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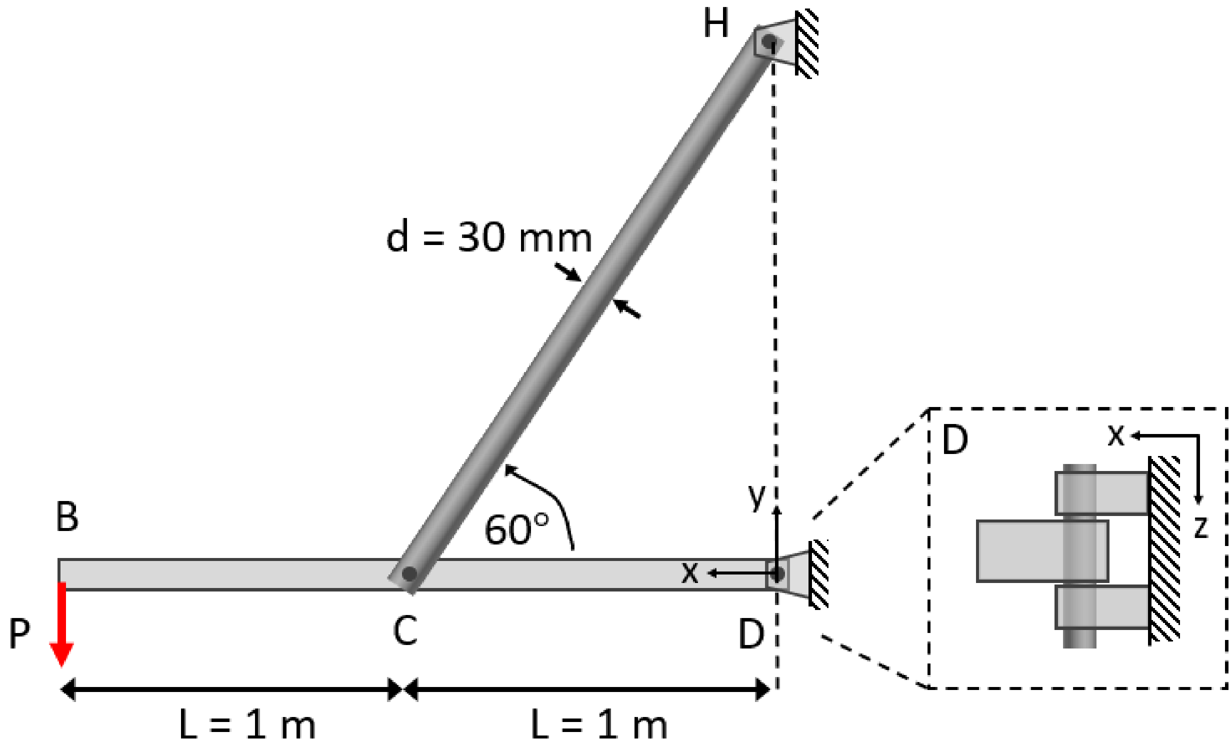
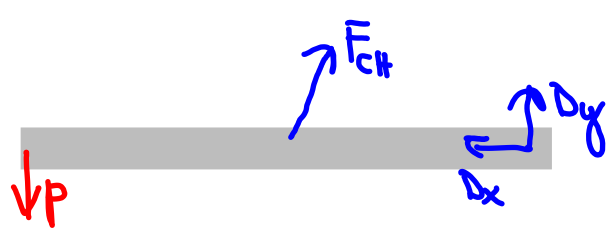
PROBLEM #1 (25 Points): BCD is a rigid beam that is connected to ground with a double pin at D and is connected to a deformable member CH at C. CH has a circular cross-section. Member CH has a **Young's modulus of 84 GPa** and a **shear modulus of 30 GPa**. A force of P = 10 kN is applied in the downward direction at B. The length L is 1 m and the diameter of CH is 30 mm in the unstressed state.



