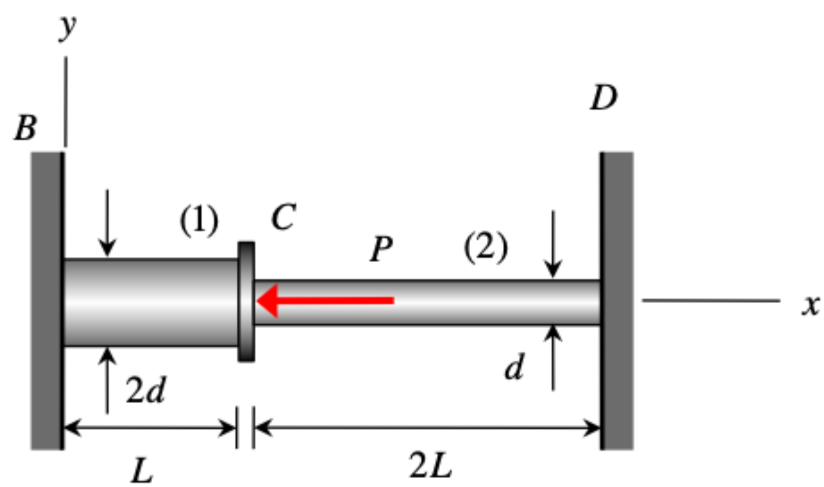
ME 323: Mechanics of Materials
Fall 2025

Homework Set 3
Due: Friday, September 19

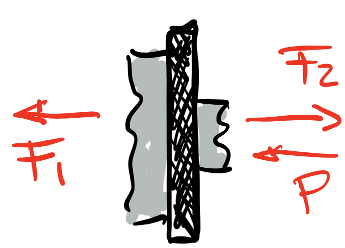
Problem 3.1 (10 points)

Two circular cross-section rod elements (1) and (2) are fixed to rigid walls. The rods are made from copper, with Young's modulus E and coefficient of thermal expansion α, and they are initially unstressed. An axial load P is then applied to the rigid connector C, the temperature of element (1) is increased by ΔT, and the temperature of element (2) is held constant.

(a) Assuming all members are under tension, draw the free body diagram of connector C.
(b) Using the free body diagrams, write down the equations of equilibrium of connector C.
(c) Is the system statically determinate or indeterminate?
(d) Write down the force-elongation equation for elements (1) and (2).
(e) Write down the key compatibility equation(s) that relate the elongation of members (1) and (2).
(f) Determine the stress in element (1). Indicate if element (1) is under tension or compression.
(g) Determine the displacement of rigid connector C along the x-axis.



* Assuming all members under tension.
* FBD.



* Equilibrium: $\overset{+}{\rightarrow} \sum F = 0 = F_2 - F_1 - P$ *1
* One equation of equilibrium and two unknowns (F_1 and F_2)

$\Rightarrow$ STATICALLY INDETERMINATE STRUCTURE

* Force-elongation equations:

$$e_1 = \frac{F_1 L}{\pi E d^2} + \alpha \Delta T L \qquad *3$$

$$e_2 = \frac{8 F_2 L}{\pi E d^2} \qquad *4$$

* Compatibility conditions

- fixed points: $u_B = 0$; $u_D = 0$
- member #1 : $e_1 = u_C - \cancel{u_B}$
- member #2 : $e_2 = \cancel{u_D} - u_C$

$\Rightarrow e_1 + e_2 = 0 \qquad *2$

* Solve for F_1

from *1 $\longrightarrow$ $0 = F_2 - F_1 - P$

from *2, using *3 and *4 $\longrightarrow$ $\dfrac{F_1 L}{E\pi d^2} + \alpha \Delta T L + \dfrac{8 F_2 L}{\pi E d^2} = 0$

$$\Rightarrow F_1 = -\frac{E\pi d^2}{9}\left[\frac{8P}{E\pi d^2} + \alpha \Delta T\right] \quad (c)$$

$$\Rightarrow \sigma_1 = \frac{F_1}{A_1} = -\frac{E}{9}\left[\frac{8P}{E\pi d^2} + \alpha \Delta T\right]$$

