ME 323 – Mechanics of Materials
Final Exam
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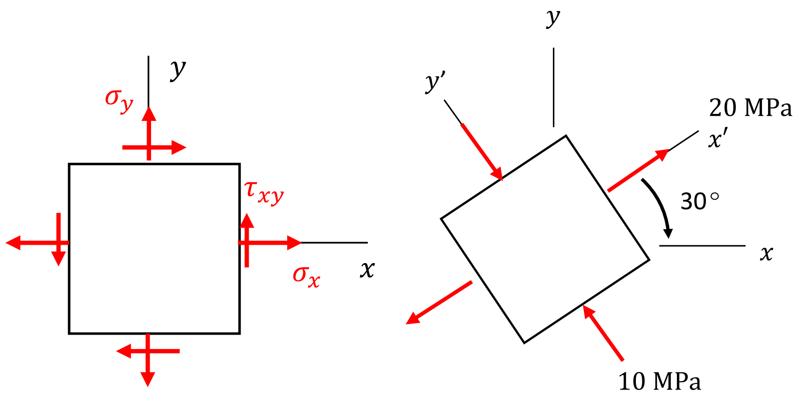
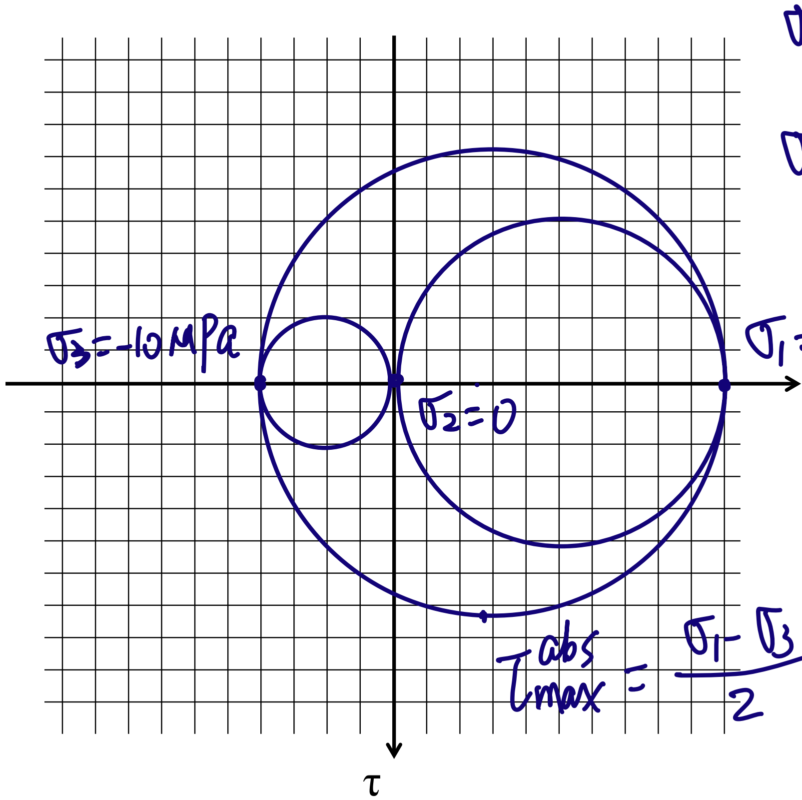
PROBLEM #1 (25 Points): A material point is subjected to the 2D stress state $(\sigma_x, \sigma_y, \tau_{xy})$, and its stresses along the x' and y' directions are shown below.



a) Draw the 3D Mohr's circle in the following diagram. Clearly label the three principal stresses and the absolute maximum shear stress.

$\sigma_{p1} = 20\ MPa$

$\sigma_{p2} = -10\ MPa$



$$\tau_{max}^{abs} = \frac{\sigma_1 - \sigma_3}{2} = 15\ MPa$$

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b) Determine the values of $(\sigma_x, \sigma_y, \tau_{xy})$.

$\sigma_x' = 20$ MPa, $\sigma_y' = -10$ MPa, $\tau_{x'y'} = 0$

$$\sigma_x = \frac{\sigma_x' + \sigma_y'}{2} + \left(\frac{\sigma_x' - \sigma_y'}{2}\right) \cos(-60^\circ)$$

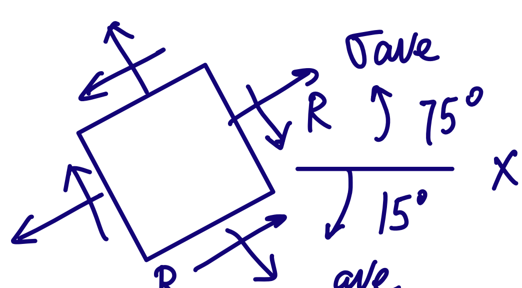
$$= \frac{20-10}{2} + \frac{20+10}{2} \cdot \frac{1}{2} = 12.5 \text{ MPa}$$

$$\tau_{xy} = -\frac{\sigma_x' - \sigma_y'}{2} \sin(-60^\circ) = \frac{20+10}{2} \cdot \frac{\sqrt{3}}{2} = 12.99 \text{ MPa}$$

$$\sigma_y = \frac{\sigma_x' + \sigma_y'}{2} + \frac{\sigma_x' - \sigma_y'}{2} \cos 120^\circ = 5 + 15 \cdot \left(-\frac{1}{2}\right) = -2.5 \text{ MPa}$$

c) Draw the maximum in-plane shear stress element. Show the stress components and its orientation relative to the x-axis.

