SOLUTION

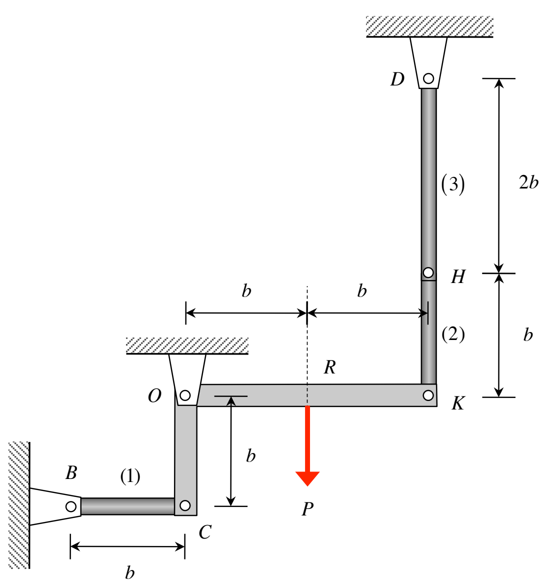
ME 323: Mechanics of Materials — ***Homework Set H09***

Summer 2025 — ***Assigned/Due: June 26/July 1***

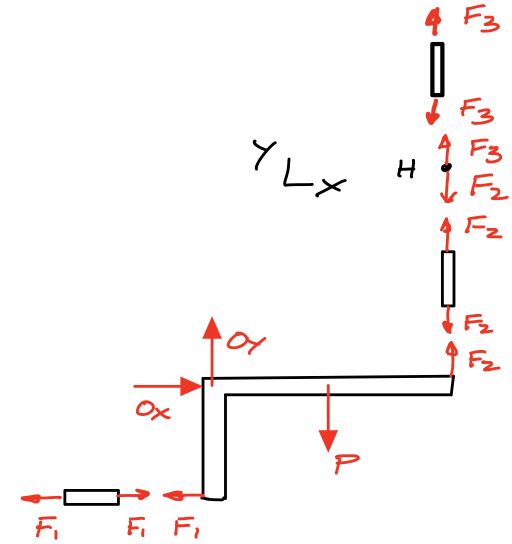
A structure is made up of a rigid member CK and three rod elements (1), (2) and (3). The cross-sectional area for each element is A. The material makeup of the three elements is such that the Young's moduli are related by $E_1 = E_3 = E$ and $E_2 = 2E$. The coefficient of thermal expansion for each member is α. A load P is applied to member CK as shown, with the temperature of elements (1) and (3) increased by ΔT. The temperature of element (2) remains unchanged. The load P is given by $P = 2\alpha\Delta TEA$.

1) ***Equilibrium***. Draw free body diagrams (FBDs) of member CK and joint H. Write down the appropriate equilibrium equations for member CK and joint H using your FBDs. Is this system determinate?
2) ***Force/elongation equations***. Write down the force/elongation equations for members (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the elongations of members (1), (2) and (3).
4) ***Solution***. Solve your equations above for the loads carried by the three members. From these, determine the axial stress in each member. State whether the *stress* in each member is tensile or compressive. State whether the *strain* in each member is tensile or compressive.

Leave your answers in terms of the given parameters of, at most: E, A, ΔT and b. Verify that your answers have appropriate units.



1) Equilibrium



(1) $\underline{COK}$: $\Sigma M_O = -F_1(\cancel{b}) - P(\cancel{b}) + F_2(2\cancel{b}) = 0$

$\hookrightarrow \quad F_1 = 2F_2 - P$

(2) $\underline{H}$: $\Sigma F_y = F_3 - F_2 = 0 \Rightarrow F_2 = F_3$

Two equations/three unknowns $\Longrightarrow$ INDETERMINATE

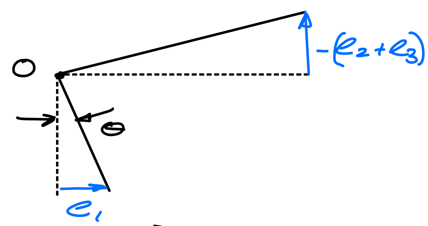
2) Force/elongation

(3) $$e_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha_1 \Delta T_1 L_1 \qquad = \frac{F_1 b}{EA} + \alpha \Delta T b$$

(4) $$e_2 = \frac{F_2 L_2}{E_2 A_2} + \alpha_2 \cancelto{0}{\Delta T_2} L_2 \qquad = \frac{F_2 b}{2EA}$$

(5) $$e_3 = \frac{F_3 L_3}{E_3 A_3} + \alpha_3 \Delta T_3 L_3 = \frac{2F_3 b}{EA} + \alpha \Delta T (2b)$$

3) Compatibility



$$\theta = \frac{e_1}{b} = \frac{-(e_2+e_3)}{2b}$$

(6) $\hookrightarrow \quad 2e_1 + e_2 + e_3 = 0$

4) Solve: 6 equations / 6 unknowns $(F_1, F_2, F_3, e_1, e_2, e_3)$

(3)–(6): $$2\left(\frac{F_1 \cancel{b}}{EA} + \alpha \Delta T \cancel{b}\right) + \frac{F_2 \cancel{b}}{2EA} + \frac{2F_3 \cancel{b}}{EA} + 2\alpha \Delta T \cancel{b}$$

(7) $\hookrightarrow \quad 4F_1 + F_2 + 4F_3 = -8\alpha \Delta T EA$

(1), (2): $4(2F_2 - P) + F_2 + 4F_2 = -8\alpha\Delta T EA$

$\hookrightarrow \quad (8+1+4)F_2 = -8\alpha\Delta TEA + 4(\alpha\Delta TEA) = 0$

$\hookrightarrow \quad F_2 = 0 = F_3 \longleftarrow F_2, F_3$

(1) $\Rightarrow \quad F_1 = 2\cancelto{0}{F_2} - P = -2\alpha\Delta TEA \longleftarrow F_1$

Stress	Strain
$\sigma_1 = \frac{F_1}{A} = -2\alpha\Delta TE$ (C)	$\varepsilon_1 = \frac{\sigma_1}{E} + \alpha\Delta T = -\alpha\Delta T$ (C)
$\sigma_2 = \frac{F_2}{A} = 0$ (Z)	$\varepsilon_2 = \frac{\sigma_2}{E} = 0$ (Z)
$\sigma_3 = \frac{F_3}{A} = 0$ (Z)	$\varepsilon_3 = \frac{\sigma_3}{E} + \alpha\Delta T = \alpha\Delta T$ (T)