SOLUTION

ME 323: Mechanics of Materials — **Homework Set H12**

Summer 2025 — **Assigned/Due: July 1/July 7**

A shaft is made up of elements (1), (2) and (3). Elements (1) and (3) have solid circular cross-sections, and element (2) has a circular tubular cross-section. Elements (1) and (2) are made up of a material with a shear modulus of G, whereas the material of element (3) has a shear modulus of 3*G*. Torque *T* acts on rigid connector C.

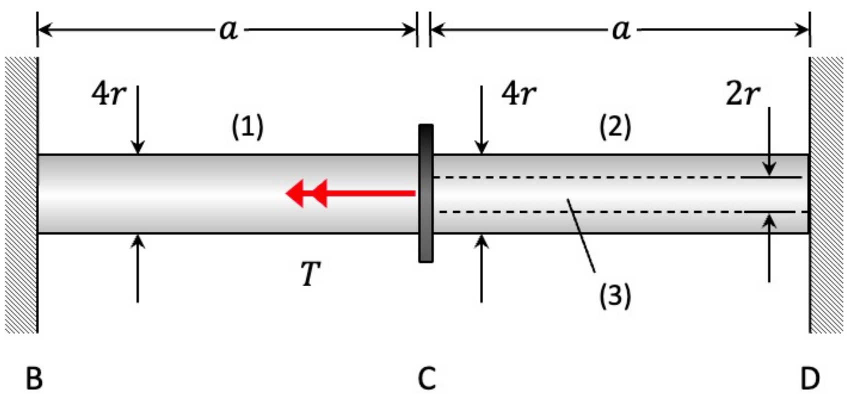
a) ***Equilibrium***. Draw free body diagrams (FBDs) of connector C. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate?

b) ***Torque/rotation equations***. Write down the torque/rotation equations for elements (1), (2) and (3).

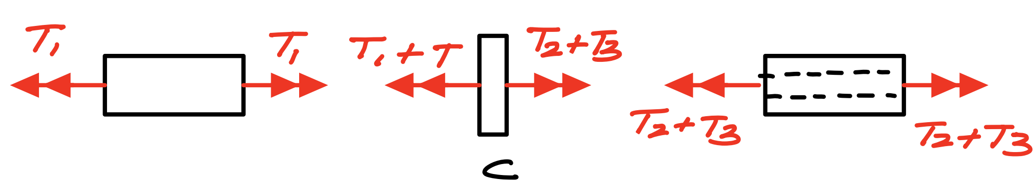
c) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the rotations of elements (1), (2) and (3).

d) ***Solution***. Solve your equations above for the torques carried by the three elements. Also, determine the maximum shear stresses in the shaft. At which location(s) does this maximum shear stress exists? Write your answers in terms of *T* and *r*.

Leave your answers in terms of the given parameters of, at most: *G*, *r* and *a*. Verify that your answers have appropriate units.



1. Equilibrium



(1) C: $\Sigma M = T_2 + T_3 - T_1 - T = 0$

1 eqn/3 unknowns $\Rightarrow$ INDETERMINATE

2. Torque/rotation

(2) $$\Delta\phi_1 = \frac{T_1 a}{G\frac{\pi}{2}(2r)^4} = \frac{1}{8\pi}\frac{T_1 a}{G r^4}$$

(3) $$\Delta\phi_2 = \frac{T_2 a}{G\left[\frac{\pi}{2}(2r)^4 - \frac{\pi}{2}r^4\right]} = \frac{2}{15\pi}\frac{T_2 a}{G r^4}$$

(4) $$\Delta\phi_3 = \frac{T_3 a}{3G\frac{\pi}{2}r^4} = \frac{2}{3\pi}\frac{T_3 a}{Gr^4}$$

3. Compatibility

$$\phi_c = \cancel{\phi_B}^{0} + \Delta\phi_1 = \Delta\phi_1$$

(5) $$\phi_D = \phi_c + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2 = 0$$

(6) $$\Delta\phi_2 = \Delta\phi_3$$

4. Solve

$$(2),(3),(5) \Rightarrow \frac{1}{8\cancel{\pi}}\frac{T_1 \cancel{a}}{\cancel{Gr^4}} + \frac{2}{15\cancel{\pi}}\frac{T_2\cancel{a}}{\cancel{Gr^4}} = 0$$

(7) $$\hookrightarrow T_1 = -\frac{16}{15}T_2$$

$$(6) \Rightarrow \frac{\cancel{2}}{15\pi}\frac{T_2\cancel{a}}{\cancel{Gr^4}} = \frac{\cancel{2}}{3\pi}\frac{T_3\cancel{a}}{\cancel{Gr^4}}$$

(8) $$\hookrightarrow T_3 = \frac{1}{5}T_2$$

$$(1),(7),(8) \Rightarrow -\left(-\frac{16}{15}T_2\right) + T_2 + \frac{1}{5}T_2 = T$$

$$\left.\begin{aligned} &\hookrightarrow T_2 = \frac{15}{34}T \\ &T_1 = -\frac{16}{15}\left(\frac{15}{34}T\right) = -\frac{16}{34}T \\ &T_3 = \frac{1}{5}\left(\frac{15}{34}T\right) = \frac{3}{34}T \end{aligned}\right\} \longleftarrow \text{torques in shaft elements}$$

Shear stress

$$\left.\begin{aligned} (\tau_1)_{max} &= \frac{T_1(\rho_1)_{max}}{I_{p1}} = \frac{\left(-\frac{16}{34}T\right)(2r)}{\frac{\pi}{2}(2r)^4} = -\frac{4}{34\pi}\frac{T}{r^3} \\ (\tau_2)_{max} &= \frac{T_2(\rho_2)_{max}}{I_{p2}} = \frac{\left(\frac{15}{34}\right)(2r)}{\frac{\pi}{2}(2r)^4 - \frac{\pi}{2}r^4} = \frac{4}{34\pi}\frac{T}{r^3} \\ (\tau_3)_{max} &= \frac{T_3(\rho_3)_{max}}{I_{p3}} = \frac{\left(\frac{3}{34}T\right)(r)}{\frac{\pi}{2}r^4} = \frac{6}{34\pi}\frac{T}{r^3} \end{aligned}\right\}$$

Units check:

$$\frac{T}{r^3} = \frac{N\cdot m}{m^3} = ^{\checkmark}\frac{N}{m^2}$$

$\therefore$ Max. torque exists on the outer surface of element 3.