SOLUTION

ME 323: Mechanics of Materials ***Homework Set H15***

Summer 2025 ***Assigned/Due: July 7/July 10***

Consider the loading on the cantilevered beam shown below.

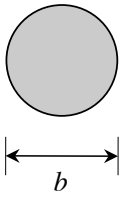
a) Sketch the shear force $V(x)$ and bending moment $M(x)$ distribution on the beam using the axes below. Provide details on your calculations.

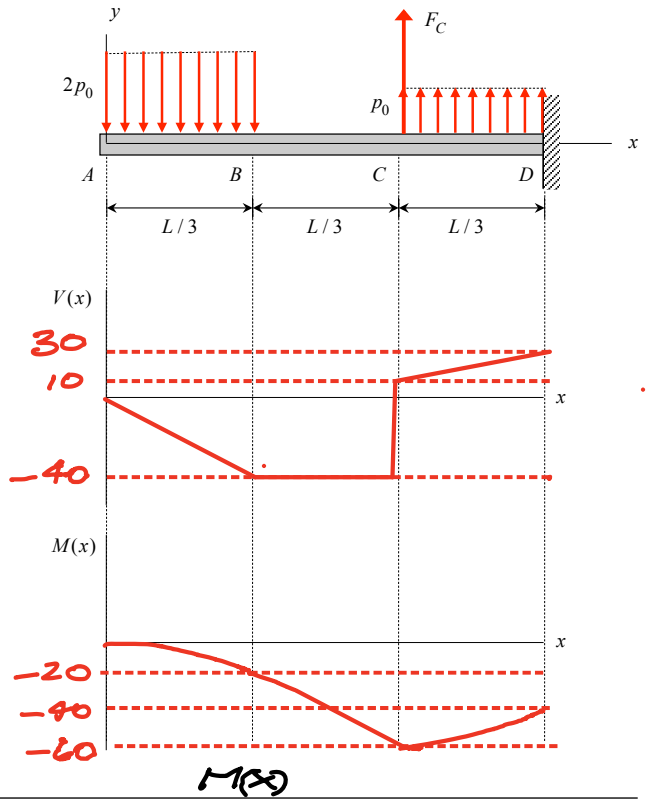
b) Determine the location(s) along the beam at which the maximum magnitude normal stress exists and location(s) along the beam at which the maximum magnitude shear stress exists.

c) Consider the circular beam cross-section shown. For this cross section, determine the maximum magnitude normal stress and its location on the cross section.

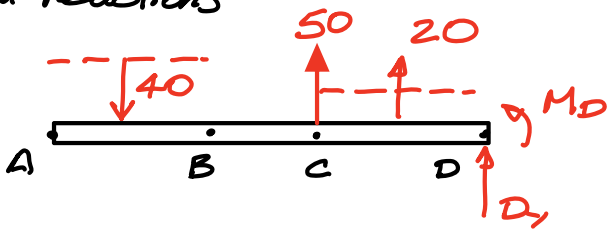
d) Also, determine the value of the maximum shear stress in the beam and its location on the neutral axis.

Use the following in your calculations: $L = 3\,m$, $p_0 = 20\,kN/m$, $F_C = 50\,kN$ and $b = 0.1\,m$.





External reactions



- $\Sigma M_D = M_D - 20\left(\frac{1}{2}\right) - 50(1) + 40\left(\frac{5}{2}\right) = 0$
  - ↳ $M_D = -40\,kN\cdot m$
- $\Sigma F_y = -40 + 50 + 20 + D_y = 0$
  - ↳ $D_y = -30\,kN$

V(x)

- $V(1) = V(0) + (-40)(1) = -40\,kN$
- $V(2) = V(1) = -40\,kN$
- $V(2^+) = V(2^-) + 50 = 10\,kN$
- $V(3) = V(2^+) + (20)(1) = 30\,kN$

Checks, since $|D_y| = 30$ pointing downward (-y)

M(x)

- $M(1) = M(0) + \frac{1}{2}(-40)(1) = -20\,kN\cdot m$
- $M(2) = M(1) + (-40)(1) = -60\,kN\cdot m$
- $M(3) = M(2) + (10)(1) + \frac{1}{2}(20)(1)$
  $= -60 + 10 + 10 = -40\,kN\cdot m$

b) • Maximum bending moment magnitude occurs at $x = 2$ m.
" normal stress " " " " "
• Maximum shear force and shear stress magnitude occurs between $2 < x < 3$ m along beam

c) $|\sigma|_{max} = \frac{|M|(b/2)}{I_o} = \frac{|M|(b/2)}{\frac{\pi}{4}(b/2)^4} = \frac{32}{\pi}\frac{M}{b^3}$

$= \frac{32}{\pi}\frac{60}{(0.1)^3} = \left(\frac{1.92}{\pi} \times 10^6\right)\frac{kN}{m^2}$

Occurs at top and bottom of cross-section

d) $|\tau|_{max} = \frac{4}{3}\frac{|V|}{A} = \frac{4}{3}\frac{|V|}{\pi(b/2)^2} = \frac{16}{3\pi}\frac{|V|}{b^3}$

$= \frac{16}{3\pi}\frac{40}{(0.1)^2} = \frac{640}{3\pi} \times 10^2 \frac{kN}{m^2}$

occurs all along the neutral axis