

Useful Equations

$$\sigma_{\text{ave}} = \frac{F_N}{A} \qquad FS = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow, member}}}$$

$$\tau_{\text{ave}} = \frac{V}{A} \qquad FS = \frac{\tau_{\text{fail}}}{\tau_{\text{allow, member}}}$$

Generalized Hooke's law:

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha \Delta T$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] + \alpha \Delta T$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha \Delta T$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy} \quad \gamma_{xz} = \frac{1}{G} \tau_{xz} \quad \gamma_{yz} = \frac{1}{G} \tau_{yz}$$

$$G = \frac{E}{2(1+\nu)}$$

Axial deformations:

$$e_{AB} = u_B - u_A$$

$$e = \int_0^L \frac{F}{AE} dx + \int_0^L \alpha \Delta T dx, \qquad e = \frac{FL}{AE} + \alpha \Delta T L$$

Planar trusses: $e = u \cos(\theta) + v \sin(\theta)$

Torsional deformations:

$$\phi_{AB} = \phi_B - \phi_A$$

$$\phi_{AB} = \int_0^L \frac{T_{AB}(x)}{G(x) I_p(x)} dx$$

$$\phi_{AB} = \frac{T_{AB} L}{G I_p}$$

$$\gamma = \rho \frac{d\phi}{dx} \qquad \tau = G \rho \frac{d\phi}{dx}$$

$$\gamma = \frac{\rho T}{G I_p} \qquad \tau = \frac{\rho T}{I_p}$$

$$\text{with} \quad I_p = \int_A \rho^2 dA, \quad I_p = \frac{\pi r^4}{2} \text{ (solid)}, \quad I_p = \frac{\pi}{2} (r_o^4 - r_i^4) \text{ (hollow)}$$

Bending deformation

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x) \qquad M = EI v'' \qquad \Delta V = P \qquad \Delta M = -M_0$$

$$\sigma(x, y) = \frac{-Ey}{\rho} = \frac{-M_{zz} y}{I_{zz}}$$

$$I_{zz} = \frac{bh^3}{12} \text{ (rectangle)}, \quad I_{zz} = \frac{\pi r^4}{4} \text{ (circle)}$$

$$\tau(x, y) = \frac{VQ}{I_{zz} t} = \frac{VA^* y^*}{I_{zz} t},$$

$$\tau_{\text{max}} = \frac{3V}{2A} \text{ (rectangle)}, \quad \tau_{\text{max}} = \frac{4V}{3A} \text{ (circle)}$$

Strain energy density:

$$\bar{u} = \frac{1}{2} [\sigma_x(\varepsilon_x - \alpha \Delta T) + \sigma_y(\varepsilon_y - \alpha \Delta T) + \sigma_z(\varepsilon_z - \alpha \Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods:

$$U = \frac{1}{2} \int_0^L \frac{F^2(x)}{EA} dx \quad U = \frac{1}{2} \int_0^L \frac{f_s V^2(x)}{GA} dx \quad U = \frac{1}{2} \int_0^L \frac{M^2(x)}{EI} dx \quad U = \frac{1}{2} \int_0^L \frac{T^2(x)}{GI_p} dx$$

Castigliano's 2nd theorem:

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \quad \theta_{M_i} = \frac{\partial U}{\partial M_i} \quad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

$$\delta_{P_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial P_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial P_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial P_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial P_i} dx$$

$$\theta_{M_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial M_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial M_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial M_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial M_i} dx$$

$$\phi_{T_i} = \int_0^L \frac{M(x)}{EI} \frac{\partial M(x)}{\partial T_i} dx + \int_0^L \frac{F(x)}{EA} \frac{\partial F(x)}{\partial T_i} dx + \int_0^L \frac{T(x)}{GI_p} \frac{\partial T(x)}{\partial T_i} dx + \int_0^L \frac{f_s V(x)}{AG} \frac{\partial V(x)}{\partial T_i} dx$$

$$f_s = 6/5 \text{ (rectangular cross section), } f_s = 10/9 \text{ (circular cross section)}$$