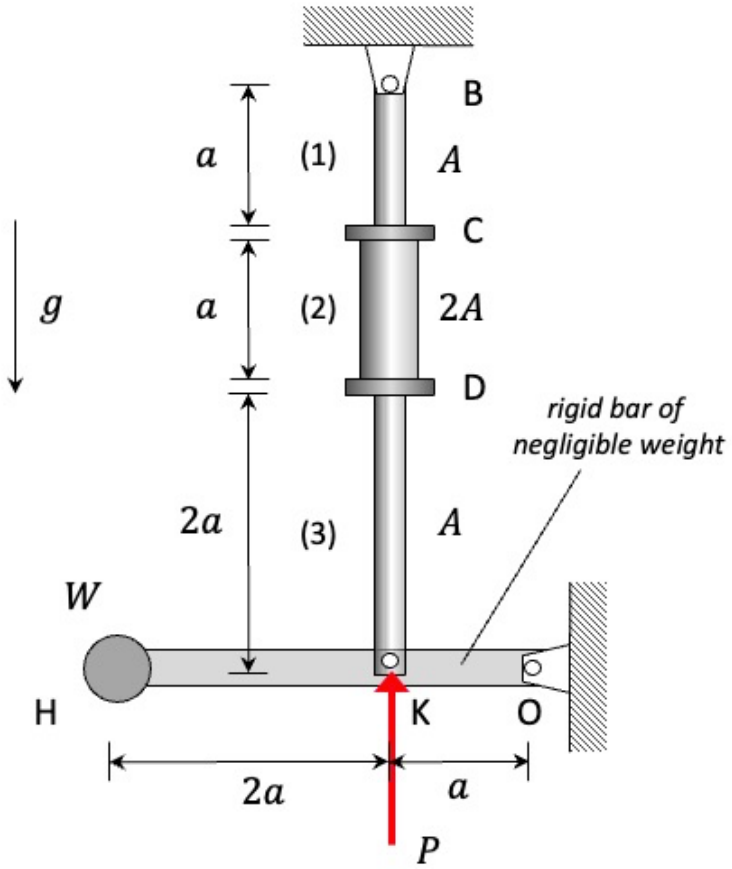
SOLUTION

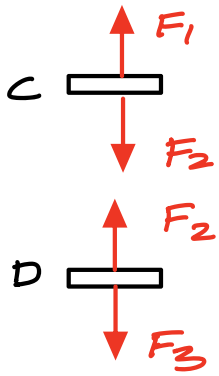
ME 323: Mechanics of Materials **Homework H19**

Summer 2025 **Assigned/due: July 14/July 17**

A horizontal rigid bar OH supports a body of weight W at end H. Bar OH, in turn, is supported by a three-member rod made up of members (1), (2) and (3), with the members having cross-sectional areas of A, $2A$ and A, and made up of a material with a Young's modulus of E. A support force P acts at connector D. The weights of the rod members and connectors can be considered to negligible. Assume small angles of rotation for bar OH.

a) Draw a free body diagram of bar OH (FBD).
b) Write down the equilibrium equations for the bar from your FBD.
c) Write down the strain energy in the system.
d) Use Castigliano's theorem to determine the displacement of pin K. Leave your answer in terms of, at most: E, A, P, W and a.





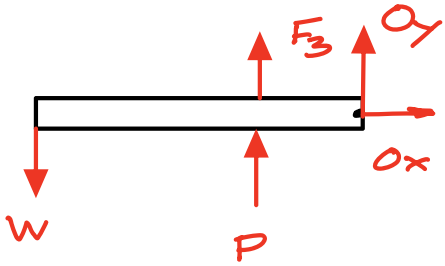
Equilibrium

Bar: $\Sigma M_O = W(3a) - Pa - F_3 a = 0$

(1) $\rightarrow F_3 = P - 3W$

(2) D: $\Sigma F_y = F_2 - F_3 = 0 \Rightarrow F_2 = F_3$

(3) C: $\Sigma F_y = F_1 - F_2 = 0 \Rightarrow F_1 = F_2 = F_3$



Strain energy

$$U_1 = \frac{1}{2}\frac{F_1^2 a}{EA}$$

(4) $$U_2 = \frac{1}{2}\frac{F_2^2 a}{2EA}$$

$$U_3 = \frac{1}{2}\frac{F_3^2 (2a)}{EA}$$

$$U = U_1 + U_2 + U_3 = \frac{1}{2}\left[F_1^2 + \frac{1}{2}F_2^2 + 2F_3^2\right]\frac{a}{EA}$$

(1)-(4):

$$U = \frac{1}{2}\left(\frac{7}{2}F_3^2\right)\frac{a}{EA} = \frac{7}{4}(P-3W)^2\frac{a}{EA}$$

Castigliano

$$V_K = \frac{\partial U}{\partial P} = \frac{7}{2}(P-3W)\frac{a}{EA}$$

$$= \frac{7}{2}\frac{a}{EA}(P-3W)$$

V_K ◄——————————