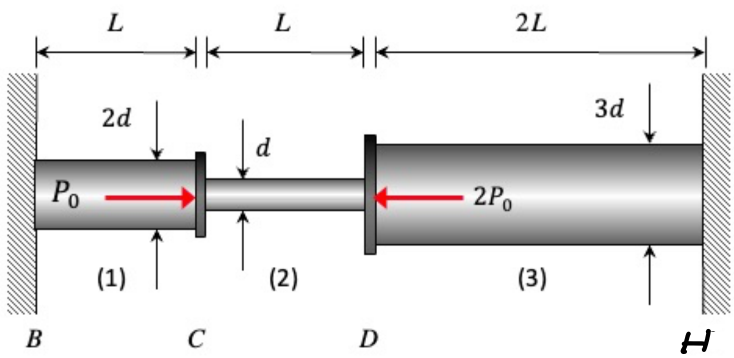
SOLUTION

ME 323: Mechanics of Materials | **Homework H22**

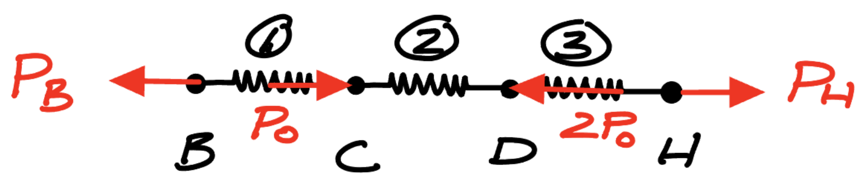
Summer 2025 | **Assigned/due: July 17/July 22**

The following rod model has loads acting on rigid connectors C and D, as shown.

a) Draw a free body diagram of the complete rod (FBD).
b) Write down the elemental stiffnesses k_1, k_2 and k_3 for the rod.
c) Write down the strain energy for the complete rod.
d) Write down the force potential energy for the loads acting on the rod.
e) Write down the total potential energy for the structure.
f) Apply the principle of minimum potential energy to set up the algebraic equations for the nodal displacements in the rod.
g) Solve for the displacements of connectors C and D.



Part a)



Part b)

$$k_1 = \frac{E\pi(2d/2)^2}{L} = \pi\frac{Ed^2}{L}$$

$$k_2 = \frac{E\pi(d/2)^2}{L} = \frac{\pi}{4}\frac{Ed^2}{L}$$

$$k_3 = \frac{E\pi(3d/2)^2}{2L} = \frac{9\pi}{8}\frac{Ed^2}{L}$$

Parts c) - e)

$$\Pi_U = \frac{1}{2}k_1 u_C^2 + \frac{1}{2}k_2(u_D - u_C)^2 + \frac{1}{2}k_3 u_D^2$$

$$\Pi_F = -\left[P_B \cancel{u_B}^{\,0} + P_0 u_C - 2P_0 u_D + P_H \cancel{u_H}^{\,0}\right] = -P_0 u_C + 2P_0 u_D$$

$\therefore \quad \Pi = \Pi_E + \Pi_U$

$$= \frac{1}{2}k_1 u_c^2 + \frac{1}{2}k_2(u_D - u_c)^2 + \frac{1}{2}k_3 u_D^2 - P_0 u_c + 2P_0 u_D$$

Part f)

- $\frac{\partial \Pi}{\partial u_c} = 0 = k_1 u_c - k_2(u_D - u_c) - P_0$

$\hookrightarrow \quad (k_1 + k_2) u_c - k_2 u_D = P_0$

(1) $\quad \hookrightarrow \quad 5u_c - u_D = \frac{4L}{\pi E d^2} P_0$

- $\frac{\partial \Pi}{\partial u_D} = 0 = k_2(u_D - u_c) + k_3 u_D + 2P_0$

$\hookrightarrow \quad -k_2 u_c + (k_2 + k_3) u_D = -2P_0$

(2) $\quad \hookrightarrow \quad -u_c + \frac{11}{2} u_D = -\frac{8L}{\pi E d^2} P_0$

Part g)

- Multiply (2) by 5 and add to (1):

$$\left(-1 + \frac{55}{2}\right) u_D = (4 - 40) \frac{L}{\pi E d^2} \Rightarrow u_D = -\frac{72}{53} \frac{P_0 L}{E d^2}$$

$\longleftarrow u_D$

- Multiply (1) by $\frac{11}{2}$ and add to (2):

$$\left(\frac{55}{2} - 1\right) u_c = (22 - 8) \frac{P_0 L}{\pi E d^2} \Rightarrow u_c = \frac{28}{53} \frac{P_0 L}{E d^2}$$

$\longleftarrow u_c$