SOLUTION

ME 323: Mechanics of Materials ***Homework H30/31***

Summer 2025 ***Assigned/due: July 29/August 1***

A circular cross-sectioned shaft having an outer radius of R is acted upon by forces F and P, and by an axial torque T and a bending moment M at its right end. The material of the shaft (aluminum alloy 2014-T6) is ductile and has a yield strength of σ_Y (see the tables in Chapter 2 for the numerical values of the yield strength of this material).

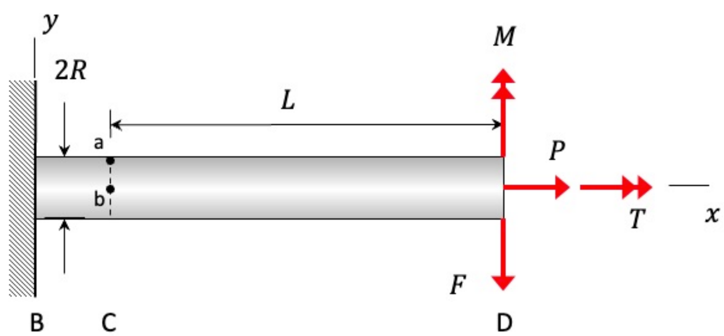
a) Determine the stress components at point "a" on the shaft, where "a" is on the top surface of the shaft along the y-axis. Show these components on the stress element provided. Leave your answers in terms of the variables defined here in the problem statement. Follow our usual procedure in this stress analysis of
   - equilibrium to determine the internal resultants,
   - showing these stress elements on the positive x-face of the cross-section,
   - showing stress distributions on the positive x-face of the cross-section,
   - calculation of the stress components in the supplied table, and
   - showing these stresses on the stress element for point "a".

b) Compute the *numerical* values for the principal components of stress and the absolute maximum shear stress at point "a".

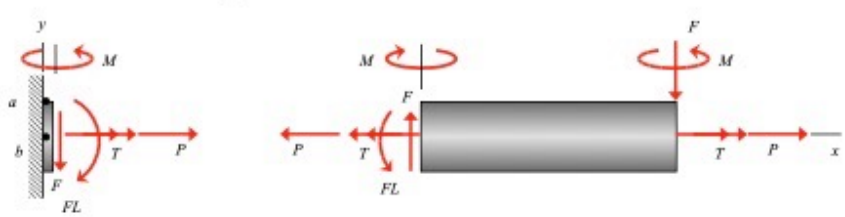
c) Compute the *numerical* value for the yielding factors of safety at point "a" using the maximum shear stress (MSS) theory.

d) Compute the *numerical* value for the yielding factors of safety at point "a" using the maximum distortional energy (MDE) theory.

For your numerical answers, use the following: P = 2 kN, F = 500 N, T = 600 N-m, M =1200 N-m, R = 20 mm and L = 1.5 m.

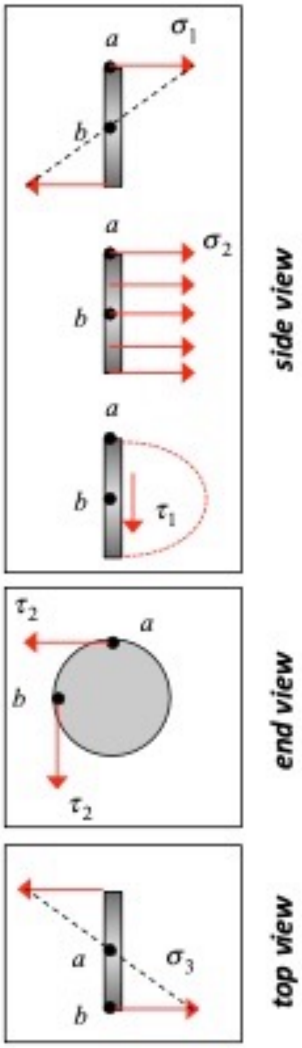


Acting on the cut surface at the wall, we have a shear force F, a bending moment $M_z = -FL$, an axial force P and a torque T.



Only the following page, we will use a table to list the components of normal and shear stress at the cut surface at points "a" and "b". These components are then shown on the stress elements above.

For location "a":



$$I = \frac{\pi}{4} R^4$$

$$A = \pi R^2$$

$$I_P = \frac{\pi}{2} R^4$$

load	location "a"	location "b"
M_z	$\sigma_{1a} = \frac{(FL)R}{I}$	$\sigma_{1b} = 0$
P	$\sigma_{2a} = \frac{P}{A}$	$\sigma_{2b} = \frac{P}{A}$
F	$\tau_{1a} = 0$	$\tau_{1b} = \frac{4}{3}\frac{F}{A}$
T	$\tau_{2a} = \frac{TR}{I_P}$	$\tau_{2b} = \frac{TR}{I_P}$
M	$\sigma_{3a} = 0$	$\sigma_{3b} = \frac{MR}{I}$

$$\sigma_x = \sigma_{1a} + \sigma_{2a} = \frac{FLR}{I} + \frac{P}{A} = \frac{FLR}{\pi R^4 / 4} + \frac{P}{\pi R^2} = \frac{F}{\pi R^2}\left(4\frac{L}{R} + \frac{P}{F}\right)$$

$$\tau_{xz} = \frac{TR}{I_P} = \frac{2T}{\pi R^3}$$

$$\sigma_{ave} = \frac{\sigma_x}{2} = \frac{F}{2\pi R^2}\left(4\frac{L}{R} + \frac{P}{F}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2} = \frac{F}{2\pi R^2}\sqrt{\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2}$$

$$\sigma_{P1} = \sigma_{ave} + R = \frac{4FL}{\pi R^3} + \frac{P}{\pi R^2} + \frac{F}{2\pi R^2}\sqrt{\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2}$$

$$= \frac{F}{2\pi R^2}\left[8\frac{L}{R} + 2\frac{P}{F} + \sqrt{\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2}\right]$$

$$\sigma_{P2} = \sigma_{ave} - R = \frac{F}{2\pi R^2}\left[8\frac{L}{R} + 2\frac{P}{F} - \sqrt{\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2}\right]$$

Since $R > \sigma_{ave}$, we know that σ_{P1} and σ_{P2} have opposite signs; therefore, $\left|\tau_{max}\right|_{abs} = R$.

Maximum shear stress (MSS) theory

For the MSS theory, we write that failure occurs when: $\left|\tau_{max}\right|_{abs} = \frac{\sigma_Y}{2}$. Therefore, the factor of safety for MSS is:

$$FS = \frac{\sigma_Y / 2}{\left|\tau_{max}\right|_{abs}} = \frac{\sigma_Y}{2R} = \frac{\pi R^2 \sigma_Y}{F}\left[\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2\right]^{-1}$$

Maximum distortional energy (MDE) theory

For the MSS theory, we write that failure occurs when: $\sigma_Y = \sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}$.
Therefore, the factor of safety for MDE is:

$$FS = \frac{\sigma_Y}{\sigma_M} = \frac{\sigma_Y}{\sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}}$$