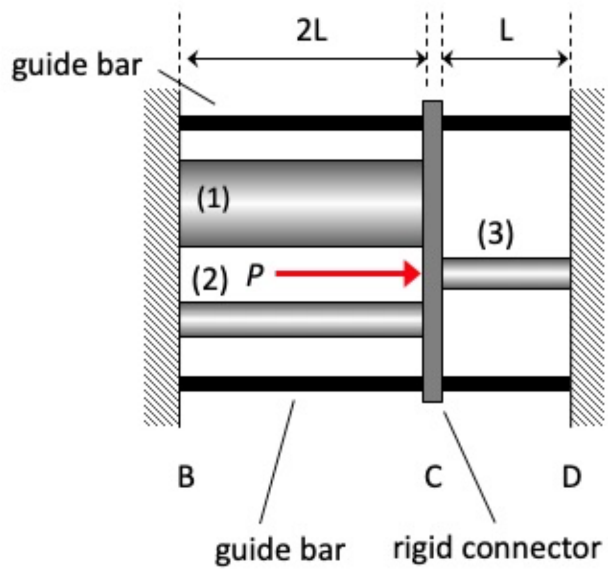
SOLUTION

ME 323: Mechanics of Materials | **Homework Set H07**

Summer 2025 | **Assigned/Due: June 24/June 27**

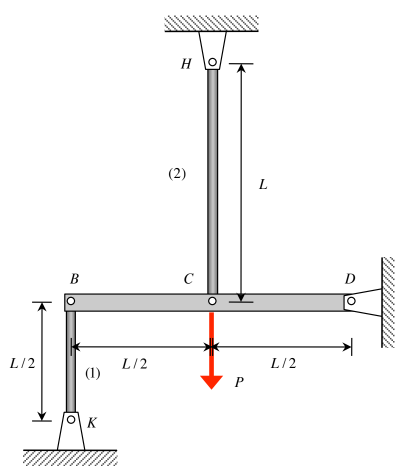
PART A (10 points): Circular cross-section rods (1), (2) and (3) are made of the same material having a Young's modulus of E, with outer diameters of $2d$, d and d, respectively, and with lengths $2L$, $2L$ and L, respectively. Connector C is able to slide freely along two parallel guide bars to maintain a vertical orientation for C. A horizontal load P is applied to C, as shown.



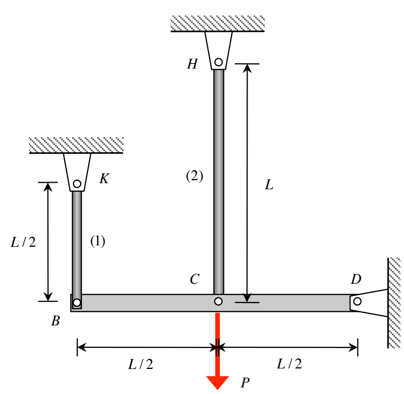
1) ***Equilibrium***. Draw a free body diagram (FBD) of connector C. Write down the appropriate equilibrium equation for connector C using your FBD. Is this system determinate?
2) ***Force/elongation equations***. Write down the force/elongation equations for rods (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the elongations of rods (1), (2) and (3).
4) ***Solution***. Solve your equations above for the loads carried by the three rods. From these, determine the axial stress in each rod.

Leave your answers in terms of the given parameters of, at most: E, d, P and L. Verify that your answers have appropriate units.

PART B (4 points): Consider the two structures below, (i) and (ii). In each case, let F_1 and F_2 represent the axial loads carried by members (1) and (2), with the sign conventions that $F_i > 0$ and $e_i > 0$ for the ith member being in tension. For each structure, write down the *compatibility equation* relating the elongations e_1 and e_2.



Structure (i)

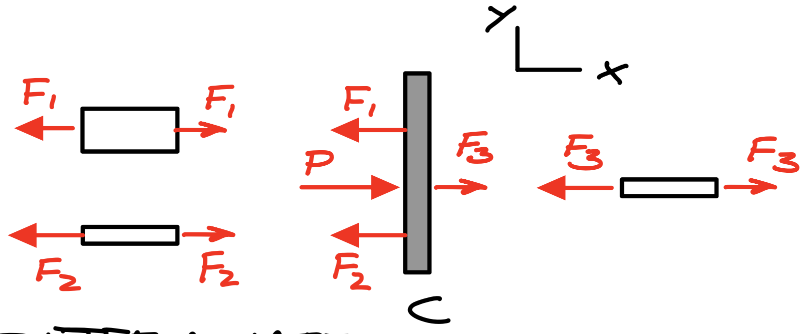


Structure (ii)

PART A

1. Equilibrium

For FBDs, assume all members in tension.