$\tau_{max}^{in\text{-}plane} = R = 15$ MPa, $\sigma_{ave} = \dfrac{\sigma_x' + \sigma_y'}{2} = 5$ MPa



d) If the normal strains in the x and x' directions are to be measured, what should their values be? Assuming Young's modulus $E = 10$ GPa and Poisson's ratio $v = 0.3$.

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x-direction: $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$

$= \frac{1}{10\times10^3}(125 + 0.3\times25)$

$= 0.1325\%$

x'-direction: $\varepsilon_{x'} = \frac{1}{E}(\sigma_{x'} - \nu\sigma_{y'})$

$= \frac{1}{10\times10^3}(20 + 0.3\times10)$

$= 0.23\%$

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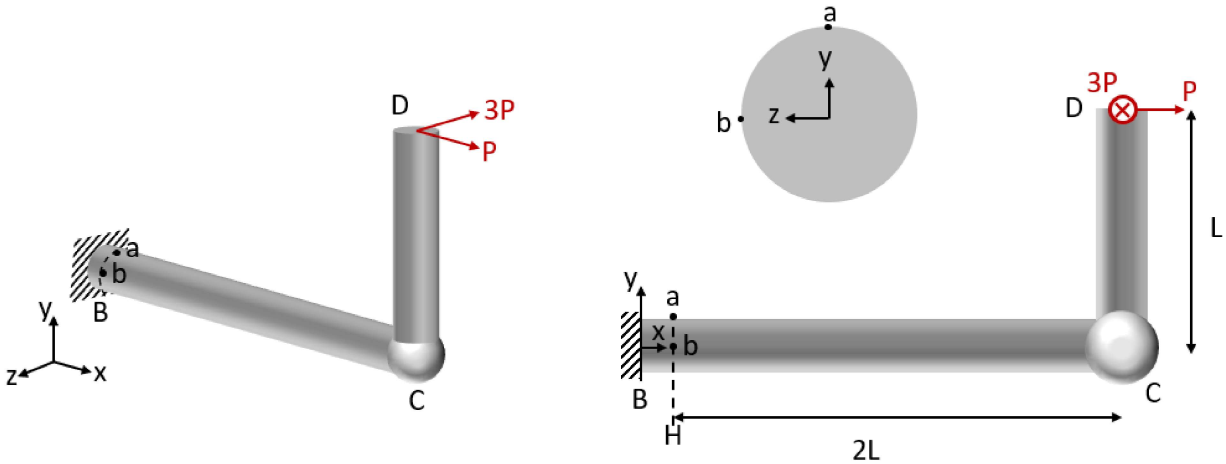
PROBLEM #2 (25 Points): BCD is an "L-shaped" structure that has a circular cross-section with a diameter of d. The structure is connected to ground at B. At D, there are applied forces of P in the x-direction and -3P in the z-direction. The beam is made of a material with a Young's modulus of E and a Poisson's ratio of 0.4. $L = 10d$.



(a) Determine the stresses at points a and b, which are located on cross-section H. a is at the most positive y location of the cross-section and b is at the most positive z location of the cross-section. Indicate the magnitude and sign of the stresses in the table below.

Loading	Stress at a	Stress at b
F_x	$\sigma_x = \frac{P}{\pi\left(\frac{d}{2}\right)^2} = \frac{4P}{\pi d^2}$	$\sigma_x = \frac{4P}{\pi d^2}$
F_z	$\tau_{xz} = -\frac{4V}{3A} = -\frac{16P}{\pi d^2}$	0
M_x	$\tau_{xz} = -\frac{TR}{I_p} = -\frac{48PL}{\pi d^3}$	$\tau_{xy} = \frac{48PL}{\pi d^2} = \frac{480P}{\pi d^2}$
M_y	$\sigma_x = 0$	$\sigma_x = \frac{M_y\left(\frac{d}{2}\right)}{I} = \frac{192PL}{\pi d^3} = \frac{1920P}{\pi d^2}$
M_z	$\sigma_x = \frac{M_z\left(\frac{d}{2}\right)}{I} = \frac{32PL}{\pi d^3}$	0

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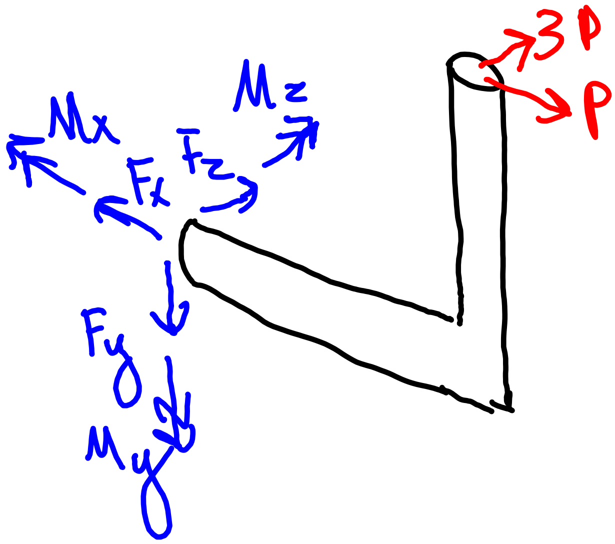


