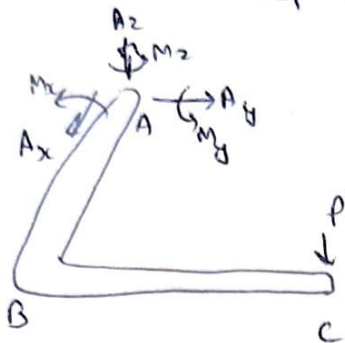


Quiz 7

FBD:



$$\sum F_z = 0 \Rightarrow A_z = P$$

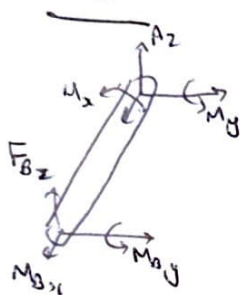
$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y = 0$$

$$\sum M_A = 0 \Rightarrow M_x \hat{i} + M_y \hat{j} + M_z \hat{k} + (L\hat{i} + L\hat{j}) \times (-P\hat{k}) = 0$$

$$M_x = PL, \quad M_y = -PL, \quad M_z = 0$$

Section AB



$$\sum F_z = 0 \Rightarrow F_{Bz} = -A_z = -P$$

$$\sum M_h = 0 \Rightarrow (M_x + M_{Bx})\hat{i} + (M_y + M_{By})\hat{j} + x\hat{i} \times (F_{Bz}\hat{k})$$

$$M_{Bx} = M_x = -PL$$

$$M_{By} = -Px - M_y = -Px + PL$$

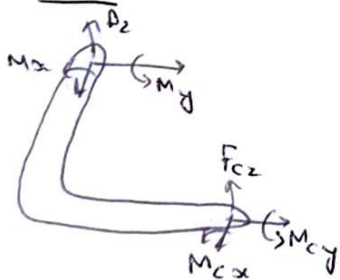
$$U_1 = \frac{1}{2} \int_0^L \frac{(-PL)^2 dx}{G I_p} + \frac{1}{2} \int_0^L \frac{P^2 (L-x)^2 dx}{EI} + \frac{1}{2} \int_0^L \frac{F_s P^2 dx}{GA}$$

$\uparrow$
 M_{Bx} is torque

$\uparrow$
 M_{By} is Bending Moment

$\uparrow$
 F_{Bz} is Shear force

Section BC



$$\sum F_z = 0 \Rightarrow F_{Cz} = -A_z = -P$$

$$\sum M_A = 0 \Rightarrow (M_x + M_{Cx})\hat{i} + (M_y + M_{Cy})\hat{j} + (L\hat{i} + y\hat{j}) \times (-P\hat{k}) = 0$$

$$\Rightarrow M_{Cx} = Py - PL, \quad M_{Cy} = PL - PL = 0$$

$$U_2 = \frac{1}{2} \int_0^L \frac{P^2 (y-L)^2 dy}{EI} + \frac{1}{2} \int_0^L \frac{F_s P^2 dy}{GA}$$

$$U_{\text{System}} = U_1 + U_2$$

$$= \frac{P^2 L^3}{2 G I_p} + \frac{P^2 L^3}{6 EI} + \frac{F_s P^2 L}{2 GA} + \frac{F_s P^2 L}{2 GA} + \frac{P^2 L^3}{6 EI}$$

$$= \left[\frac{P^2 L^3}{2 G I_p} + \frac{P^2 L^3}{3 EI} + \frac{F_s P^2 L}{GA} \right]$$