ME 323: Mechanics of Materials — SOLUTION — **Homework Set H10**

Summer 2025 — **Assigned/Due: June 27/July 2**

A shaft system is made up of components (1) and (2), with each component having a solid circular cross-section. Shaft components (1) and (2) have outer diameters of $2d$ and d, respectively, with (1) and (2) connected by rigid meshing gears B and C (having diameters of $2D$ and D, respectively), as shown. Component (2) is attached to a fixed wall at end H. Torques $2T$, $2T$ and T are applied to gears A, B and C, respectively.

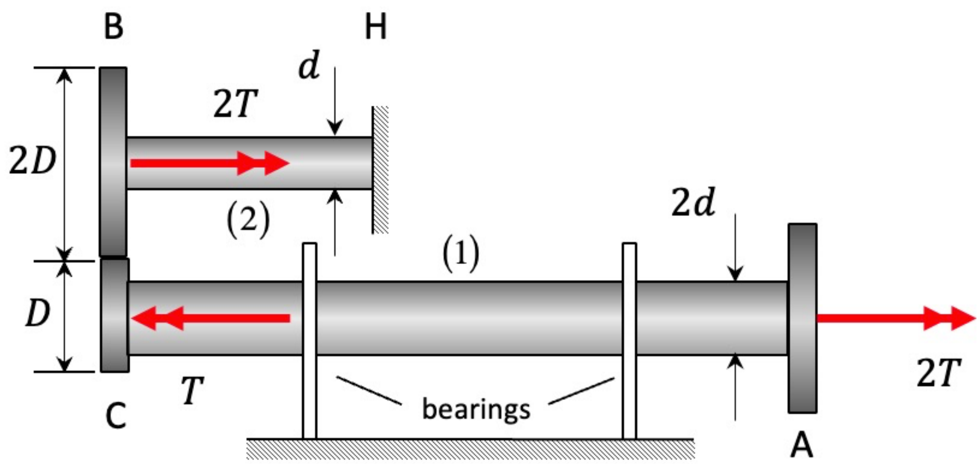
a) Draw free body diagrams of gears A, B and C.

b) Determine the torque carried by each shaft component.

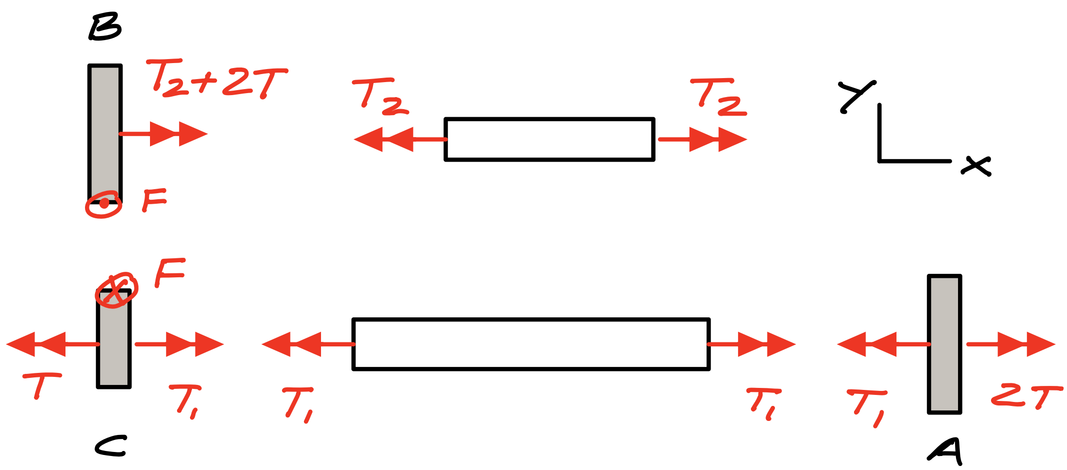
c) Determine the maximum shear stress in each shaft component.

d) Where (in which shaft component and at which location(s) on the shaft component cross-section) is the largest shear stress experienced in the shaft system?

Leave your answers in terms of, at most: T and d. Verify that your answers have appropriate units.



Equilibrium



(1) $A: \Sigma M = 2T - T_1 = 0 \Rightarrow T_1 = 2T$

T_1

(2) $B: \Sigma M = T_2 + 2T - FD = 0$

(3) $C: \Sigma M = T_1 - T + F\frac{D}{2} = 0$

(2) $\Rightarrow$ $F = \frac{1}{D}(T_2 + 2T)$

(3) $\Rightarrow$ $F = \frac{2}{D}(T - T_1)$

Equating:

$$\frac{1}{\cancel{D}}(T_2 + 2T) = \frac{2}{\cancel{D}}(T - T_1)$$

(4) $\hookrightarrow$ $T_2 = -2T_1 = -4T$ $\longleftarrow$ T_2

<u>Shear stress</u>

$$(\tau_1)_{max} = \frac{T_1 (\rho_1)_{max}}{I_{p1}} \quad ; \quad I_{p1} = \frac{\pi}{2}\left(\frac{2D}{2}\right)^4 = \frac{\pi}{2}D^4$$

$$= \frac{(2T)(2\cancel{D}/2)}{\frac{\pi}{2}D^{\cancel{4}\,3}} = \frac{4}{\pi}\,\frac{T}{D^3} \quad \left(= \frac{N\cdot m}{m^3} = \frac{N}{m^2}\right) \longleftarrow (\tau_1)_{max}$$

$$(\tau_2)_{max} = \frac{T_2 (\rho_2)_{max}}{I_{p2}} \quad ; \quad I_{p2} = \frac{\pi}{2}\left(\frac{D}{2}\right)^4 = \frac{\pi}{32}D^4$$

$$= -\frac{(4T)(\cancel{D}/2)}{\frac{\pi}{32}D^{\cancel{4}\,3}} = \frac{64}{\pi}\,\frac{T}{D^3} \quad \left(= \frac{N\cdot m}{m^3} = \frac{N}{m^2}\right) \longleftarrow (\tau_2)_{max}$$

Maximum shear stress occurs on outer surface of shaft component (2).