$$\vec{F} = -F_x\hat{i} - F_y\hat{j} - F_z\hat{k} + P\hat{i} - 3P\hat{k} = 0$$

$$F_x = P$$

$$F_y = 0$$

$$F_z = -3P$$

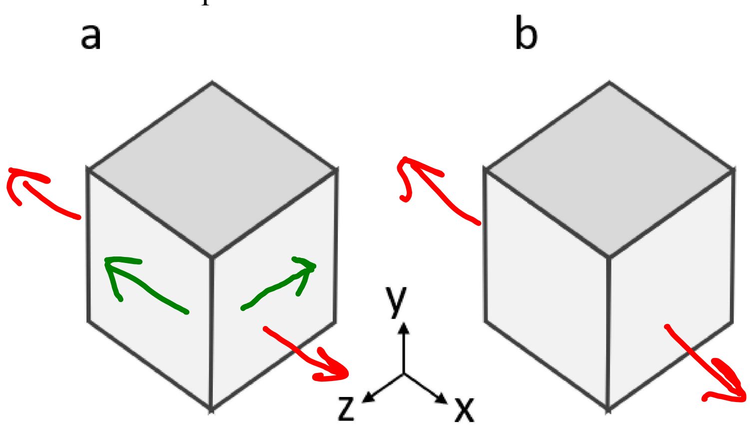
$$\Sigma M = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} + \vec{r}\times\vec{F} = 0$$

$$\vec{r} = (2L, L, 0) \qquad \vec{F} = (P, 0, -3P)$$

$$M_x = -3P$$

$$M_y = 6P \qquad M_z = -P$$

(b) Draw the stress elements for point a and b below.



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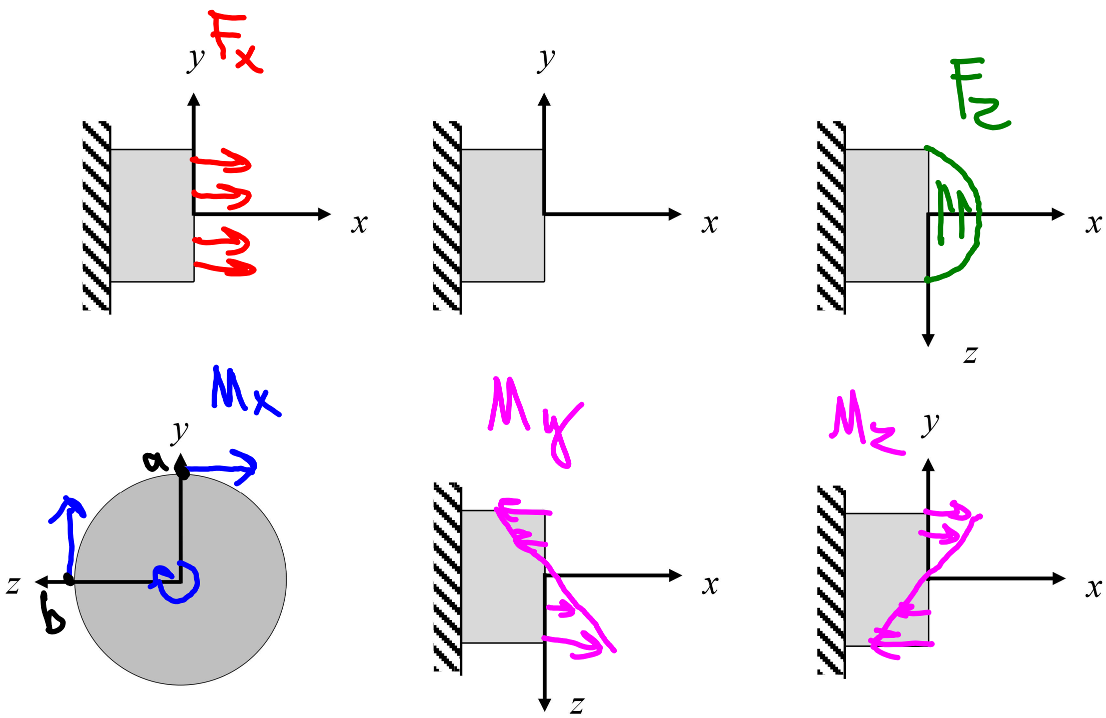
(c) Show the distribution of stresses on the cross-section H due to each loading in part (a). If non-zero, indicate their maximum and minimum values



(d) Determine the axial strains ($\epsilon_x, \epsilon_y, \epsilon_z$) for the material **point at a** in terms of P, d, and E.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\cancelto{0}{\sigma_y} + \cancelto{0}{\sigma_z}\right)\right]$$

$$\varepsilon_x = \frac{1}{E}\left(\frac{4P}{\pi d^2} + \frac{320P}{\pi d^2}\right) = \frac{324P}{E\pi d^2}$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.4\left[\frac{324P}{E\pi d^2}\right] = -\frac{129.6P}{E\pi d^2}$$

