SOLUTION

ME 323: Mechanics of Materials **Homework H26**

Summer 2025 **Assigned/due: July 23/July 28**

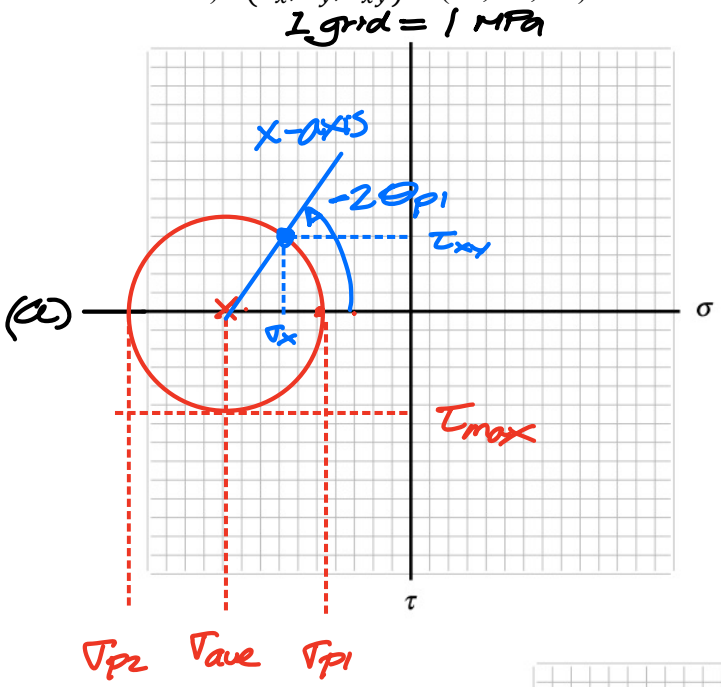
Consider the three states of stress given below in a), b) and c). For each state of stress, do the following:

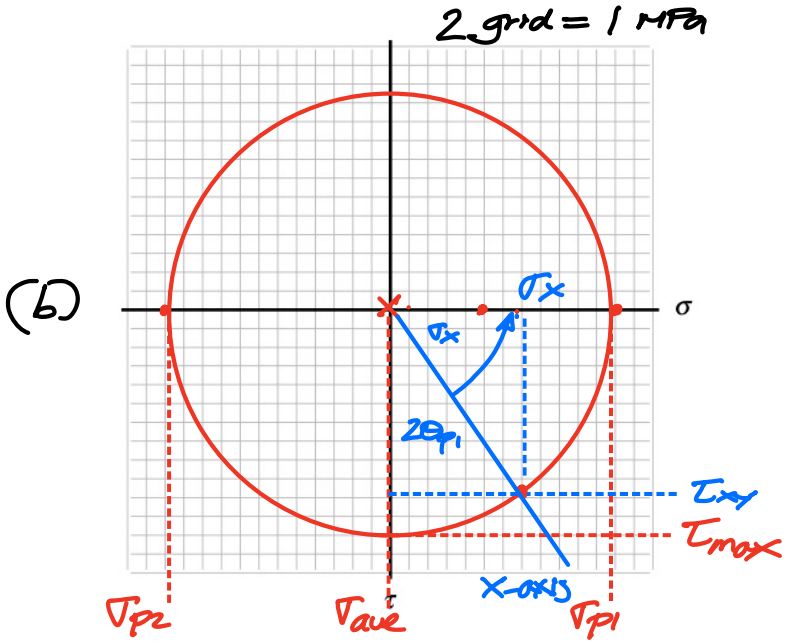
- Draw the in-plane *Mohr's circle*.
- Determine the two principal components of stress.
- Locate the *x-axis* on the in-plane Mohr's circle.
- Determine the rotation angle for the stress element that produces the principal stress of σ_{P1}.
- Determine the maximum *in-plane* shear stress.

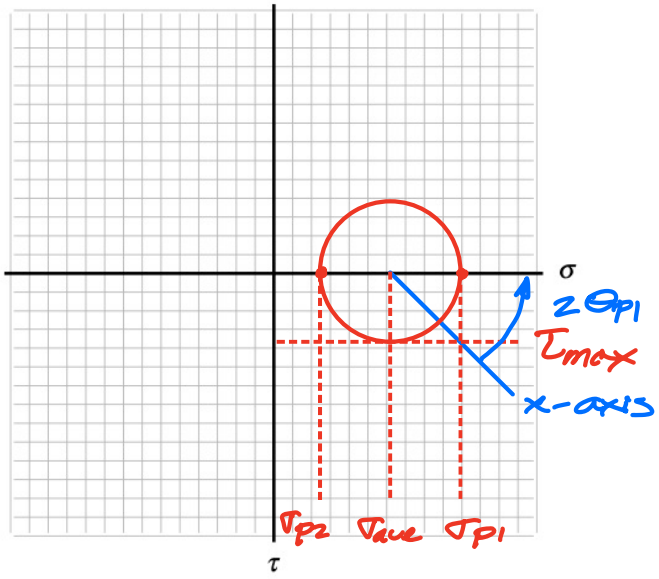
a) $(\sigma_x, \sigma_y, \tau_{xy})$ = (-7, -13, -4) MPa

b) $(\sigma_x, \sigma_y, \tau_{xy})$ = (15, -15, 20) MPa

c) $(\sigma_x, \sigma_y, \tau_{xy})$ = (88, 40, 32) MPa







Part a)

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-7 + (-13)}{2} = -10 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left[\frac{-7 - (-13)}{2}\right]^2 + (-4)^2} = 5 \text{ MPa}$$

$$\therefore \sigma_{p1} = \sigma_{ave} + R = -10 + 5 = -5 \text{ MPa}$$

$$\sigma_{p2} = \sigma_{ave} - R = -10 - 5 = -15 \text{ MPa}$$

$$\tau_{max} = R = 5 \text{ MPa}$$

$$x\text{-axis}: (\sigma_x, \tau_{xy}) = (-7, -4) \text{ MPa}$$

$$-2\theta_{p1} = \tan^{-1}\left(\frac{4}{10-7}\right) = 53.1^\circ \Rightarrow \theta_{p1} = -26.6^\circ \text{ (CW)}$$

Part b)

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{15 - 15}{2} = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{15 - (-15)}{2}\right)^2 + 20^2} = 25 \text{ MPa}$$

$$\therefore \sigma_{p1} = \sigma_{ave} + R = 25 \text{ MPa}$$

$$\sigma_{p2} = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\tau_{max} = R = 25 \text{ MPa}$$

$$x\text{-axis}: (\sigma_x, \tau_{xy}) = (15, 20) \text{ MPa}$$

$$-2\theta_{p1} = \tan^{-1}\left(\frac{20}{15}\right) = 53.13^\circ \Rightarrow \theta_{p1} = 26.6^\circ \text{ (CCW)}$$

Part c)

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{88 + 40}{2} = 64 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{88 - 40}{2}\right)^2 + 32^2} = 40 \text{ MPa}$$

$$\therefore \sigma_{p1} = \sigma_{ave} + R = 104 \text{ MPa}$$

$$\sigma_{p2} = \sigma_{ave} - R = 24 \text{ MPa}$$

$$\tau_{max} = R = 40 \text{ MPa}$$

$$x\text{-axis}: (\sigma_x, \tau_{xy}) = (88, 32) \text{ MPa}$$

$$-2\theta_{p1} = \tan^{-1}\left(\frac{32}{88-64}\right) = 53.13^\circ \Rightarrow \theta_{p1} = 26.6^\circ \text{ (CCW)}$$