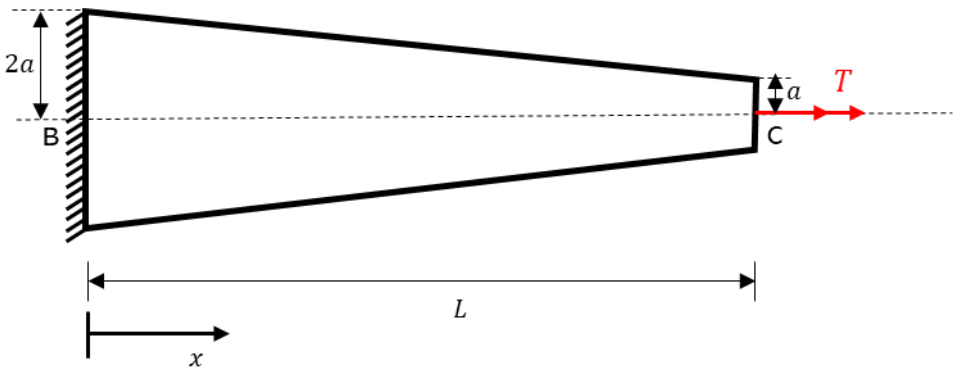
ME 323: Mechanics of Materials **Fall 2025**

Homework Set 4 **Due: Friday, October 3**

Problem 4.1 (10 points)

A tapered shaft of length L is fixed at one end and subjected to a torque T at the other. The radius at the fixed end is $2a$, while the radius at the free end is a. The shaft is made of steel with a shear modulus G.

1) Draw the free body diagram of the tapered rod, write down the equation(s) of equilibrium, and determine if the problem is statically determined or indeterminate.
2) Determine the polar area moment I_P using the coordinate system provided.
3) Write down the torque-angle of twist relation.
4) Determine the angle of twist at point C.
5) Determine the maximum shear stress at B and C. Indicate which end develops the highest shear stress.



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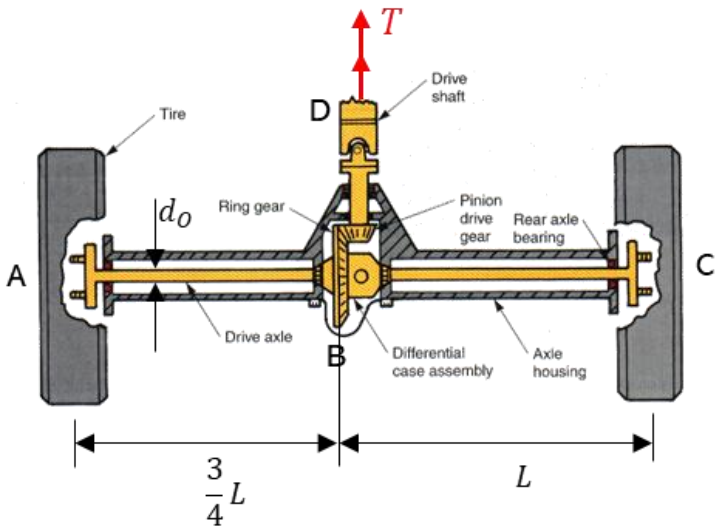
Problem 4.2 (10 points)

A schematic of a vehicle's differential rear axle subjected to a torque T applied through the drive shaft is shown below. For the design of the drive axle, the condition is assumed that the **tires are constrained from rotating**. The drive axle has a diameter d_o and is made of steel with shear modulus G. The gear ratio of the differential rear axle is 1:10 and the axial force produced in the bevel gears can be neglected.

1) Draw the free body diagrams for the pinion drive gear, the ring gear, and the shaft segments AB and BC. Use the schematic provided below.
2) Using the free body diagrams, write the equations of equilibrium of the ring gear.
3) Demonstrate if the system statically determinate or indeterminate.
4) Write down the torque-angle of twist equations for shaft segments AB and BC.
5) Determine the angle of twist at B.
6) If the drive axle is made hollow to lightweight the differential rear axle while keeping the outer diameter d_o unchanged. Determine the largest inner diameter d_i that can be used if the allowed shear stress τ_{all} and the maximum torque applied in the drive shaft is T_{Max}.