$$\varepsilon_z = \varepsilon_y$$

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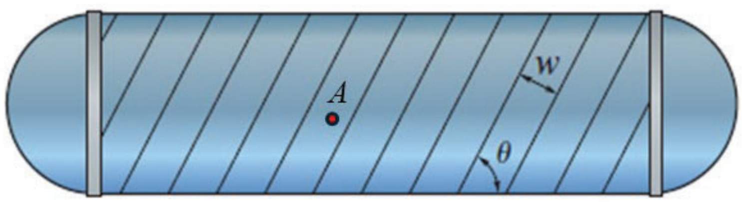
PROBLEM #3 (25 Points): The cylindrical tank is fabricated by welding a strip of thin plate helically, making an angle $\boldsymbol{\theta}$ with the longitudinal axis of the tank. The strip has a width $\boldsymbol{w}$ and thickness $\boldsymbol{t}$ and is made from a ductile aluminum alloy. However, welds between plates are known to be brittle. The gas within the tank of diameter **d** is pressured to **P**. The yield strength of the aluminum alloy used in the construction of the tank is **145 MPa**, and the tensile strength parallel and perpendicular to the welds is **100 MPa**. The following parameters were used in the design of the tank:
$\boldsymbol{\theta = 35^\circ, w = 0.1\,m,\ t = 0.03\,m, d = 0.75\,m, P = 10\,MPa.}$

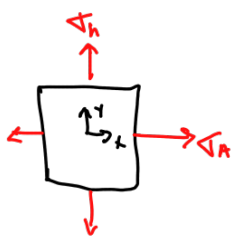


a) Determine the hoop and axial stresses at point A.

$$\sigma_h = \frac{Pr}{t} = \frac{(10\ \text{MPa})(0.375\text{m})}{0.03\text{ m}} = 125\ \text{MPa} \quad (2\ \text{Pts})$$

$$\sigma_A = \frac{Pr}{2t} = \frac{(10\ \text{MPa})(0.375\text{m})}{0.06\text{ m}} = 62.5\ \text{MPa} \quad (2\ \text{Pts})$$

b) Determine the three principal stresses and absolute maximum shear stress at point A.



$$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + 62.5}{2} = 93.75\ \text{MPa}$$

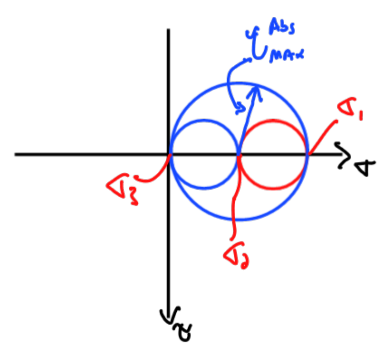
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 31.25\ \text{MPa}$$

$$\sigma_{P_1} = \sigma_{Avg} + R = 125\ \text{MPa} \quad (2\ \text{Pts})$$

$$\sigma_{P_2} = \sigma_{Avg} - R = 62.5\ \text{MPa} \quad (2\ \text{Pts})$$

$$\sigma_3 = 0 \quad (2\ \text{Pts})$$

$$\tau_{max}^{Abs} = \frac{\sigma_1 - \sigma_3}{2} = 62.5\ \text{MPa} \quad (2\ \text{Pts})$$



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c) Determine if the tank fails within the strips of ductile material using the **Maximum Distortional Energy** failure theory.

$$\sigma_M = \sqrt{\sigma_{p_1}^2 - \sigma_{p_1}\sigma_{p_2} + \sigma_{p_2}^2}$$

$$= \sqrt{(125\ \text{MPa})^2 - (125\ \text{MPa})(62.5\ \text{MPa}) + (62.5\ \text{MPa})^2}$$

$$\sigma_M = 108.25\ \text{MPa}$$

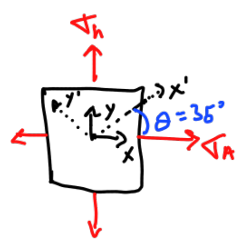
$$\sigma_M = 108.25\ \text{MPa} < \sigma_y = 145\ \text{MPa}$$

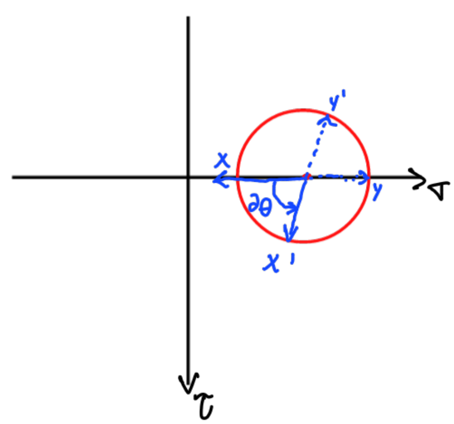
(4 pts)

The tank does not fail!

d) Consider the welds can fail only under normal stresses acting either parallel or perpendicular to the weld bead. Determine whether any of the welds under consideration fail in either of these directions.

$\sigma_{\parallel}$ = stress parallel to weld bead





$$\sigma_{\parallel} = \sigma_{avg} - R\cos(2\theta)$$

$$\sigma_{\parallel} = 93.75\ \text{MPa} - 31.25\cos(70)$$

$$\sigma_{\parallel} = 83.06\ \text{MPa}$$

$\sigma_{\parallel} = 83.06\ \text{MPa} < \sigma_U = 100\ \text{MPa}$

∴ Does not fail parallel to weld bead!

