SOLUTION

ME 323: Mechanics of Materials — **Homework Set H14**

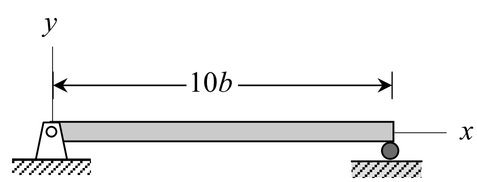
Summer 2025 — **Assigned/Due: July 3/July 9**

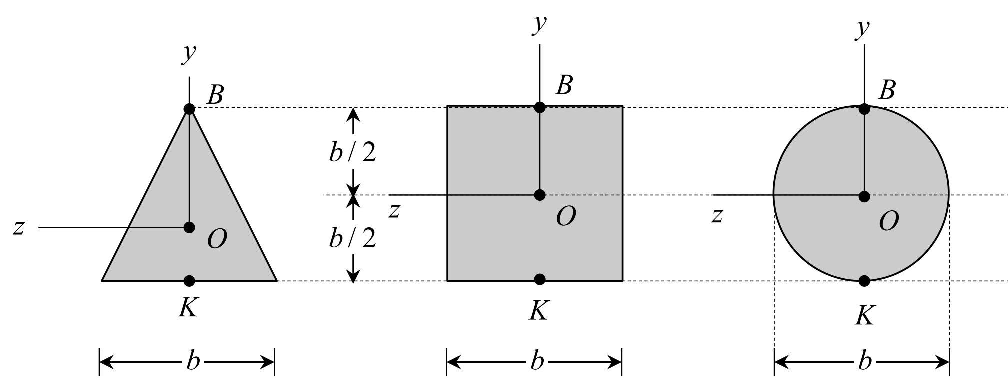
At a critical location along the length of a beam, the bending M on the cross-section of the beam is known. On each of the three beam cross-sections shown below, point O is the centroid, point B is the top-most point and point K is the bottom-most point on the cross-section. The beam is made of a material having a specific weight of ρ (N/m^3). The dimension b is in meters.

a) Determine the weight of the beam for each of the three cross-sections shown below. Leave these answers in terms of ρ and b. Rank order these weights from smallest to largest.

b) Determine the magnitude of the normal stress at points B and K for each cross-section. Leave these answers in terms of M and b. Rank order the maximum of these magnitudes for each point from smallest to largest.

c) Comment on the cost (weight) vs. maximum stress of these beam cross-sections.





beam cross section (1) **beam cross section (2)** **beam cross section (3)**

Beam 1

$W_1 = \text{weight of } 1 = \rho\left[\tfrac{1}{2}(b)(b)\right][10b] = 5\rho b^3$

$I_{O1} = \frac{1}{36}(b)(b^3) = \frac{b^4}{36}$

σ_{B1} = normal stress at B for 1

$$= -\frac{M\left(\frac{2b}{3}\right)}{I_{O1}} = -24\frac{M}{b^3}$$

σ_{K1} = normal stress at K for 1

$$= -\frac{M\left(-b/3\right)}{I_{O1}} = 12\frac{M}{b^3}$$

$$|\sigma_1|_{max} = 24\frac{M}{b^3}$$

Beam 2

$$W_2 = \rho[(b)(b)][10b] = 10\rho b^3$$

$$I_{02} = \frac{1}{12}(b)(b^3) \quad = \frac{1}{12}b^4$$

$$\left.\begin{aligned} \sigma_{B2} &= -\frac{M\left(\frac{b}{2}\right)}{I_{02}} = -6\frac{M}{b^3} \\ \sigma_{K2} &= -\frac{M\left(-\frac{b}{2}\right)}{I_{02}} = 6\frac{M}{b^3} \end{aligned}\right\} |\sigma_2|_{max} = 6\frac{M}{b^3}$$

Beam 3

$$W_3 = \rho\pi\left(\frac{b}{2}\right)^2[10b] = \frac{5}{2}\pi\rho b^3$$

$$I_{02} = \frac{\pi}{4}\left(\frac{b}{2}\right)^4 = \frac{\pi}{64}b^4$$

$$\left.\begin{aligned} \sigma_{B3} &= \frac{-M(b/2)}{I_{03}} = -\frac{32}{\pi}\frac{M}{b^3} \\ \sigma_{K3} &= -\frac{M(-b/2)}{I_{03}} = \frac{32}{\pi}\frac{M}{b^3} \end{aligned}\right\} |\sigma_3|_{max} = \frac{32}{\pi}\frac{M}{b^3}$$

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a) $W_2 > W_3 > W_1$

b) $|\sigma_1|_{max} > |\sigma_3|_{max} > |\sigma_2|_{max}$

c) The triangle has the smallest weight, but largest stress.
The rectangle has the largest weight, but smallest stress.
The circle has the intermediate weight and stress.