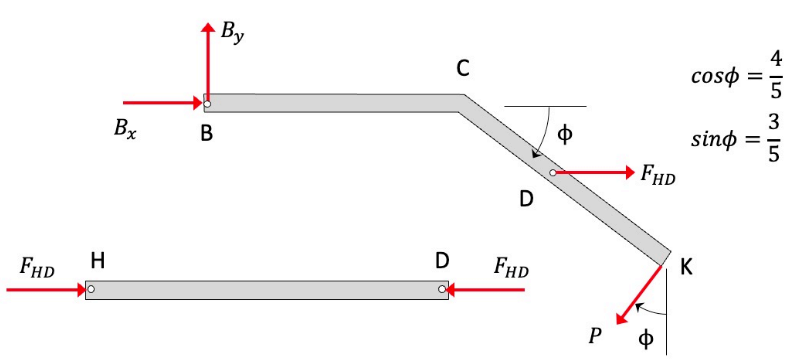


A frame is made up of two members: BCK and HD, with these two members pinned to ground at ends B and H, respectively, with pins having a diameter of *d* with double-sided pin connections. You are asked here to find the stress in pins B and H, along with the shear stress in HD along the plane shown at G. Please show the following steps in your work. The grading of your work will be based on these steps.

1. Draw individual free body diagrams (FBDs) of the two frame members.
2. Determine the reaction on BCK at B and the reaction on HD at H. Write your answers as vectors.
3. Determine the magnitude of the shear stress in pin B.
4. Determine the magnitude of the shear stress in pin H.
5. Determine the magnitude of the shear stress in HD along the plane shown at G.

Leave your answers in terms of, at most: *P*, *b* and *d*.

Part 1 – FBDs shown below.



Part 2

$$\sum \vec{M}_B = F_{HD} b\hat{k} + [(3b + 8b/3)\hat{\imath} - 2b\hat{\jmath}] \times \left(-\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath}\right) P = b\left(F_{HD} - \frac{86}{15}P\right)\hat{k} = \vec{0}$$

$$\Rightarrow \quad F_{HD} = \frac{10}{3}P$$

$$\sum F_x = F_{HD} - Psin\phi + B_x = 0 \quad \Rightarrow \quad B_x = Psin\phi - F_{HD} = \frac{3}{5}P - \frac{10}{3}P = -\frac{41}{15}P$$

$$\sum F_y = -Pcos\phi + B_y = 0 \quad \Rightarrow \quad B_y = Pcos\phi = \frac{4}{5}P$$

Therefore:

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} = -\frac{41}{15}P\hat{\imath} + \frac{4}{5}P\hat{\jmath} \quad \Rightarrow \quad \left|\vec{B}\right| = P\sqrt{\left(\frac{41}{15}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{P}{3}\sqrt{73}$$

$$\vec{H} = F_{HD}\hat{\imath} = \frac{10}{3}P\hat{\imath}$$

Part 3

$$\tau_B = \frac{\left|\vec{B}\right|/2}{A} = \frac{1}{2}\frac{P}{3}\sqrt{73}\left(\frac{1}{\pi d^2 / 4}\right) = \frac{2}{3\pi}\frac{P}{d^2}\sqrt{73}$$

Part 4

$$\tau_H = \frac{\left|\vec{H}\right|/2}{A} = \frac{1}{2}P\sqrt{\frac{10}{3}}\left(\frac{1}{\pi d^2 / 4}\right) = \frac{2}{\pi}\frac{P}{d^2}\sqrt{\frac{10}{3}}$$

Part 5

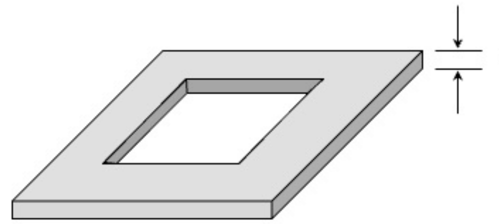
$$\sigma_{HD} = \frac{\left|\vec{H}\right|}{A} = \frac{4}{\pi}\frac{P}{d^2}\sqrt{\frac{10}{3}}$$

From Chapter 3 of the course lecture book:

$$\tau = \frac{\sigma_{HD}}{2}\sin[(2)(40°)] = \frac{2sin80°}{\pi}\frac{P}{d^2}\sqrt{\frac{10}{3}}$$

Problem 1.4 (4 points)

It is desired to punch a hole in a sheet of metal with the thickness shown in the figure above. Consider the four hole shapes presented below of A, B, C and D. Here we want to determine which hole shape will require the largest and smallest punching force.



sheet of metal with hole punched

Problem 1.4.1

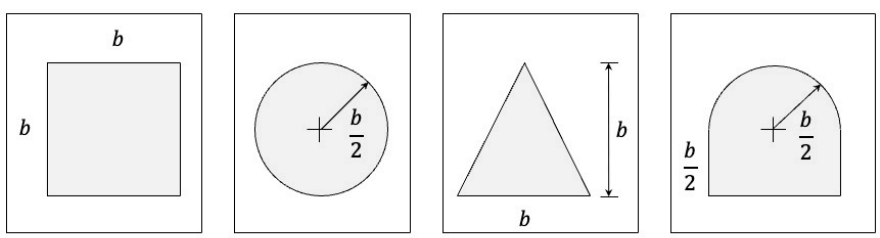
Choose the hole shape that will require the *largest* punching force:

A. Hole shape A
B. Hole shape B
C. Hole shape C
D. Hole shape D

Problem 1.4.2

Choose the hole shape that will require the *smallest* punching force:

A. Hole shape A
B. Hole shape B
C. Hole shape C
D. Hole shape D



A: square ***B: circle*** ***C: triangle*** ***D: composite***

Let τ_P be the shear stress required to punch the hole in the metal. Therefore, the punching force is given by: $P_P = A\tau_P$

A: $A_A = 4bt \;\Rightarrow\; P_P = A_A\tau_P = (4bt)\tau_P$ (LARGEST)
B: $A_B = \pi bt \;\Rightarrow\; P_P = A_B\tau_P = (\pi bt)\tau_P$ (SMALLEST)
C: $A_C = \left[b + 2\left(\frac{\sqrt{5}}{2}\right)b\right]t = (1+\sqrt{5})bt \Rightarrow P_P = A_C\tau_P = [(1+\sqrt{5})bt]\tau_P$
D: $A_D = (\pi b/2 + 2b)t = (\pi/2 + 2)bt \;\Rightarrow\; P_P = A_D\tau_P = [(\pi/2+2)bt]\tau_P$