(3 pts)

$\sigma_{\perp}$: stress perpendicular to weld bead

$$\sigma_{\perp} = \sigma_{avg} + R\cos(2\theta)$$

$$= 93.75\ \text{MPa} + 31.25\cos(70)$$

$$\sigma_{\perp} = 104.44\ \text{MPa}$$

$\sigma_{\perp} = 104.44\ \text{MPa} > \sigma_U = 100\ \text{MPa}$

∴ Fails perpendicular to weld bead!

(3 pts)

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e) The aluminum tank is heat treated after manufacturing to increase the tensile strength at the welds from 100 MPa to **120 MPa**. Determine the minimum angle θ at which the strips should be welded to avoid failure. Note: Consider the welds can fail only under normal stresses acting either parallel or perpendicular to the weld bead.

Parallel to weld bead

$$\sigma_{\parallel} = \sigma_{Avg} - R\cos(2\theta)$$

$$\sigma_U < \sigma_{Avg} - R\cos(2\theta)$$

$$\frac{\sigma_{Avg} - \sigma_U}{R} > \cos(2\theta)$$

$$\cos^{-1}\left(\frac{\sigma_{Avg} - \sigma_U}{R}\right) > 2\theta$$

$$\cos^{-1}\left(\frac{93.75\text{ MPa} - 120\text{ MPa}}{31.25\text{ MPa}}\right) > 2\theta$$

$$\theta_{\parallel} < 73.57^\circ$$

Perpendicular to weld bead

$$\sigma_{\perp} = \sigma_{Avg} + R\cos(2\theta)$$

$$\sigma_U < \sigma_{Avg} + R\cos(2\theta)$$

$$\cos^{-1}\left(\frac{\sigma_U - \sigma_{Avg}}{R}\right) < 2\theta$$

$$\cos^{-1}\left(\frac{120 - 93.75}{31.25}\right) < 2\theta$$

$$\theta > 16.43^\circ$$

To prevent failure parallel or perpendicular to bead welds:

$$73.57^\circ > \theta > 16.43^\circ$$

$$\boxed{\theta_{min} = 16.43^\circ}$$

(3 pts)

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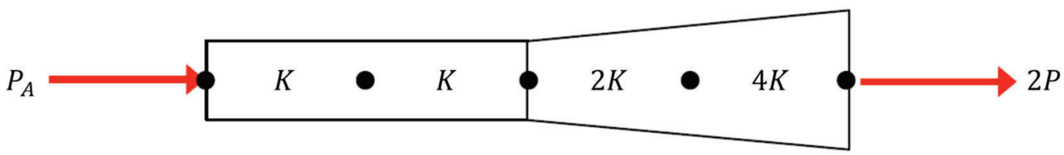
PROBLEM #4.1 (5 Points):

A rod with a non-uniform cross section is subjected to a load $2P$ on one end. The rod is fixed on the opposite end with an effective force P_A. Four finite elements of equal length were used to discretize the rod. The stiffness of each finite element is indicated in the figure below. Note: The finite elements and nodes are numbered starting from the fixed end on the left.



4.1-A.
Provide the entries missing in the stiffness matrix $[K]$:

$$[K] = \begin{bmatrix} K & -K & 0 & 0 & 0 \\ -K & 2K & -K & 0 & 0 \\ 0 & -K & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} \\ 0 & 0 & -2K & 6K & -4K \\ 0 & 0 & 0 & -4K & 4K \end{bmatrix}$$

A) $3K$
B) $-2K$
C) 0

4.1-B.
Complete the load vector $\{F\}$:

$$\{F\} = \begin{Bmatrix} \boldsymbol{D} \\ 0 \\ 0 \\ \boldsymbol{E} \\ 2P \end{Bmatrix}$$

D) P_A
E) 0

4.1-C.
Enforce the fixed boundary condition by striking through the corresponding rows and columns.

$$\begin{bmatrix} \cancel{K} & \cancel{-K} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{-K} & 2K & -K & 0 & 0 \\ \cancel{0} & -K & A & B & C \\ \cancel{0} & 0 & -2K & 6K & -4K \\ \cancel{0} & 0 & 0 & -4K & 4K \end{bmatrix} \begin{Bmatrix} \cancel{u_1} \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} \cancel{D} \\ 0 \\ 0 \\ E \\ 2P \end{Bmatrix}$$

