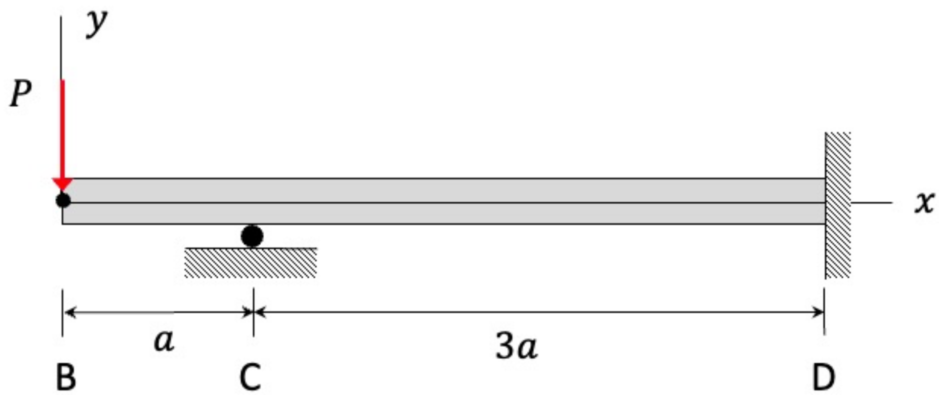
SOLUTION

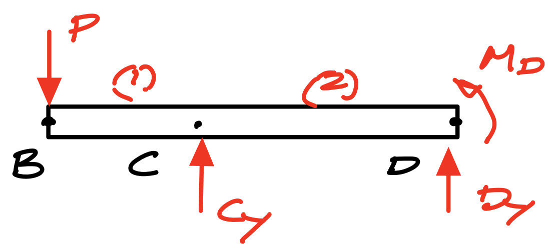
ME 323: Mechanics of Materials — **Homework H20**

Summer 2025 — **Assigned/due: July 15/July 18**

The propped-cantilevered beam BD is made up of a material with a Young's modulus of E and the cross-section of the beam has a second area moment of I. A point load P acts at end B of the beam.

a) Draw a free body diagram of the beam (FBD).
b) Write down the equilibrium equations for the bar from your FBD. Is this beam determinate or indeterminate?
c) Choose a redundant reaction for the beam. (It is suggested that you use the reaction force at C as your redundant load.)
d) Write down the strain energy in the system that involves your redundant load. You may ignore the effects of shear in your strain energy function.
e) Use Castigliano's theorem to determine the redundant reaction that you chose earlier. Leave your answer in terms of, at most: E, I, P and a.
f) Determine the reactions on the beam at locations C and D.





Equilibrium

(1) $\Sigma M_D = -C_y(3a) + P(4a) + M_D = 0$

(2) $\Sigma F_y = C_y - P + D_y = 0$

2 eqns / 3 unknowns $\Rightarrow$ INDETERMINATE

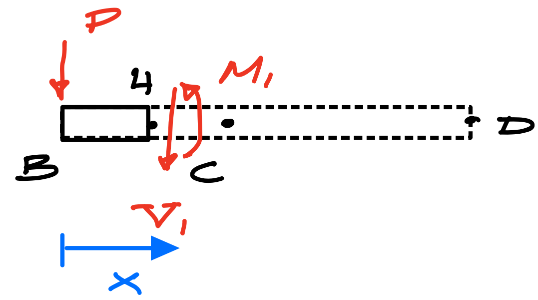
Redundant load

Will choose C_y as the redundant load

$\therefore$

(1a): $M_D = 3C_y a + 4Pa$

(1b): $D_y = P - C_y$



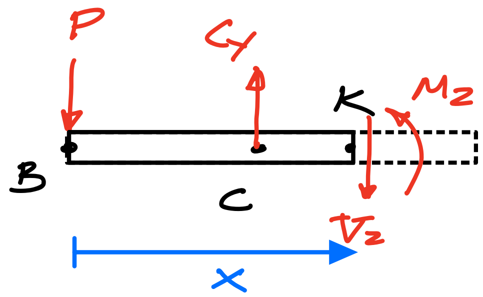
Strain energy

Section BC

$$\Sigma M_4 = M_1 + Px = 0$$

$$\hookrightarrow M_1(x) = -Px$$

$$\therefore \quad \frac{\partial M_1}{\partial C_y} = 0$$



Section CD

$$\Sigma M_K = M_2 + Px - C_y(x-a) = 0$$

$$\hookrightarrow M_2(x) = C_y(x-a) - Px$$

$$\therefore \quad \frac{\partial M_2}{\partial C_y} = x-a$$

$$\therefore \quad U = U_1 + U_2$$

$$= \frac{1}{2EI}\int_0^a M_1^2\,dx + \frac{1}{2EI}\int_a^{4a} M_2^2\,dx$$

Castigliano

$$\frac{\partial U}{\partial C_y} = 0 = \frac{1}{\cancel{EI}}\int_0^a M_1 \cancelto{0}{\frac{\partial M_1}{\partial C_y}}\,dx + \frac{1}{\cancel{EI}}\int_a^{4a} M_2 \frac{\partial M_2}{\partial C_y}\,dx$$

$$= \int_a^{4a}\left[C_y(x-a)^2 - Px(x-a)\right]dx$$

$$= \int_a^{4a}\left[C_y(x^2-2ax+a^2) - P(x^2-ax)\right]dx$$

$$= \left\{\frac{1}{3}\left[(4a)^3 - a^3\right] - \frac{2a}{2}\left[(4a)^2 - a^2\right] + a^2(4a-a)\right\} C_y$$

$$- \left\{\frac{1}{3}\left[(4a)^3 - a^3\right] - \frac{a}{2}\left[(4a)^2 - a^2\right]\right\} P$$

$$= 9C_y a^3 - \frac{27}{2}Pa^3$$

$$\therefore \quad C_y = \frac{3}{2}P \quad \longleftarrow \quad C_y$$

$$(1a) \Rightarrow \quad M_D = 3C_y a + 4Pa = \frac{17}{2}Pa \quad \longleftarrow \quad M_D$$

$$(1b) \Rightarrow \quad D_y = P - C_y = -\frac{1}{2}P \quad \longleftarrow \quad D_y$$