* $u_C = e_1 = \dfrac{F_1 L}{E\pi d^2} + \alpha \Delta T L = \dfrac{8L}{9}\left[\alpha \Delta T - \dfrac{P}{E\pi d^2}\right]$

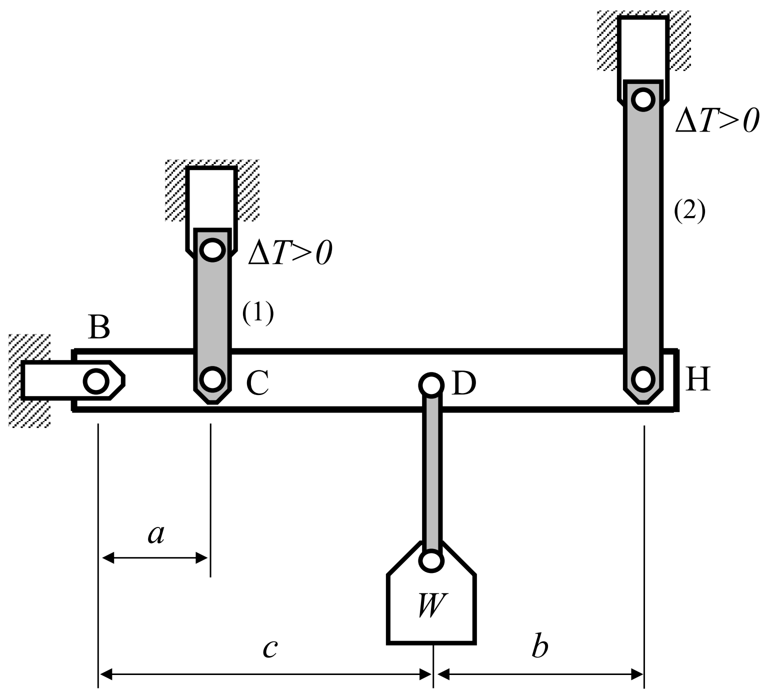
ME 323: Mechanics of Materials
Fall 2025

Homework Set 3
Due: Friday, September 19

Problem 3.2 (10 points)

A rigid bar BCDH is suspended using two deformable rods as shown in the figure. A weight W of 80 kN is supported from the bar. Both rods undergo the same change in temperature ΔT. The coefficient of thermal expansion of steel is $11 \cdot 10^{-6}\,°C^{-1}$ and of bronze is $19 \cdot 10^{-6}\,°\mathrm{C}^{-1}$. Neglecting the weight of the rigid bar,

(a) Assuming that all deformable rods are under tension, draw the free body diagram of the rigid bar BCDH.
(b) Using the free body diagram, write down the equation(s) of equilibrium of the rigid bar.
(c) Is the system statically determinate or indeterminate?
(d) Write down the force-elongation equation for rods (1) and (2).
(e) Write down the key compatibility equation(s) that relate the elongation of rods (1) and (2).
(f) Determine the change in temperature that will cause the tensile stress in the steel rod to be equal to 75 MPa.

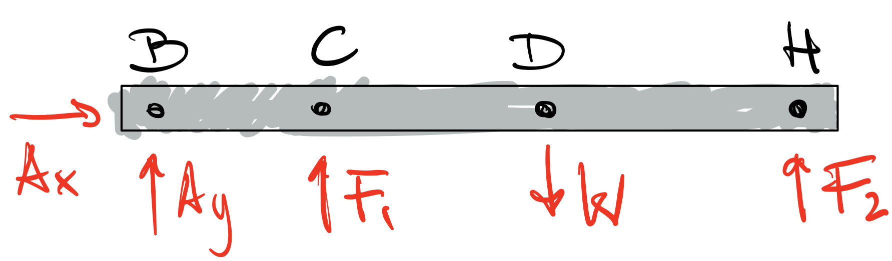


Steel: $E_1 = 210$ GPa, $L_1 = 1.5$ m, $A_1 = 300$ mm^2

Bronze: $E_2 = 80$ GPa, $L_2 = 3$ m, $A_1 = 1200$ mm^2

$a = 1$ m, $b = 2$ m, $c = 3$ m

* FBD



* Equilibrium:

