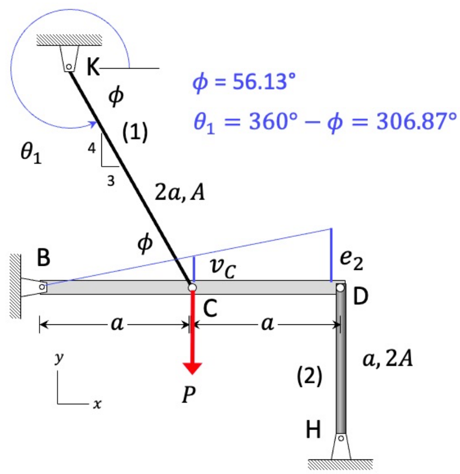
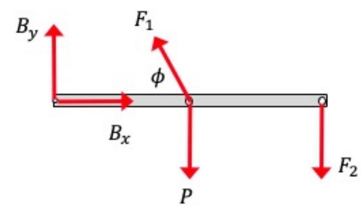
ME 323: Mechanics of Materials **Midterm exam - Summer 2024**

Problem No. 1 – 20 pts

Elements (1) and (2) have lengths of $2a$ and a, respectively, cross-sectional areas of A and $2A$, respectively, and identical Young's moduli E. The elements are pinned to the rigid bar BD as shown in the figure. A force P acts at pin C on BD. Determine the load (force) carried by elements (1) and (2). ***You are asked to clearly label the four steps in your solution.***





1. ***Equilibrium***

(1) $\Sigma M_B = -Pa - F_2(2a) + (F_1 sin\phi)a = 0 \;\Rightarrow\; \frac{4}{5}F_1 - 2F_2 = P$

2. ***Load/deformation***

(2) $e_1 = \frac{F_1(2a)}{EA}$

(3) $e_2 = \frac{F_2 a}{E(2A)}$

3. ***Compatibility***

$e_2 = 2v_C$; similar triangles

$e_1 = u_C cos\theta_1 + v_C sin\theta_1 = \frac{3}{5}u_C - \frac{4}{5}v_C$

$u_C = 0$

Therefore:

(4) $e_1 = -\frac{2}{5}e_2$

4. ***Solve***

Combining (2)-(4):

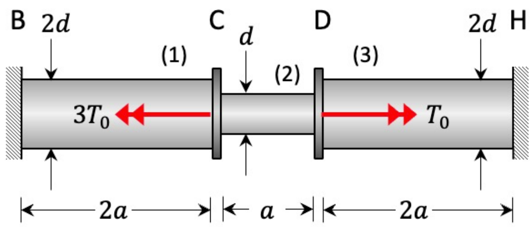
$$\frac{2F_1 a}{EA} = -\frac{2}{5}\left(\frac{F_2 a}{2EA}\right) \;\Rightarrow\; F_2 = -10F_1$$

Substituting into (1):

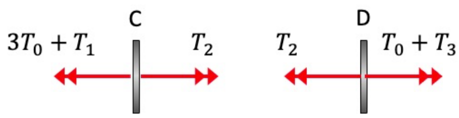
$$\frac{4}{5}F_1 - 2(-10F_1) = P \;\Rightarrow\; \boxed{F_1 = \frac{5}{104}P} \;\Rightarrow\; \boxed{F_2 = -\frac{50}{104}P}$$

Problem No. 2 – 20 pts

The shaft shown is made up of three elements having solid circular cross-sections, each of a material having a shear modulus of G. Determine the torque carried by each element. ***You are asked to clearly label the four steps in your solution.***



1. Equilibrium



(1) Connector C: $\Sigma M = -3T_0 - T_1 + T_2 = 0 \quad \Rightarrow \quad T_1 = T_2 - 3T_0$

(2) Connector D: $\Sigma M = T_0 - T_2 + T_3 = 0 \quad \Rightarrow \quad T_3 = T_2 - T_0$

2. Torque/rotation angle

(3) $\Delta\phi_1 = \frac{T_1(2a)}{GI_{P1}}$ where $I_{P1} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4$