(a) Draw the free body diagram and write the equilibrium equations.



$$(\Sigma M)_D = P(2L) - F_{CH}\sin(60°)(L) = 0 \Rightarrow F_{CH} = \frac{2P}{\sin 60°} = \frac{4P}{\sqrt{3}}$$

$$\Sigma F_x = D_x - F_{CH}\cos(60°) = 0 \Rightarrow D_x = \frac{4P}{\sqrt{3}}\left(\frac{1}{2}\right) = \frac{2P}{\sqrt{3}}$$

$$\Sigma F_y = D_y - P + F_{CH}\sin(60°) = 0 \qquad D_y = P - \frac{2P}{\sin 60°}\sin 60° = -P$$

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(b) Member CH has a yield stress of 98 MPa ($\sigma_Y = 98\ MPa$) and a diameter of 30 mm (0.03 m). What is the factor of safety when P = 10 kN?

$$\sigma_{CH} = \frac{F_{CH}}{A} = \frac{4(10\ 000\ N)}{\sqrt{3}\,\pi(0.015)^2} = 32.67\ MPa$$

$$FS = \frac{\sigma_Y}{\sigma_{CH}} = \frac{98\ MPa}{32.67\ MPa} = 3.0$$

(c) The pin at D has an ultimate shear stress of 30 MPa ($\tau_U = 30\ MPa$). Determine the minimum diameter of pin to achieve a factor of safety of 4.0 when P = 10 kN.

$$|D| = \sqrt{\left(\frac{2P}{\sqrt{3}}\right)^2 + P^2} = 1.527P = 15\ 270\ N. \qquad \tau_D = \frac{|D|}{2A}$$

$$FS = \frac{\tau_U}{\tau_{allow}} = 4 = \frac{\tau_U}{\frac{|D|}{2\pi r^2}} \qquad r^2 = \frac{4|D|}{2\pi(\tau_U)} = \frac{4(15\ 270\ N)}{2\pi(30\times10^6\ Pa)}$$

$$r = 0.018\ m$$

$$d = 0.036\ m = 36\ mm.$$

(d) If the value of P is increased until the system failed, which would fail first: member CH or the pin at D?

Member CH

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(e) Find the change in diameter that occurs as P is increased from 0 to 10 kN. (Remember that the starting diameter of CH is 30 mm with no stress).

$$e = \frac{F_{CH} L_{CH}}{E_{CH} A_{CH}}$$

$$\varepsilon_{axial} = \frac{e}{L_{CH}} = \frac{\left(\frac{4}{\sqrt{3}}\right) 10000}{(84 \times 10^9)(\pi (0.015)^2)}$$

$$\varepsilon_{axial} = 3.89 \times 10^{-4}$$

$$2(1+\nu)G = E$$

$$\nu = \frac{E}{2G} - 1$$

$$\nu = \frac{84}{2(30)} - 1 = 0.4$$

$$\varepsilon_{radial} = -\nu \, \varepsilon_{axial}$$

$$\varepsilon_{radial} = -0.4(3.89 \times 10^{-4})$$

$$\frac{\Delta d}{d} = -0.4(3.89 \times 10^{-4})$$

$$\Delta d = -4.67 \times 10^{-6} \text{ m}$$

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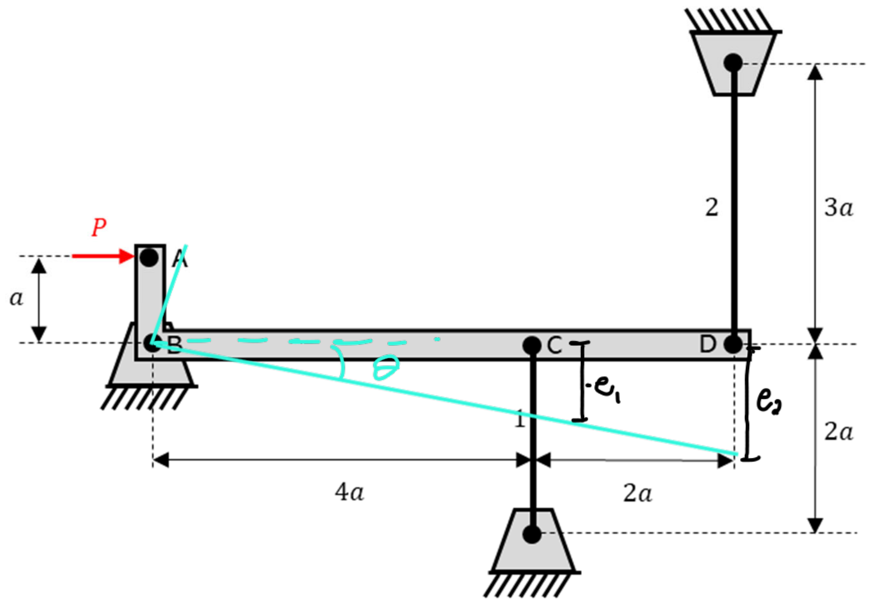
PROBLEM #2 (25 Points): The structure shown below is initially at the ambient temperature T_{RT}. The rigid element AD is subjected to a load P at point A, connected with a rigid pin at point B, and connected with cylindrical rods 1 and 2 at points C and D, respectively. The rods are made from low carbon steel with an elastic modulus E. The area of rod 1 is $2A$ and the area rod 2 is A.