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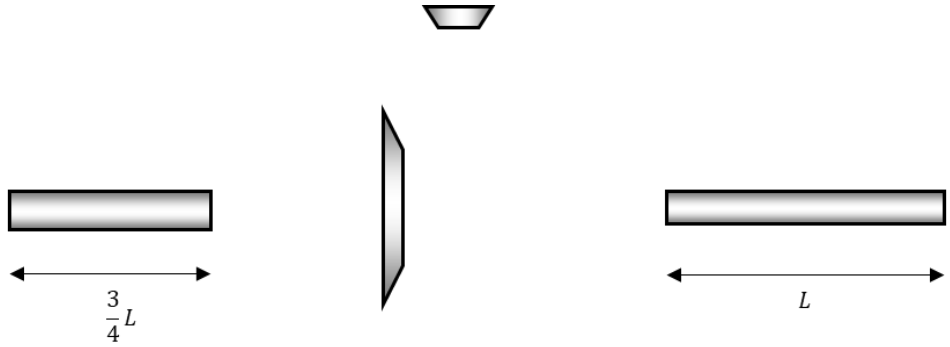

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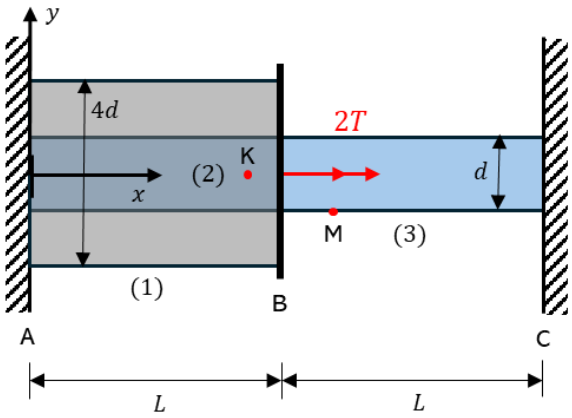
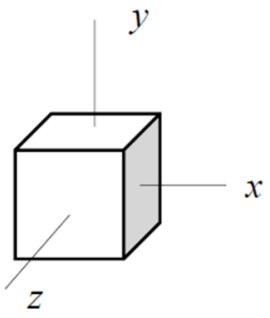
Problem 4.3 (10 points)

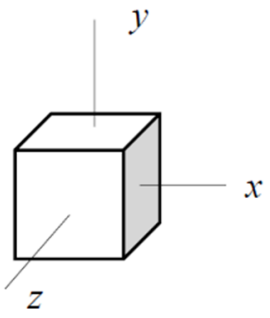
A coaxial torsional shaft is assembled from three rods (1–3). Rod 1 is a hollow sleeve with inner diameter d and outer diameter $4d$. Rod 2 is a solid rod of diameter d that passes through Rod 1; it is brazed to Rod 1 over their overlap and continues beyond the sleeve. Rod 3 is attached to Rod 2 with a connector at point B. The shear moduli of the rods are $G_1 = G, G_2 = G_3 = 4G$. The shaft is constrained at points A and C and a torque $2T$ is applied at connector B.

1) Draw the free body diagrams at the fixed end A and at connector B.
2) Write down the equations of equilibrium at connector B.
3) Write down the torque-angle of twist equations for rods 1, 2 and 3.
4) Determine the reaction torque at fixed end A.
5) Using the stress elements provided below, draw the shear stresses and indicate the magnitude of the shear stress at points K and M on the shaft. Point K is located on the outer surface of rod 1.





Stress element at point K



Stress element at point M

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Problem 4.4 (10 points)

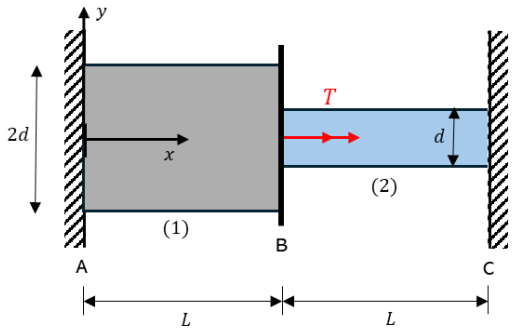
1) A shaft consists of two segments of the same material and equal length but different diameters, connected by a rigid plate at point B. If a torque T is applied at B, the magnitude of the reaction torques at the fixed ends A and C are:

A) $|T_A| > |T_C|$

B) $|T_A| < |T_C|$

C) $|T_A| = |T_C|$

D) More information is required to answer this question.



2) A shaft consists of two segments of different materials ($G_1 = 3G_2$) but equal length and diameter, connected by a rigid plate at point B. If a torque T is applied at B, the magnitude of the reaction torques at the fixed ends A and C are:

A) $|T_A| > |T_C|$

B) $|T_A| < |T_C|$

C) $|T_A| = |T_C|$

D) More information is required to answer this question.