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\oplus \uparrow \sum F_y = 0 = A_y + F_1 + F_2 - W \qquad *1$$

$$\oplus \circlearrowleft \left(\sum M\right)_B = 0 = F_1 - 3W + 5F_2$$

2 equations of equilibrium and 3 unknowns

$\left(\sum F_y = 0;\ \sum M = 0\right)$ $\quad$ (A_y, F_1, F_2)

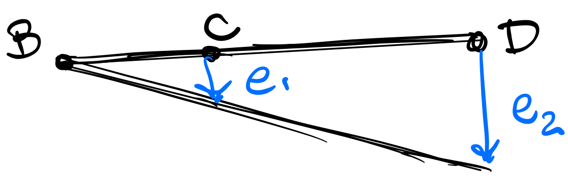
⇒ **STATICALLY INDETERMINATE STRUCTURE**

* Force-elongation equations

$$e_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha_1 L_1 \Delta T \qquad *3$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} + \alpha_2 L_2 \Delta T \qquad *4$$

* Compatibility conditions



$$\Rightarrow \frac{e_1}{1\,m} = \frac{e_2}{5\,m}$$

$$\Rightarrow \boxed{e_2 = 5 e_1} \qquad *2$$

* Solve for F_1

from *1 $\longrightarrow$ $0 = F_1 - 3W + 5F_2$

from *2 using *3 and *4 $\rightarrow$ $\dfrac{F_2 L_2}{E_2 A_2} + \alpha_2 \Delta T L_2 = 5\left[\dfrac{F_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1\right]$

$$\Rightarrow \left[\frac{3W - F_1}{5}\right]\frac{L_2}{E_2 A_2} + \alpha_2 \Delta T L_2 = 5\left[\frac{F_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1\right]$$

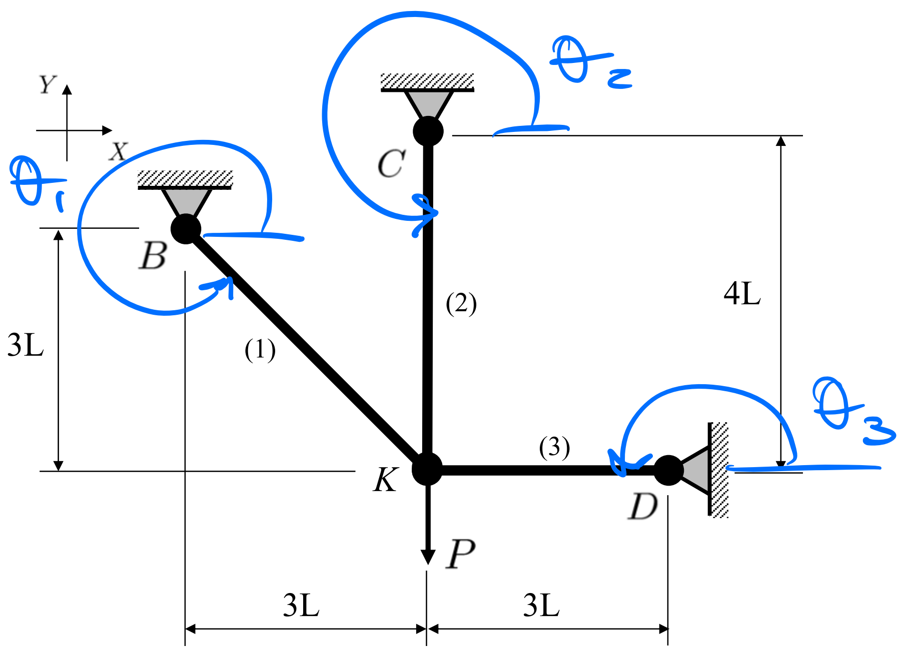
If $\sigma_1 = \frac{F_1}{A_1} = 75\ MPa$, then solve for ΔT

$$\Rightarrow \quad \Delta T = \frac{\left[\frac{3W - F_1}{5}\right]\frac{L_2}{E_2 A_2} - 5\frac{F_1 L_1}{E_1 A_1}}{5\alpha_1 L_1 - \alpha_2 L_2}$$