For the following questions, provide your symbolic answers in terms of $a, E, P, A, \Delta T$.



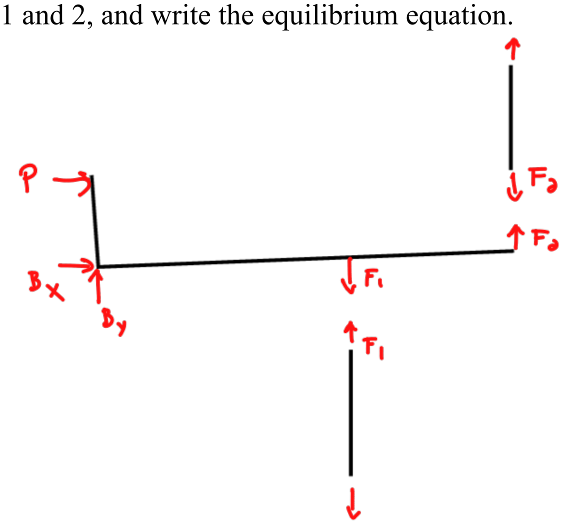
a) Draw the free body diagram of the rigid element AD, rods 1 and 2, and write the equilibrium equation.

$\Sigma F_y : 0 = B_y - F_1 + F_2$

$\Sigma F_x : 0 = B_x + P = 0 \rightarrow B_x = -P$

$\Sigma M_B \ (\circlearrowleft +) = 0 = -4a F_1 + 6a F_2 - Pa$

$F_1 = \frac{6\cancel{a} F_2 - P\cancel{a}}{4\cancel{a}} = \frac{3}{2} F_2 - \frac{P}{4}$



b) Is the system statically determinate or indeterminate?

3 unknowns and 2 Equations → Indeterminate.

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c) Determine the stress in rods 1 and 2. Indicate if stresses are tensile or compressive.

$$\tan(\theta) \approx \theta = \frac{-e_1}{4a} = \frac{e_2}{6a}$$

$$e_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{\cancel{2}F_1 a}{2AE}$$

$$e_2 = \frac{F_2 L_2}{A_2 E_2} = \frac{3F_2 a}{AE}$$

$$\frac{-F_1 \cancel{a}}{4\cancel{a}\cancel{AE}} = \frac{3F_2 \cancel{a}}{6\cancel{a}\cancel{AE}}$$

$$F_1 = -2F_2$$

$$\frac{3}{2}F_2 - \frac{P}{4} = -2F_2$$

$$\boxed{F_2 = \frac{P}{14}}$$

$$\boxed{F_1 = -2F_2 = -\frac{P}{7}}$$

$$\boxed{\sigma_1 = \frac{F_1}{2A} = \frac{-P}{14A} \text{ (compression)}}$$

$$\boxed{\sigma_2 = \frac{F_2}{A} = \frac{P}{14A} \text{ (Tensile)}}$$

d) Determine the vertical displacement at point D.

$$\boxed{e_2 = \frac{3F_2 a}{AE} = \frac{3Pa}{14AE}}$$

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e) If rod 2 is cooled by a $\Delta T < 0$ while the rest of the structure remains at the ambient temperature T_{RT}, what would be the stress in rods 1 and 2?