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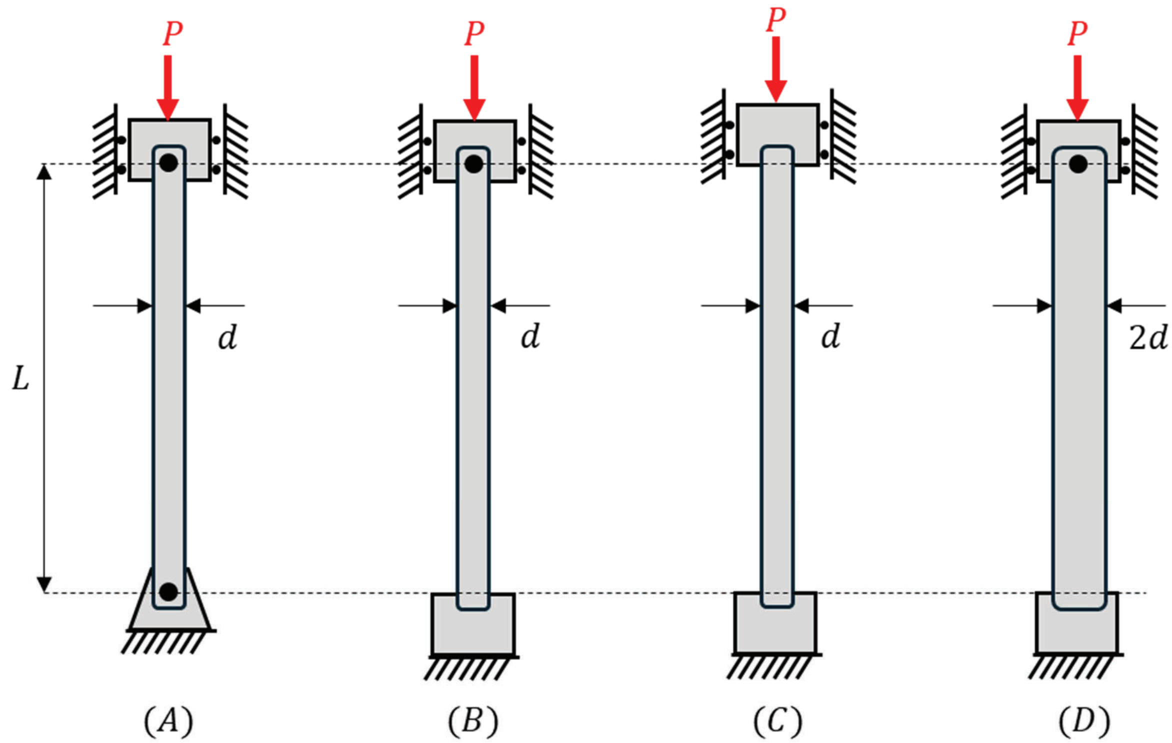
PROBLEM #4.2 (5 Points):

Consider the four columns of length L shown below. Each column has a circular cross-section and is made of a material with a Young Modulus E. Let P_{cr} represent the largest load that can be applied to the column without buckling.



4.2-A.
Which column has the largest P_{cr}?

(A) (B) (C) (D)

A) $P_{cr} = \frac{\pi^2 E I}{L} = \frac{\pi^3 E d^4}{64 L}$

B) $P_{cr} = \frac{\pi^2 E I}{0.7 L} = \frac{\pi^3 E d^4}{44.8 L}$

C) $P_{cr} = \frac{\pi^2 E I}{0.5 L} = \frac{\pi^3 E d^4}{32 L}$

D) $P_{cr} = \frac{\pi^2 E I}{0.7 L} = \frac{\pi^3 E d^4}{2.8 L}$

$P_{cr}^{D} > P_{cr}^{C} > P_{cr}^{B} > P_{cr}^{A}$

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4.2-B.
Order the columns by the largest P_{cr} that can carry?

1) $C > D > B > A$
2) $D > C > B > A$
3) $A > B > C > D$
4) $D > B > C > A$

4.2-C.
If column D is made hollow with an inner diameter d and outer diameter $2d$, how would P_{cr} change?

1) P_{cr} increases by more than 10%
2) P_{cr} decreases by more than 10%
3) P_{cr} decreases by less than 10%
4) P_{cr} increases by less than 5%

$$\% \text{ Change} = \frac{P_{cr}^{h} - P_{cr}^{s}}{P_{cr}^{s}} \cdot 100 = \frac{\frac{\cancel{\pi^2 E} I^h}{\cancel{0.7L}} - \frac{\cancel{\pi^2 E} I^s}{\cancel{0.7L}}}{\frac{\cancel{\pi^2 E} I^s}{\cancel{0.7L}}} \cdot 100$$

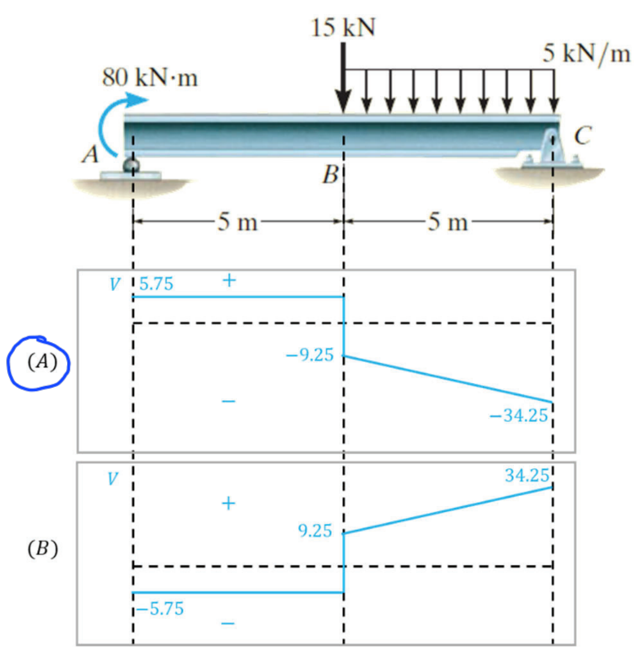
$$= \frac{I^h - I^s}{I^s} \cdot 100 = \frac{\cancel{\frac{\pi}{4}}\left(d^4 - \frac{d^4}{16}\right) - \cancel{\frac{\pi}{4}} d^4}{\cancel{\frac{\pi}{4}} d^4}$$

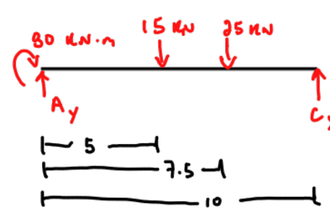
$$= \left(\frac{15}{16} - 1\right) \cdot 100 = \boxed{-6.25\ \%}$$

PROBLEM #4.3 (4 Points):

Consider a simply supported beam subjected to the loads and bending moments shown in the figure below.

