SOLUTION

ME 323: Mechanics of Materials — ***Homework Set H16***

Summer 2025 — ***Assigned/Due: July 8/July 14***

Consider the loading on the beam shown below where the beam is supported by a pin joint at B and a roller support at C. Using integration techniques, determine the slope of the displacement at B and the displacement of end D of the beam. In your work, explicitly show the following steps:

a) Draw FBD of beam and determine the reactions at B and C.

b) Make a mathematical cut in beam between B and C (at a location "x").

   i. Draw an FBD of the beam between B and the cut at "x".

   ii. Determine the bending moment, M(x), in the beam at "x".

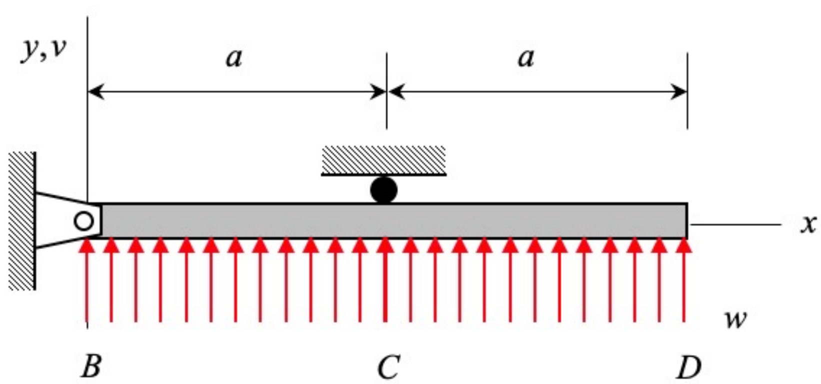
   iii. Integrate M(x) to find $\theta(x)$.

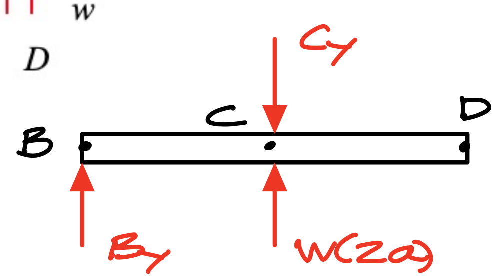
   iv. Integrate $\theta(x)$ to find v(x).

   v. Evaluate $\theta(0)$.

c) Make a mathematical cut in the beam between C and D. Repeat Steps i.-iv. above to find $\theta(x)$ and v(x) for "x" between C and D.

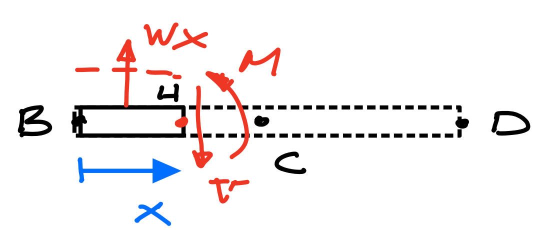
d) Evaluate v(2a).





External reactions

$$\Sigma M_B = -C_y a + (2aw)a = 0$$
$$\hookrightarrow C_y = 2aw$$
$$\Sigma F_y = B_y - C_y + 2aw = 0$$
$$\hookrightarrow B_y = 0$$



Section BC

$$\Sigma M_H = M - (wx)\left(\frac{x}{2}\right) \Rightarrow M(x) = \frac{1}{2} w x^2$$

$$\theta(x) = \theta(0) + \frac{1}{EI}\int_0^x \left(\frac{1}{2} w x^2\right) dx = \theta_B + \frac{1}{6EI} w x^3$$

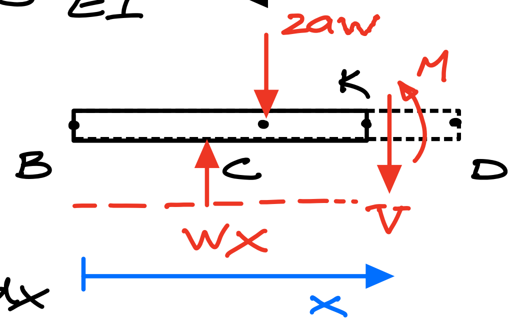
$$v(x) = \cancel{v(0)}^{\,0} + \int_0^x \left[\theta_B + \frac{1}{6EI} w x^3\right] dx = \theta_B x + \frac{1}{24EI} w x^4$$

$\therefore \quad V_c = V(a) = 0 = \theta_B a + \frac{1}{24EI} Wa^4$

$\hookrightarrow \theta_B = -\frac{Wa^3}{24EI} \longleftarrow \theta_B$

$\theta_c = \theta(a) = -\frac{1}{24EI}a^3 + \frac{1}{6EI}Wa^3 = \frac{1}{8}\frac{Wa^3}{EI} \longleftarrow \theta_c$

<u>Section CD</u>



$\Sigma M_K = (2aw)(x-a) - (wx)\frac{x}{2} + M = 0$

$\hookrightarrow M(x) = 2a^2w - 2awx + \frac{1}{2}wx^2$

$\theta(x) = \theta(a) + \frac{1}{EI}\int_a^x [2a^2w - 2awx - \frac{1}{2}wx^2]dx$

$= \theta_c + \frac{1}{EI}[2a^2w(x-a) - aw(x^2-a^2) - \frac{1}{6}w(x^3-a^3)]$

$= \theta_c + \frac{1}{EI}[\frac{19}{6}a^3w + 2a^2wx - awx^2 - \frac{1}{6}wx^3]$

$V(x) = \cancel{V(a)}^{0} + \int_a^x [\theta_c + \frac{1}{EI}(\frac{19}{6}a^3w + 2a^2wx - awx^2 - \frac{1}{6}wx^3]dx$

$= \theta_c(x-a) + \frac{1}{EI}[\frac{19}{6}a^3w(x-a) + a^2w(x^2-a^2)$

$- \frac{1}{3}aw(x^3-a^3) - \frac{1}{24}w(x^4-a^4)]$