$$\tan(\theta) \approx \theta = \frac{-e_1}{4a} = \frac{e_2}{6a}$$

$$e_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{\cancel{2} F_1 a}{\cancel{2} A E}$$

$$e_2 = \frac{F_2 L_2}{A_2 E_2} - L_2 \alpha \Delta T = \frac{3 F_2 a}{A E} - 3a \alpha \Delta T$$

$$-\frac{F_1 \cancel{a}}{4\cancel{a} A E} = \frac{3 F_2 \cancel{a}}{6 \cancel{a} A E} - \frac{3\cancel{a} \alpha \Delta T}{6\cancel{a}}$$

$$-\frac{F_1}{4} = \frac{F_2}{2} - \frac{A E \alpha \Delta T}{2}$$

$$F_1 = 2 A E \alpha \Delta T - 2 F_2$$

$$\frac{3}{2} F_2 - \frac{P}{4} = 2 A E \alpha \Delta T - 2 F_2$$

$$\frac{7}{2} F_2 = 2 A E \alpha \Delta T + \frac{P}{4}$$

$$\boxed{F_2 = \frac{4}{7} A E \alpha \Delta T + \frac{P}{14}}$$

$$F_1 = 2 A E \alpha \Delta T - \frac{8}{7} A E \alpha \Delta T - \frac{P}{7}$$

$$\boxed{F_1 = \frac{6}{7} A E \alpha \Delta T - \frac{P}{7}}$$

$$\boxed{\sigma_1 = \frac{F_1}{2A} = \frac{3}{7} E \alpha \Delta T - \frac{P}{14A}}$$ Compressive if $\frac{P}{A} > 6 E \alpha \Delta T$

$$\boxed{\sigma_2 = \frac{F_2}{A} = \frac{4}{7} E \alpha \Delta T + \frac{P}{14A}}$$ Tensile.

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PROBLEM #3 (25 Points): Shafts (1), (2) and (3) in Fig. 3(a) are connected by a rigid connector at B, and are fixed to the rigid walls at the ends A and C. Shafts (1) and (2) have length $2L$, and shaft (3) has length L. The shear moduli of shafts (1), (2), and (3) are $G_1=15G$, $G_2=G$ and $G_3=8G$, respectively. Shaft (1) has a solid cross section of diameter $2d$, shaft (2) has a hollow cross section of outer diameter $4d$ and inner diameter $2d$, and shaft (3) has a hollow section of outer diameter $2d$ and inner diameter d, as shown in Figs. 3(b) and 3(c). An external torque T_B is applied at the connect B.
For the following questions, express your symbolic results in terms of T_B, d, L, G, and π.

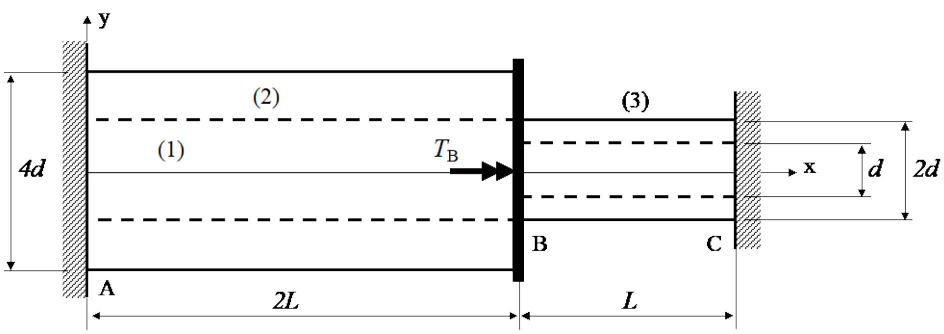


Fig. 3(a)

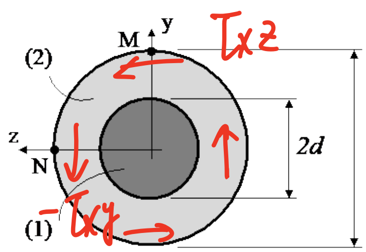


Fig. 3(b)

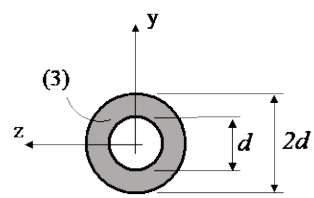
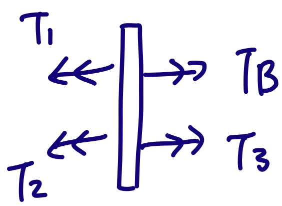


Fig. 3(c)

a) Draw the free body diagram of the rigid connector B and write its equilibrium equation.

