SOLUTION

ME 323: Mechanics of Materials — ***Homework Set H17***

Summer 2025 — ***Assigned/Due: July 9/July 15***

Consider the loading on the beam shown below where the beam is fixed to a wall at B and a roller support at C. Using integration techniques, determine the reaction on the beam at C and the displacement of end D of the beam. In your work, explicitly show the following steps:

a) Draw FBD of beam and write down the equilibrium equations for the beam. Is this a determinate problem?

b) Make a mathematical cut in beam between B and C (at a location "x").

   i. Draw an FBD of the beam between B and the cut at "x".

   ii. Determine the bending moment, M(x), in the beam at "x".

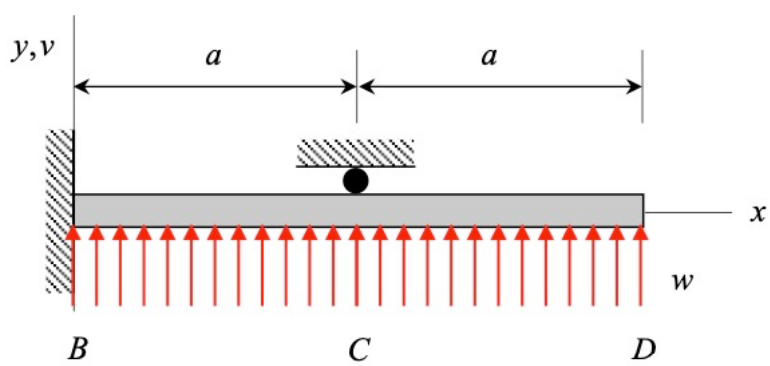
   iii. Integrate M(x) to find $\theta(x)$.

   iv. Integrate $\theta(x)$ to find v(x).

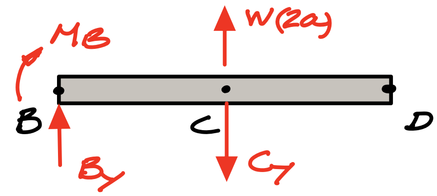
   v. Evaluate the reaction on the beam at C.

c) Make a mathematical cut in the beam between C and D. Repeat Steps i.-iv. above to find $\theta(x)$ and v(x) for "x" between C and D.

d) Evaluate v(2a).



1. Equilibrium – ext. reactions



$$\sum M_B = -C_y a + (2Wa)a - M_B = 0$$

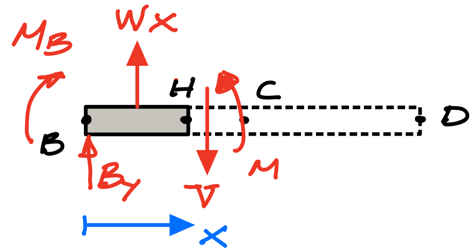
(1) $\hookrightarrow \quad M_B = 2Wa^2 - C_y a$

$$\sum F_y = 2Wa - C_y + B_y = 0$$

(2) $\hookrightarrow \quad B_y = C_y - 2Wa$

2 equations / 3 unknowns $\Rightarrow$ INDETERMINATE

2. Load/deformation



- Section BC:

$$\sum M_H = -M_B + M - Wx\left(\frac{x}{2}\right) - B_y x = 0$$

$$\hookrightarrow \quad M(x) = M_B + B_y x + \frac{1}{2}Wx^2$$

$\therefore$

$$\theta(x) = \cancel{\theta(0)}^{\,0} + \frac{1}{EI}\int_0^x \left(M_B + B_y x + \frac{1}{2}Wx^2\right)dx$$

wall

$$= \frac{1}{EI}\left(M_B x + \frac{1}{2}B_y x^2 + \frac{1}{6}Wx^3\right)$$

$$V(x) = \cancel{V(0)}^{\,0} + \int_0^x \frac{1}{EI}\left(M_B x + \frac{1}{2}B_y x^2 + \frac{1}{6}Wx^3\right)dx$$

wall

$$= \frac{1}{EI}\left(\frac{1}{2}M_B x^2 + \frac{1}{6}B_y x^3 + \frac{1}{24}Wx^4\right)$$

3. Compatibility: enforce boundary conditions on deflection function

$$V_c = V(a) = 0 = \frac{1}{EI}\left(\frac{1}{2}M_B a^2 + \frac{1}{6}B_y a^3 + \frac{1}{24}Wa^4\right)$$

(3) $\hookrightarrow \quad 12M_B + 4B_y a + Wa^2 = 0$

4. Solve

(1) and (2) into (3):

$$12(2Wa^2 - C_y a) + 4(C_y - 2Wa)a + Wa^2 = 0$$

$$\hookrightarrow \quad (12a - 4a)C_y = 17Wa^2$$

$$\hookrightarrow \quad C_y = \frac{17}{8}Wa \longleftarrow C_y$$

$$(1) \Rightarrow \quad M_B = 2Wa^2 - \left(\frac{17}{8}Wa\right)a = -\frac{1}{8}Wa^2 \longleftarrow M_B$$

$$(2) \Rightarrow \quad B_y = \frac{17}{8}Wa - 2Wa = \frac{1}{8}Wa \longleftarrow B_y$$