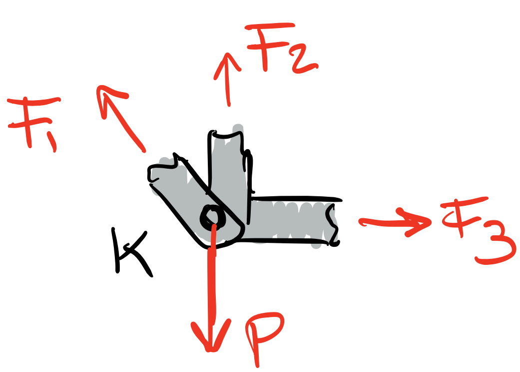
Problem 3.3 (10 points)

A planar truss structure is composed of three elastic truss members (1), (2), and (3) of uniform cross-sectional area A, as shown the figure. Members (1), (2), and (3) are pinned together at K. Members (1) and (2) are composed of an elastic material whose Young's modulus is E, whereas Young's modulus for (3) is $2E$. The structure is acted upon by a concentrated load P at joint K as shown. Orientation of the x-y axes is indicated.

(a) Assuming that all deformable rods are under tension, draw the free body diagram of the joint K.
(b) Using this free body diagram, write down the equilibrium equations of joint K.
(c) Is the system statically determinate or indeterminate?
(d) Write down the force-elongation equation for truss members (1), (2) and (3).
(e) Write down the compatibility equations for the structure. Specifically, write down the equations relating the displacement of joint K in the x and y directions to the axial elongation of each member (i.e., u_K and v_K).
(f) Determine the angular orientation θ for each member used in the compatibility equations (i.e., θ_1, θ_2, and θ_3).
(g) Determine the axial load in each truss element. Indicate if the element is under tension or compression.



* FBD



* Equilibrium

$\overset{+}{\rightarrow} \Sigma F_x = 0 = F_3 - F_1/\sqrt{2}$ (*1)

$\oplus \uparrow \Sigma F_y = 0 = F_2 + F_1/\sqrt{2} - P$ (*2)

2 equations of equilibrium and 3 unknowns
$(\Sigma F_x = 0 ; \Sigma F_y = 0)$ $(F_1 \; F_2 \; F_3)$

$\Rightarrow$ STATICALLY INDETERMINATE STRUCTURE

* Force-elongation equations

$$e_1 = \frac{F_1(\sqrt{2} \cdot 3L)}{EA} \; ; \quad e_2 = \frac{F_2(4L)}{EA} \; ; \quad e_3 = \frac{F_3(3L)}{2EA}$$

* Compatibility conditions

$$e_1 = u_k \cos\theta_1 + v_k \sin\theta_1 = u_k/\sqrt{2} - v_k/\sqrt{2}$$

$$e_2 = u_k \cos\theta_2 + v_k \sin\theta_2 = -v_k$$

$$e_3 = u_k \cos\theta_3 + v_k \sin\theta_3 = -u_k$$

From Figure : $\theta_1 = -45^\circ$; $\theta_2 = -90^\circ$; $\theta_3 = 180^\circ$

$$\Rightarrow \quad e_1 = -e_3/\sqrt{2} + e_2/\sqrt{2}$$ (*3)