FBD:



Equilibrium: $T_1 + T_2 = T_B + T_3$ (1)

b) Is the system statically determinate or indeterminate?

statically indeterminate

c) Determine the torque carried in each shaft.

Compatibility Conditions: $\phi_A = 0$, $\phi_C = 0$

$$\phi_1 = \phi_2 = \phi_B - \phi_A = \phi_B$$

$$\phi_3 = \phi_C - \phi_B = -\phi_B$$

$$\phi_1 = -\phi_3$$

$$\Rightarrow \quad \frac{T_1 L_1}{G_1 I_{p1}} = \frac{T_2 L_2}{G_2 I_{p2}} \quad (2)$$

$$\frac{T_1 L_1}{G_1 I_{p1}} = -\frac{T_3 L_3}{G_3 I_{p3}} \quad (3)$$

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$G_1 = 15G, \; L_1 = 2L, \; I_{p1} = \frac{\pi d^4}{2}$

$G_2 = G, \; L_2 = 2L, \; I_{p2} = \frac{\pi((2d)^4 - d^4)}{2} = \frac{15\pi d^4}{2}$

$G_3 = 8G, \; L_3 = L, \; I_{p3} = \frac{\pi}{2}\left[d^4 - \left(\frac{d}{2}\right)^4\right] = \frac{15\pi d^4}{32}$

substitude into ②: $T_1 = T_2$

substitude into ③: $T_1 = -T_3$

plug into ①: $T_1 = T_2 = \frac{1}{3}T_B, \; T_3 = -\frac{1}{3}T_B$

d) Determine the change in angle of connector B.

$$\phi_B = \phi_1 = \frac{T_1 L_1}{G_1 I_{p1}} = \frac{\frac{1}{3}T_B \cdot 2L}{15G \cdot \frac{\pi d^4}{2}} = \frac{4T_B L}{45\pi G d^4}$$

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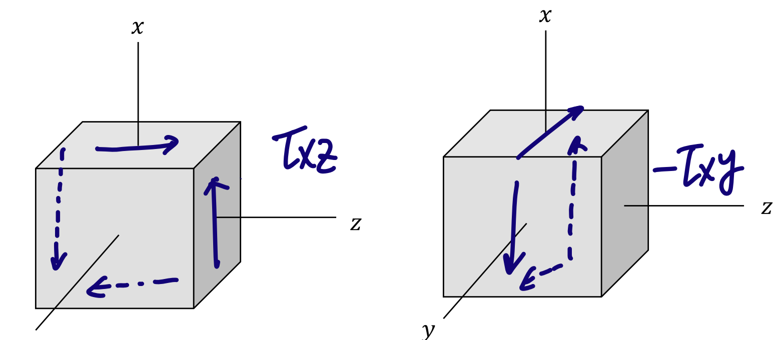
e) Consider the points "M" and "N" on the outer radius of shaft (2). Draw the stress elements to represent the stress states at "M" and "N".



Stress Element M

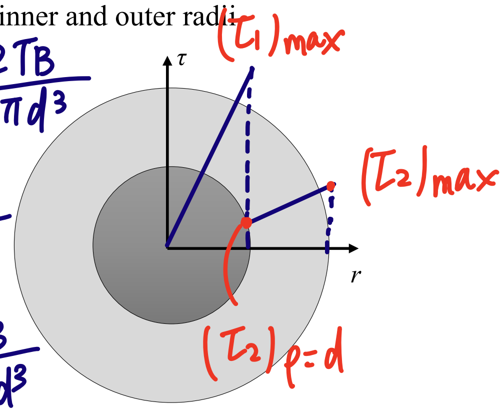
Stress Element N

f) Draw the magnitude of shear stress along the radial direction in the cross sections of the shafts. Mark their values at the inner and outer radii.

