SOLUTION

ME 323: Mechanics of Materials **Homework Set H11**

Summer 2025 **Assigned/Due: June 30/July 3**

A shaft system is made up of components (1), (2) and (3), with all three components having solid circular cross-sections and are made up of the same material having a shear modulus of G. Components (1) and (2) have outer diameters of $2d$, whereas component (3) has an outer diameter of d. Component (1) is attached to a fixed wall at end B. The three components are joined at the rigid connectors of C and D, as shown. Torques $3T$ and T are applied to the shaft system as shown.

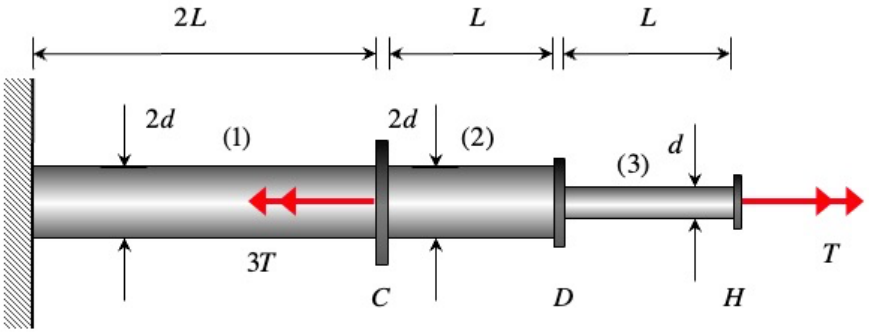
a) Draw free body diagrams (FBDs) of connectors C, D and H.

b) Determine the torque carried by each shaft component.

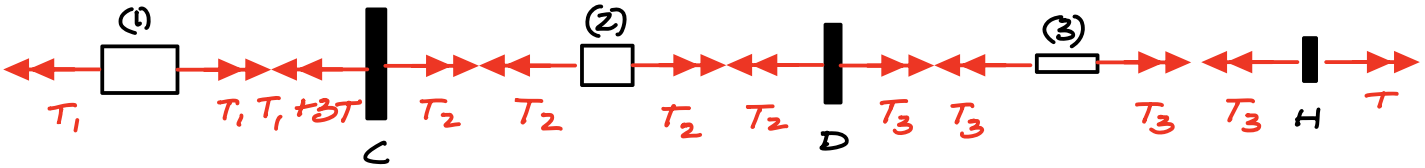
c) Determine the angles of rotation for connectors C, D and H. Include appropriate signs on your answers.

d) Determine the maximum shear stress in the shaft system. Determine the location(s) of this maximum shear stress in terms of position(s) along the length of the shaft and position(s) on the cross-section.

Leave your answers in terms of the given parameters of, at most: G, d and L. Verify that your answers have appropriate units.



1. Equilibrium



H: $\Sigma M = T - T_3 = 0$ $\Rightarrow T_3 = T$

D: $\Sigma M = T_3 - T_2 = 0$ $\Rightarrow T_2 = T_3 = T$

C: $\Sigma M = -T_1 - 3T + T_2 = 0 \Rightarrow T_1 = T_2 - 3T = -2T$

Determinate. Solved for torques T_1, T_2, T_3 using equilibrium.

2. Torque/rotation

$\Delta\phi_1 = \frac{T_1(2L)}{G I_{p1}}$; $I_{p1} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4$

$= -\frac{8}{\pi}\frac{TL}{Gd^4}$

$$\Delta\phi_2 = \frac{T_2 L}{G I_{p2}} \quad ; \; I_{p2} = I_{p1} = \frac{\pi}{2} d^4$$

$$= \frac{2}{\pi} \frac{TL}{Gd^4}$$

$$\Delta\phi_3 = \frac{T_3 L}{G I_{p3}} \quad ; \; I_{p3} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

$$= \frac{32}{\pi} \frac{TL}{Gd^4}$$

(c)

$$\phi_C = \cancel{\phi_B^{\,0}} + \Delta\phi_1 = -\frac{8}{\pi} \frac{TL}{Gd^4}$$

$$\phi_D = \phi_C + \Delta\phi_2 = -\frac{6}{\pi} \frac{TL}{Gd^4}$$

$$\phi_H = \phi_D + \Delta\phi_3 = \frac{26}{\pi} \frac{TL}{Gd^4}$$

$$\frac{TL}{Gd^4} = \frac{(N\cdot m)m}{\frac{N}{m^2} m^4} \overset{\checkmark}{=} \text{dimensionless} \quad \text{checks}$$

(d) $$|\tau_1|_{max} = \frac{|T_1|(2d/2)}{I_{p1}} = \frac{4}{\pi} \frac{T}{d^3}$$

$$|\tau_2|_{max} = \frac{|T_2|\left(\frac{2d}{2}\right)}{I_{p2}} = \frac{2}{\pi} \frac{T}{d^3}$$

$$|\tau_3|_{max} = \frac{|T_3|\left(\frac{d}{2}\right)}{I_{p3}} = \frac{16}{\pi} \frac{T}{d^3}$$

$|\tau|_{max} = |\tau_3|_{max} = \frac{16}{\pi}\frac{T}{d^3}$

$|\tau|_{max}$ for shaft occurs on outer surface of member 3.

$$\frac{T}{d^3} = \frac{N\cdot m}{m^3} \overset{\checkmark}{=} \frac{N}{m^2} \quad \text{checks}$$