4.3-A.
Select the shear force diagram resulting from the loads and bending moments acting on the simply supported beam.





$\Sigma F_y = 0 = A_y + C_y - 15\text{ kN} - 25\text{ kN}$

$A_y + C_y = 40\text{ kN}$

$\Sigma M_A \circlearrowleft + = 0 = -80\text{ kN·m} - (15\text{ kN})(5) - (25\text{ kN})(7.5) + 10\, C_y$

$\boxed{C_y = 34.25}$

$\boxed{A_y = 40\text{ kN} - 34.25 = 5.75\text{ kN}}$

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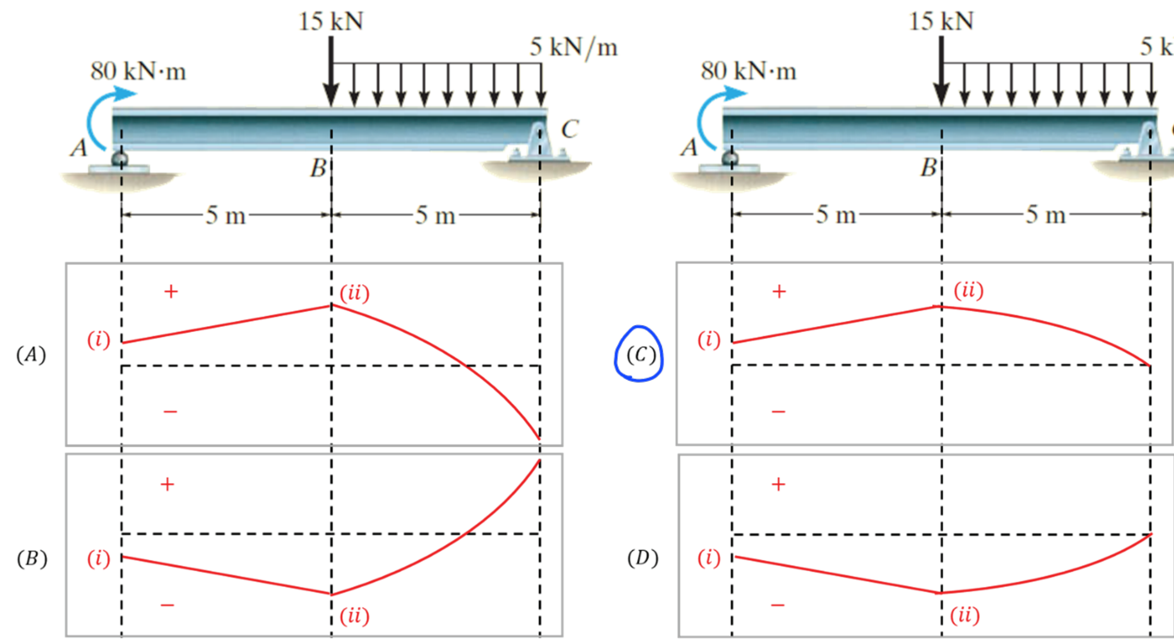
4.3-B.

Select the bending moment diagram resulting from the loads and bending moments acting on the simply supported beam.



4.3-C.

Indicate the magnitude of the bending moment at locations i and ii shown in the figure of question 4.3-B.

$|M_i|$ = 80 $(kN \cdot m)$
$|M_{ii}|$ = 108.75 $(kN \cdot m)$

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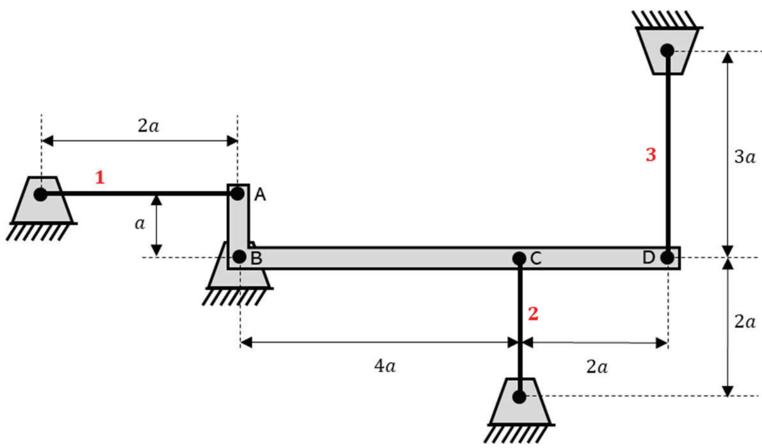
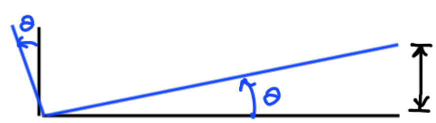
PROBLEM #4.4 (5 Points):