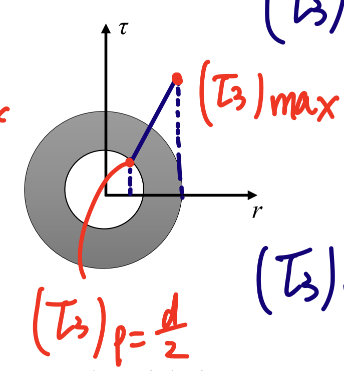
$$(\tau_1)_{max} = \frac{T_1 d}{I_{p1}} = \frac{2T_B}{3\pi d^3}$$

$$(\tau_2)_{\rho=d} = \frac{T_2 \cdot d}{I_{p2}} = \frac{2T_B}{45\pi d^3}$$

$$(\tau_2)_{max} = \frac{T_2 \cdot 2d}{I_{p2}} = \frac{4T_B}{45\pi d^3}$$



Cross section of shafts 1&2



Cross section of shaft 3

$$(\tau_3)_{\rho=\frac{d}{2}} = \frac{T_3 \cdot \frac{d}{2}}{I_{p3}} = -\frac{16T_B}{45\pi d^3}$$

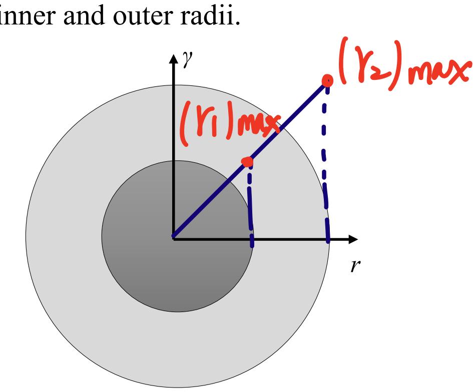
$$(\tau_3)_{max} = \frac{T_3 \cdot d}{I_{p3}} = -\frac{32T_B}{45\pi d^3}$$

g) Draw the magnitude of shear strain along the radial direction in the cross sections of the shafts. Mark their values at the inner and outer radii.

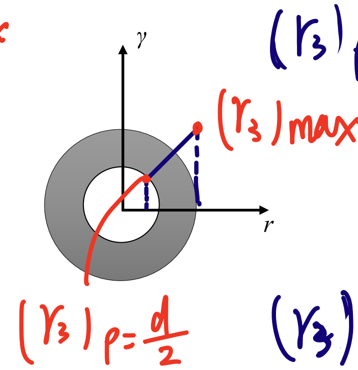
$$(\gamma_1)_{max} = \frac{(\tau_1)_{max}}{G_1} = \frac{2T_B}{45\pi G d^3}$$

$$(\gamma_2)_{\rho=\frac{d}{2}} = \frac{(\tau_2)_{\rho=d}}{G_2} = \frac{2T_B}{45\pi G d^3}$$

$$(\gamma_2)_{max} = \frac{(\tau_2)_{max}}{G} = \frac{4T_B}{45\pi G d^3}$$



Cross section of shafts 1&2



Cross section of shaft 3

$$(\gamma_3)_{\rho=\frac{d}{2}} = \frac{(\tau_3)_{\rho=\frac{d}{2}}}{G_3} = -\frac{2T_B}{45\pi G d^3}$$

$$(\gamma_3)_{max} = \frac{(\tau_3)_{max}}{G_3} = -\frac{4T_B}{45\pi G d^3}$$

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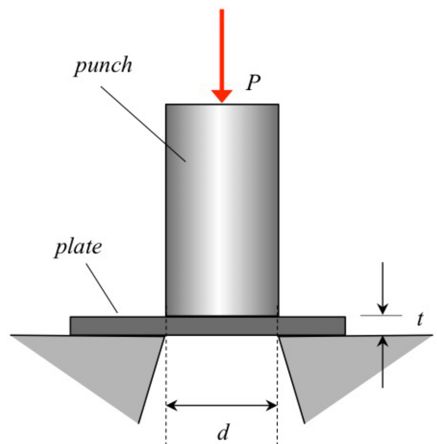
PROBLEM # 4 (25 points – No need to justify your answers, partial credit may not be granted)

PROBLEM # 4 – PART A (3 points)

A hydraulic punch of diameter d is used to punch circular holes in a plate of thickness t. Upon applying a punch force of P, what is the magnitude of the shear stress τ of the plate?