* Solve for F_1, F_2, F_3

$$0 = F_3 - F_1/\sqrt{2}$$

$$0 = F_2 + F_1/\sqrt{2} - P$$

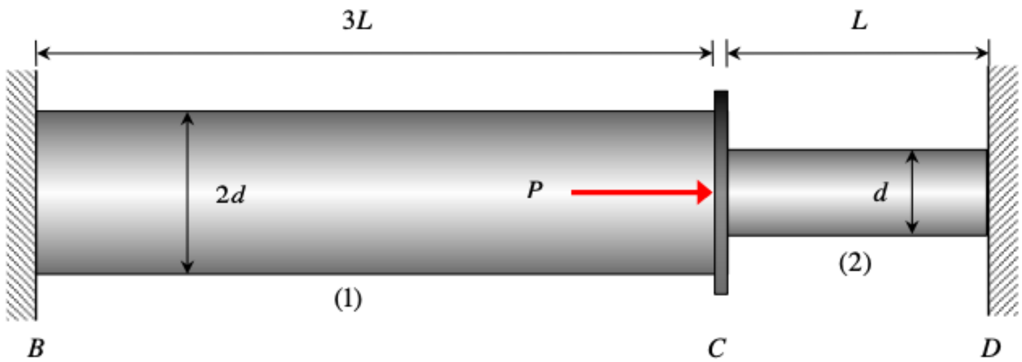
$$\frac{F_1(\sqrt{2}\cdot 3L)}{E\cancel{A}} = -\frac{1}{\sqrt{2}}\frac{F_3(3L)}{2E\cancel{A}} + \frac{1}{\sqrt{2}}\frac{F_2(4L)}{E\cancel{A}}$$

Next, write in terms of F_1 ....

ME 323: Mechanics of Materials
Fall 2025

Homework Set 3
Due: Friday, September 19

Problem 3.4 (10 points)



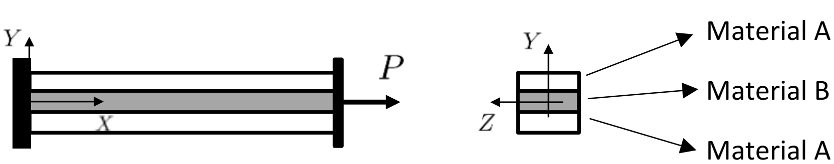
(a) A rod is made up of solid elements (1) and (2) joined by a rigid connector C, with the material of (1) and (2) having the same modulus of elasticity. An axial load P is applied to C with no thermal loads being present. Let F_1 and F_2 represent the axial loads in elements (1) and (2), respectively. Circle the response below which most accurately describes the relative sizes of F_1 and F_2:

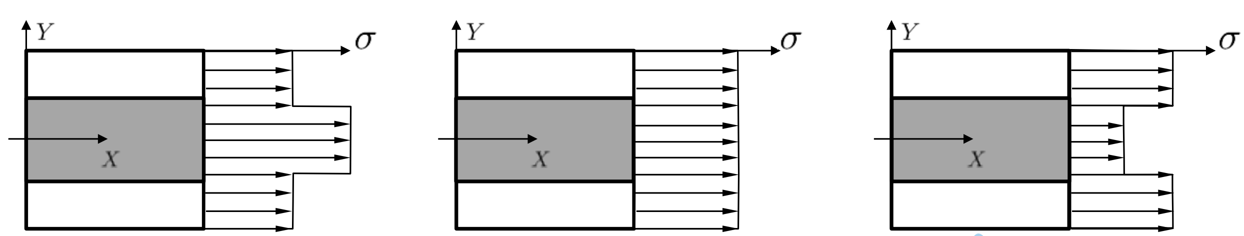
i) $|F_1| > |F_2|$
ii) $|F_1| = |F_2|$
iii) $|F_1| < |F_2|$
iv) More information is needed to answer this question.

$$e_1 + e_2 = 0 \Rightarrow |e_1| = |e_2| \Rightarrow \frac{|F_1|\,3L}{E\pi\left(\frac{2d}{2}\right)^2} = \frac{|F_2|\,L}{E\pi\left(\frac{d}{2}\right)^2} \Rightarrow |F_2| = \frac{3}{4}|F_1|$$

(b) A bimetallic bar with square cross section comprised of two elastic materials is subjected to an axial for P. Material A, depicted using white, is stiffer than material B, depicted using gray. Specifically, the Young's modulus of material A is two times larger than the Young's modulus of material B, and both materials have the same Poisson's ratio.

(b)





(1) (2) (3)

$$\epsilon_A = \epsilon_B = \epsilon \Rightarrow \sigma_A = E_A\epsilon \,,\; \sigma_B = E_B\epsilon \Rightarrow \sigma_A > \sigma_B$$

with $E_A > E_B$