The structure shown below is made from a rigid element $ABCD$ connected with pins to three elastic members 1, 2, and 3. The temperature of element 3 is decreased by ΔT, which has a positive coefficient of thermal expansion α. Consider strains ϵ_1, ϵ_2 and ϵ_3, and stresses σ_1, σ_2 and σ_3 in members 1, 2 and 3. Circle the correct responses below:

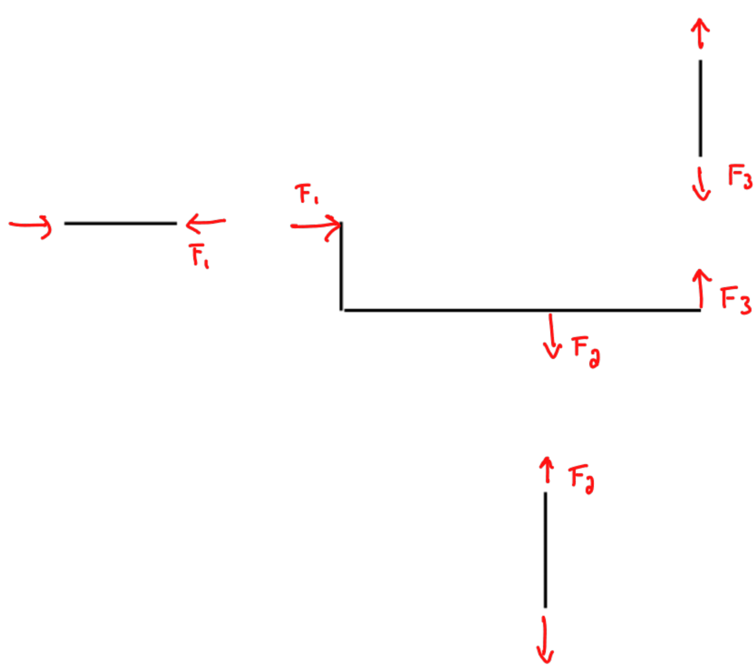


4.4-A.
A. $\epsilon_1 > 0$
B. $\epsilon_1 < 0$
C. $\epsilon_1 = 0$



$\delta_3^T = |3a\,\alpha\,\Delta T|$

4.4-B.
A. $\epsilon_2 > 0$
B. $\epsilon_2 < 0$
C. $\epsilon_2 = 0$



4.4-C.
A. $\sigma_1 > 0$
B. $\sigma_1 < 0$
C. $\sigma_1 = 0$

4.4-D.
A. $\sigma_2 > 0$
B. $\sigma_2 < 0$
C. $\sigma_2 = 0$

4.4-E.
A. $\sigma_3 > 0$
B. $\sigma_3 < 0$
C. $\sigma_3 = 0$

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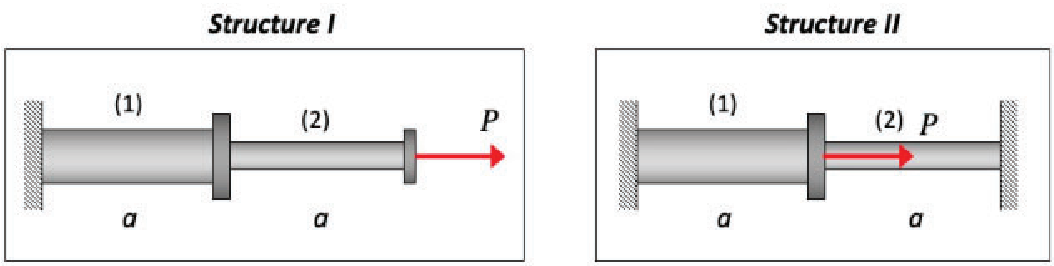
PROBLEM #4.5 (6 Points):

Consider the two structures shown below. Structure I consists of two elements of length L, each made of a material with an elastic modulus E. Structure II consists of the same two elements, made of the same material. Let F_1 and F_2 be the internal forces of elements 1 and 2, respectively. Circle the correct responses below.



4.5-A.
Suppose the material used in element 2 in Structure I is replaced by a material with an elastic modulus of $\mathbf{3E}$. As a result of this change:

A. The value of $|F_2|$ is increased
B. The value of $|F_2|$ is decreased
C. The value of $|F_2|$ is unchanged
D. More information is needed to answer this question.

4.5-B.
Suppose the length of element 1 in Structure I is increased to $\mathbf{3L}$. As a result of this change:

A. The value of $|F_1|$ is increased
B. The value of $|F_1|$ is decreased
C. The value of $|F_1|$ is unchanged
D. More information is needed to answer this question.

4.5-C.
Suppose the length of element 1 in Structure II is increased to $\mathbf{3L}$. As a result of this change:

A. The value of $|F_1|$ is increased
B. The value of $|F_1|$ is decreased
C. The value of $|F_1|$ is unchanged
D. More information is needed to answer this question.

$F_1 \leftarrow | \rightarrow P + F_2$

$P + F_2 - F_1 = 0$

$F_1 = P + F_2$

$K_1^A U_1 = P + K_2^A U_1$

$K_1^A = \frac{E_1 A_1}{L_1} > \frac{E_1 A_1}{3L_1}$

$F_1 = K_1^A U_1 > \frac{K_1^A}{3} U_1$