(1) C: $\sum F_x = -F_1 - F_2 + F_3 + P = 0$

1 eqn / 3 unknowns $\Rightarrow$ INDETERMINATE

2. Load / Elongation

(2) $e_1 = \dfrac{F_1(2L)}{E\pi(2d/2)^2} = \dfrac{2}{\pi}\dfrac{F_1 L}{Ed^2}$

(3) $e_2 = \dfrac{F_2(2L)}{E\pi(d/2)^2} = \dfrac{8}{\pi}\dfrac{F_2 L}{Ed^2}$

(4) $e_3 = \dfrac{F_3 L}{E\pi(d/2)^2} = \dfrac{4}{\pi}\dfrac{F_3 L}{Ed^2}$

3. Compatibility

$u_C = \cancel{u_B}^{0} + e_1$

(5) $u_D = u_C + e_3 = e_1 + e_3 = 0$

(6) Also: $e_1 = e_2$

4. Solve

- (2),(3),(6) $\Rightarrow \dfrac{2}{\cancel{\pi}}\dfrac{F_1\cancel{L}}{\cancel{Ed^2}} = \dfrac{8}{\cancel{\pi}}\dfrac{F_2\cancel{L}}{\cancel{Ed^2}} \Rightarrow$

(7) $F_1 = 4F_2$

- (2),(4),(5) $\Rightarrow \dfrac{2}{\cancel{\pi}}\dfrac{F_1\cancel{L}}{\cancel{Ed^2}} + \dfrac{4}{\cancel{\pi}}\dfrac{F_3\cancel{L}}{\cancel{Ed^2}} \Rightarrow$

(8) $\hookrightarrow F_3 = -\frac{1}{2}F_1 = -2F_2$

- (1),(7),(8) $\Rightarrow -4F_2 - F_2 - 2F_2 = -P$

$$\left.\begin{aligned} &\hookrightarrow F_2 = \tfrac{1}{7}P \\ &F_1 = 4F_2 = \tfrac{4}{7}P \\ &F_3 = -2F_2 = -\tfrac{2}{7}P \end{aligned}\right\}$$

member loads
units check: N

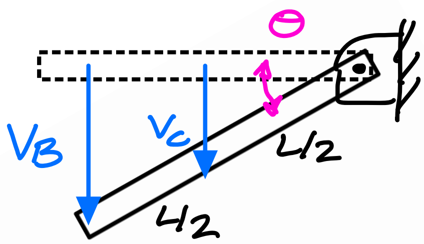
$\therefore$

$$\left.\begin{aligned} \sigma_1 &= \frac{F_1}{A_1} = \frac{\frac{4}{7}P}{\pi(2d/2)^2} = \frac{4}{7\pi}\frac{P}{d^2} \\ \sigma_2 &= \frac{F_2}{A_2} = \frac{\frac{1}{7}P}{\pi(d/2)^2} = \frac{4}{7\pi}\frac{P}{d^2} \\ \sigma_3 &= \frac{F_3}{A_3} = -\frac{\frac{2}{7}P}{\pi(d/2)^2} = -\frac{8}{7\pi}\frac{P}{d^2} \end{aligned}\right\}$$

Stresses
units check: $\dfrac{N}{m^2}$

PART B

Structure (i)

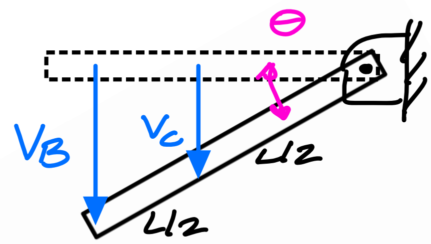


$$\begin{cases} V_B = -e_1 = L\sin\theta \\ V_C = e_2 = \frac{L}{2}\sin\theta \end{cases}$$

$\div \Rightarrow -\frac{e_1}{e_2} = 2$

$\hookrightarrow e_1 = -2e_2$

Structure (ii)



$$\begin{cases} V_B = e_1 = L\sin\theta \\ V_C = e_2 = \frac{L}{2}\sin\theta \end{cases}$$

$\div \Rightarrow \frac{e_1}{e_2} = 2$

$\hookrightarrow e_1 = 2e_2$