(a) $\boldsymbol{\tau = \frac{P}{\frac{\pi}{4}d^2}}$

(b) $\boldsymbol{\tau = \frac{P}{\pi d t}}$

(c) $\boldsymbol{\tau = \frac{P/2}{\pi d t}}$

(d) $\boldsymbol{\tau = \frac{P/2}{\frac{\pi}{4}d^2}}$

(e) None of the above

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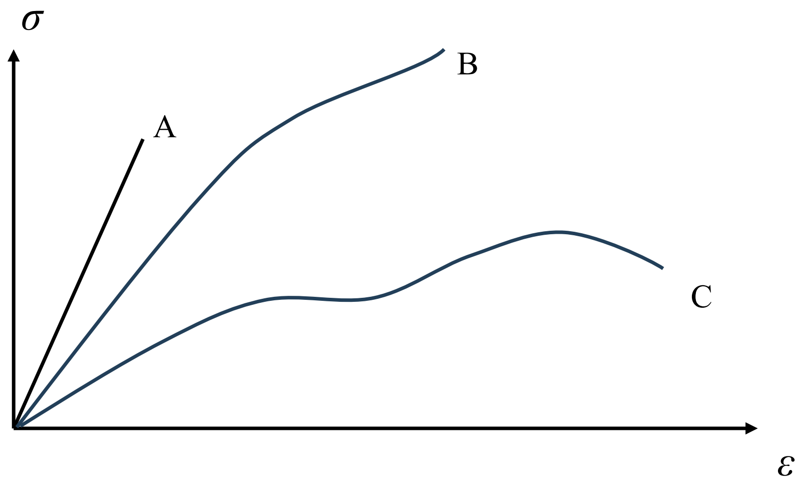
PROBLEM # 4 – PART B (3 points)

Three materials, A, B, and C are tested in the mechanics laboratory. Their stress-strain curves are shown below. Circle the correct answer for the following questions.



(a) Which material is strongest? **A** **B** **C**

(b) Which material is most ductile? **A** **B** **C**

(c) Which material is stiffest? **A** **B** **C**

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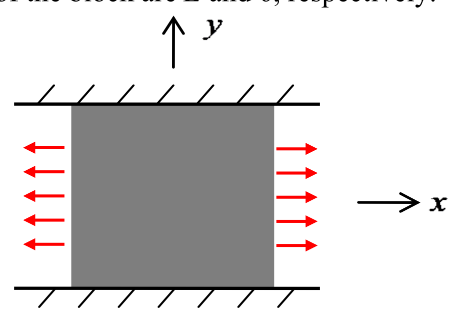
PROBLEM # 4 – PART C (3 points)

A block is fully constrained in the *y* direction and is free to expand in the *x* and *z* (out of paper) directions. The block is subject to an applied stress σ in the *x* direction. Which of the following statements about stresses and strains is correct? Young's modulus and Poisson's ratio of the block are *E* and υ, respectively.



(a) $\sigma_x = \sigma,\ \varepsilon_x = \sigma/E,\ \sigma_y = 0,\ \varepsilon_y = 0$

(b) $\sigma_x = -\sigma,\ \varepsilon_x = -\sigma/E,\ \sigma_y = 0,\ \varepsilon_y = \upsilon\varepsilon_x$

(c) $\sigma_x = \sigma,\ \varepsilon_x = (1-\upsilon^2)\sigma/E,\ \sigma_y = \upsilon\sigma,\ \varepsilon_y = 0$

(d) $\sigma_x = \sigma,\ \varepsilon_x = \frac{(\upsilon^2+1)\sigma}{E},\ \sigma_y = -\upsilon\sigma,\ \varepsilon_y = -\upsilon\varepsilon_x$

