Solution

ME 323: Mechanics of Materials | ***Homework Set H04***

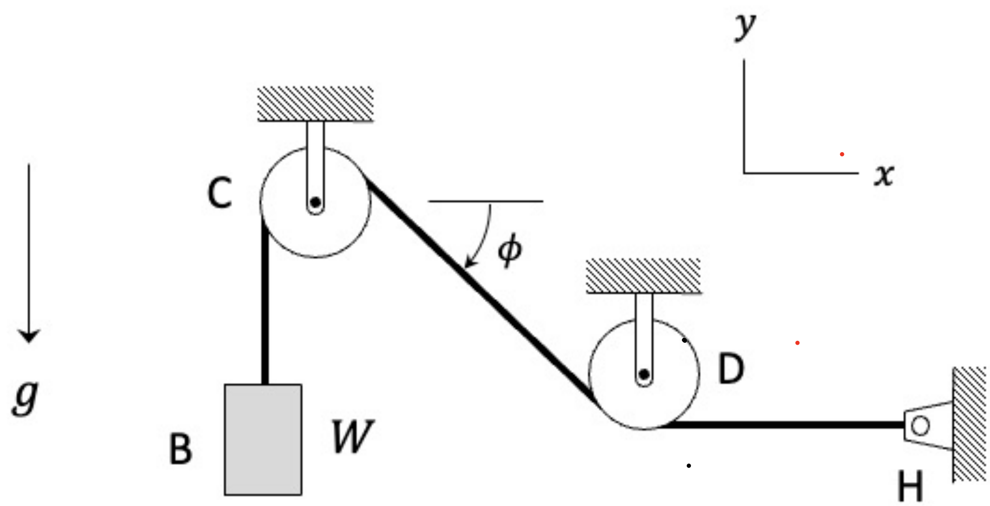
Summer 2025 | ***Assigned/Due: June 19/June 24***

The wire-pulley system shown supports block B of weight W. The diameters of the double-sided pulley pins C and D are d_C and d_D, respectively. The diameter of the wire is d_w. The pulley pins and wire are manufactured from the same grade of steel having a tensile yield strength of σ_Y and a shear strength in yielding of τ_Y. The minimum design factors of safety for the pins and the wire are FS_{pin}, and FS_{cable}, respectively. The weight of the wire can be neglected compared to that of the block. The pulleys are to be considered to be ideal.

a) Determine the diameters of the pulley pins that satisfy their design factor of safety.

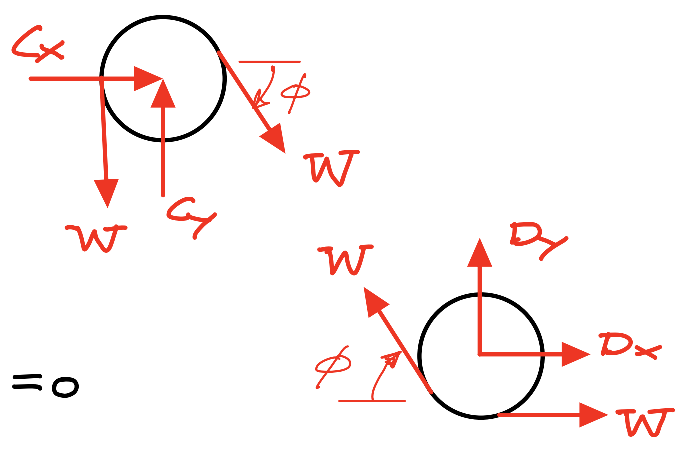
b) Determine the diameter of the wire that satisfies its design factor of safety.

Use the following parameter values in your analysis: $\phi = 36.87°$, $W = 2$ kN, $\sigma_Y = 220\ MPa$, $\tau_Y = 0.5\sigma_Y$, $FS_{pin} = 4$ and $FS_{wire} = 3$. Please substitute in these numerical values in the last step of your work.



FBDs of pulleys

Since pulleys are ideal, the tension in cable is a constant value of W throughout.



Equilibrium of pulleys

$$C: \quad \Sigma F_x = C_x + W\cos\phi = 0$$

$$\hookrightarrow C_x = -W\cos\phi$$

$$\Sigma F_y = -W + C_y - W\sin\phi = 0$$

$$\hookrightarrow C_y = W(1+\sin\phi)$$

$$\therefore |\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(W\cos\phi)^2 + W^2(1+\sin\phi)^2}$$

$$= W\sqrt{\cos^2\phi + 1 + 2\sin\phi + \sin^2\phi}$$

$$= \sqrt{2}\,W\sqrt{1+\sin\phi}$$

V_c = shear force in pin C $= \dfrac{|\vec{C}|}{2}$

τ_c = shear stress in pin C $= \dfrac{V_c}{A_c} = \dfrac{|\vec{C}|}{2\pi(d_c/2)^2} = \dfrac{2}{\pi}\dfrac{|\vec{C}|}{d_c^2}$

D: $\sum F_x = D_x - W\cos\phi + W = 0$

$\quad \hookrightarrow D_x = W(-1+\cos\phi)$

$\sum F_y = D_y + W\sin\phi = 0$

$\quad \hookrightarrow D_y = -W\sin\phi$

$$\therefore\ |\vec{D}| = \sqrt{D_x^2 + D_y^2} = \sqrt{W^2(-1+\cos\phi)^2 + (-W\sin\phi)^2}$$

$$= W\sqrt{1-2\cos\phi+\cos^2\phi+\sin^2\phi}$$

$$= \sqrt{2}\,W\sqrt{1-\cos\phi}$$

τ_D = shear stress in pin D $= \dfrac{2}{\pi}\dfrac{|\vec{D}|}{d_D^2}$

(a) For $\phi = 36.87°$

$|\vec{C}| = \sqrt{2}\,W\sqrt{1+0.6} = \sqrt{3.2}\,W$

$|\vec{D}| = \sqrt{2}\,W\sqrt{1-0.8} = \sqrt{0.8}\,W$

$$\therefore\ \begin{cases} \tau_c = \dfrac{|\vec{C}|/2}{\pi(d_c/2)^2} = \dfrac{2|\vec{C}|}{\pi d_c^2} \\[2ex] \tau_D = \dfrac{|\vec{D}|/2}{\pi(d_D/2)^2} = \dfrac{2|\vec{D}|}{\pi d_c^2} \end{cases}$$

and $\bullet\ FS_c = \dfrac{\tau_Y}{\tau_c} = \dfrac{\pi d_c^2}{2|\vec{C}|}\tau_Y$

$\quad \hookrightarrow d_c = \sqrt{\dfrac{2FS_c|\vec{C}|}{\pi\tau_Y}} = \sqrt{\dfrac{(2)(FS_c)\sqrt{3.2}\,W}{\pi\tau_Y}}$ $\longleftarrow$ d_c

$\bullet\ FS_D = \dfrac{\tau_Y}{\tau_D} = \dfrac{\pi d_D^2}{2|\vec{D}|}\tau_Y$

$\quad \hookrightarrow d_D = \sqrt{\dfrac{(2)(FS_D)\sqrt{0.8}\,W}{\pi\tau_Y}}$ $\longleftarrow$ d_D

b) $\sigma_W = \dfrac{W}{A_W} = \dfrac{W}{\pi(d_W/2)^2} = \dfrac{4W}{\pi d_W^2}$

$\Rightarrow FS_W = \dfrac{\sigma_Y}{\sigma_W} = \dfrac{\pi d_W^2}{4W}\sigma_Y \Rightarrow d_W = \sqrt{\dfrac{4W(FS_W)}{\pi\sigma_Y}}$ $\longleftarrow$ d_W