SOLUTION

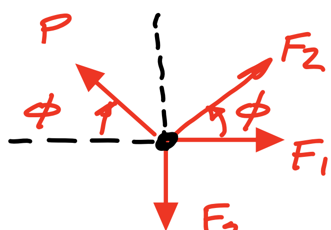
ME 323: Mechanics of Materials — **Homework Set H08**

Summer 2025 — **Assigned/Due: June 25/June 30**

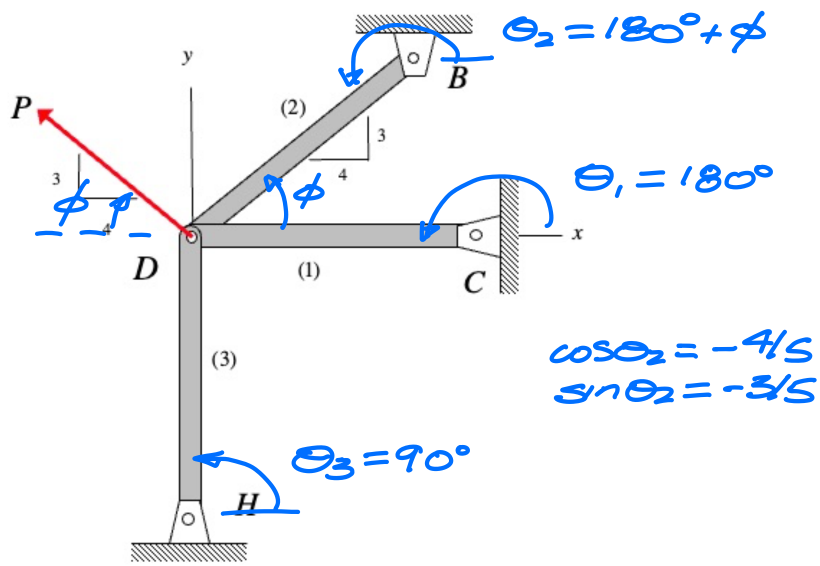
A truss is constructed using three identical members (each of length L, cross-sectional area A and made up of a material having a Young's modulus of E). A load P acts on joint D.

1) ***Equilibrium***. Draw a free body diagram (FBD) of Joint D. Write down the appropriate equilibrium equations for joint D using your FBD. Is this system determinate?
2) ***Force/elongation equations***. Write down the force/elongation equations for members (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the elongations of members (1), (2) and (3).
4) ***Solution***. Solve your equations above for the loads carried by the three members. From these, determine the axial stress in each member.

Leave your answers in terms of the given parameters of, at most: E, A, P and L. Verify that your answers have appropriate units.



$\cos\phi = 4/5$
$\sin\phi = 3/5$



1. Equilibrium

(1) D: $\Sigma F_x = F_1 + F_2\cos\phi - P\cos\phi = 0 \Rightarrow F_1 = (-4F_2 + 4P)\frac{1}{5}$

(2) $\Sigma F_y = F_2\sin\phi + P\sin\phi - F_3 = 0 \Rightarrow F_3 = (3F_2 + 3P)\frac{1}{5}$

2. Force/elongation

(3) $e_1 = \dfrac{F_1 L}{EA}$

(4) $e_2 = \dfrac{F_2 L}{EA}$

(5) $e_3 = \dfrac{F_3 L}{EA}$

3. Compatibility

(6) $e_1 = u_D \cancel{\cos\theta_1}^{-1} + v_D \cancel{\sin\theta_1}^{0} = -u_D$

(7) $e_2 = u_D \cos\theta_2 + v_D \sin\theta_2 = -\frac{4}{5}u_D - \frac{3}{5}v_D$

(8) $e_3 = u_D \cancel{\cos\theta_3}^{0} + v_D \cancel{\sin\theta_3}^{1} = v_D$

4. Solve: 8 equations / 8 unknowns

(6) $\Rightarrow u_D = -e_1$

(8) $\Rightarrow v_D = e_3$

(7) $\Rightarrow e_2 = -\frac{4}{5}(-e_1) - \frac{3}{5}e_3$

(9) $\hookrightarrow 4e_1 - 5e_2 - 3e_3 = 0$

(3),(4),(5),(9) $\Rightarrow 4\frac{F_1 \cancel{L}}{\cancel{EA}} - 5\frac{F_2 \cancel{L}}{\cancel{EA}} - 3\frac{F_3 \cancel{L}}{\cancel{EA}} = 0$

(10) $\hookrightarrow 4F_1 - 5F_2 - 3F_3 = 0$

(1),(2),(9) $\Rightarrow$

$$\frac{4}{5}(-4F_2 + 4P) - 5F_2 - \frac{3}{5}(3F_2 + 3P) \Rightarrow F_2 = \frac{7}{50}P$$

$\therefore$

$$F_1 = \frac{4}{5}(-F_2 + P) = \frac{86}{125}P$$

$$F_3 = \frac{3}{5}(F_2 + P) = \frac{171}{250}P$$

Stresses

$$\left.\begin{aligned} \sigma_1 &= \frac{F_1}{A} = \frac{86}{125}\frac{P}{A} \quad (T) \\ \sigma_2 &= \frac{F_2}{A} = \frac{7}{50}\frac{P}{A} \quad (T) \\ \sigma_3 &= \frac{F_3}{A} = \frac{171}{250}\frac{P}{A} \quad (T) \end{aligned}\right\}$$

units: $\frac{P}{A} = \frac{N}{m^2}$ ✓