(e) None of the above

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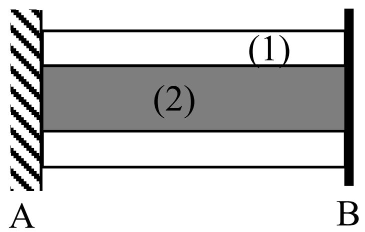
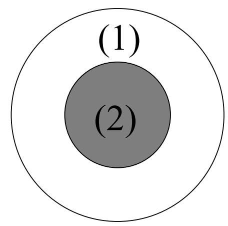
PROBLEM # 4 – PART D (7 points)

A circular bimetallic structure is composed of a tubular shell (1) and core (2). The structure is fixed to the wall at A and has a free end at B. The diameter of the shell (1) and core (2) is $2d$ and d, respectively. The Young's modulus of the shell and the core is the same. The coefficient of thermal expansion of the two members are α_1 and α_2, respectively, where $\alpha_1 > \alpha_2$. The structure is free of stress at the initial temperature T. Upon applying a uniform temperature increase ΔT to both the core and the shell, circle TRUE or FALSE for the following statements:





Cross section view

(d) **TRUE** or **FALSE**: The two members have the same internal force.

(e) **TRUE** or **FALSE**: The internal forces of the two members are both 0.

(f) **TRUE** or **FALSE**: The internal forces in the two members are of equal magnitude but different signs.

(g) **TRUE** or **FALSE**: The internal stresses in the two members are of equal magnitude but different signs.

(h) **TRUE** or **FALSE**: The strains in the two members are of different signs.

(i) **TRUE** or **FALSE**: The two members have the same elongation.

(j) **TRUE** or **FALSE**: The sum of elongations of the two members is zero.

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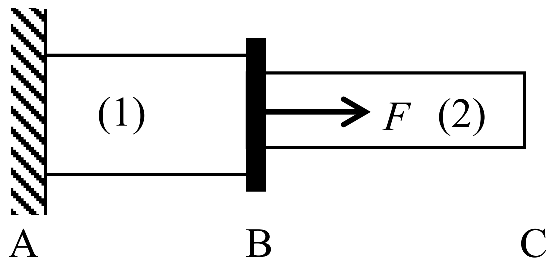
PROBLEM # 4 – PART E (6 points)

A stepped shaft is composed of two members (1) and (2). The two members are connected by a rigid connector at B on which an external force *F* is applied. The shaft is fixed to the wall at A and has a free end at C. Circle TRUE or FALSE for the following statements:



(a) **TRUE** or **FALSE**: The two members have the same internal force.

(b) **TRUE** or **FALSE**: The member (2) is free of stress.

(c) **TRUE** or **FALSE**: The internal forces in the two members are of equal magnitude but different signs.

(d) **TRUE** or **FALSE**: The shaft is a statically indeterminate structure.

(e) **TRUE** or **FALSE**: The two members have the same elongation.

(f) **TRUE** or **FALSE**: The sum of elongations of the two members is zero.

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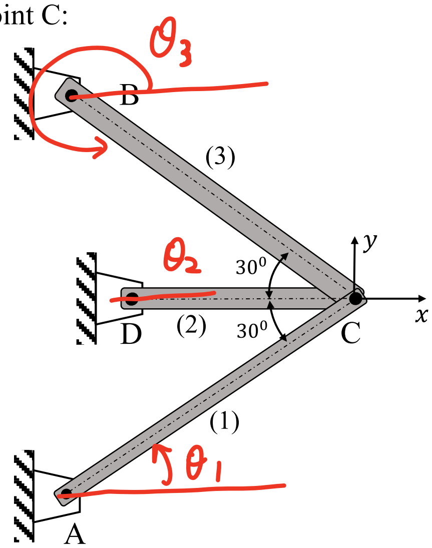
PROBLEM # 4 – PART F (3 points)

The geometry of deformation in planar truss is given by $e = u\cos\theta + v\sin\theta$, where e represents the elongation of a truss member, u and v are the displacement of the joint C in the x and y directions, respectively. Determine the θ value, in **radian**, for the members (1), (2), and (3) in the following truss for an arbitrary force applied at the joint C:



$\boldsymbol{\theta_1}$ = $\frac{\pi}{6}$

$\boldsymbol{\theta_2}$ = 0

$\boldsymbol{\theta_3}$ = $\frac{11\pi}{6}$