(4) $\Delta\phi_2 = \frac{T_2(a)}{GI_{P2}}$ where $I_{P2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$

(5) $\Delta\phi_3 = \frac{T_3(2a)}{GI_{P3}}$ where $I_{P3} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4$

3. Compatibility

$\phi_C = \phi_B + \Delta\phi_1 = \Delta\phi_1$

$\phi_D = \phi_C + \Delta\phi_2 = \Delta\phi_1 + \Delta\phi_2$

(6) $\phi_H = \phi_D + \Delta\phi_3 = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3 = 0$

4. Solve

(3)-(6) $\Rightarrow \frac{4}{\pi}\frac{T_1 a}{Gd^4} + \frac{32}{\pi}\frac{T_2 a}{Gd^4} + \frac{4}{\pi}\frac{T_3 a}{Gd^4} = 0 \quad \Rightarrow \quad T_1 + 8T_2 + T_3 = 0$

Combining with equations (1) and (2):

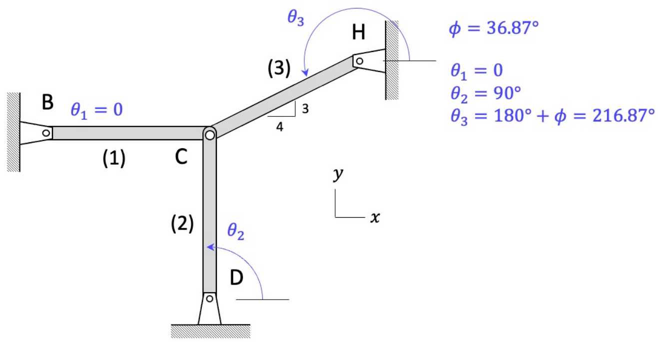
$T_2 - 3T_0 + 8T_2 + T_2 - T_0 = 0 \quad \Rightarrow \quad \boxed{T_2 = \frac{2}{5}T_0}$

Also:

$$\boxed{\begin{aligned} T_1 &= T_2 - 3T_0 = -\frac{13}{5}T_0 \\ T_3 &= T_2 - T_0 = -\frac{3}{5}T_0 \end{aligned}}$$

Problem No. 3 – 20 pts

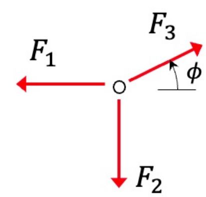
The truss shown is made up of three identical members, each has a Young's modulus of E, a cross-sectional area A, a length of a and thermal coefficient of expansion of α. The temperature of members (1) and (3) is each increased by an amount of ΔT, with the temperature of (2) remaining unchanged. Find the load (force) carried by each member, and state whether the member is in tension or compression. ***You are asked to clearly label the four steps in your solution. You must also clearly state the member angles for each member used in Step 3.***



1. ***Equilibrium***

   (1) $\Sigma F_x = F_3 cos\phi - F_1 = 0 \quad \Rightarrow \quad F_1 = \frac{4}{5}F_3$

   (2) $\Sigma F_y = F_3 sin\phi - F_2 = 0 \quad \Rightarrow \quad F_2 = \frac{3}{5}F_3$



2. ***Load/deformation***

   (3) $e_1 = \frac{F_1 a}{EA} + \alpha\Delta T a$

   (4) $e_2 = \frac{F_2 a}{EA}$

   (5) $e_3 = \frac{F_3 a}{EA} + \alpha\Delta T a$

3. ***Compatibility***

   (6) $e_1 = u_C cos\theta_1 + v_C sin\theta_1 = u_C$

   (7) $e_2 = u_C cos\theta_2 + v_C sin\theta_2 = v_C$

   (8) $e_3 = u_C cos\theta_3 + v_C sin\theta_3 = -\frac{4}{5}u_C - \frac{3}{5}v_C$

4. ***Solve***

   Using (3)-(8):

$$e_3 = -\frac{4}{5}e_1 - \frac{3}{5}e_2 \quad \Rightarrow \quad 5\left(\frac{F_3 a}{EA} + \alpha\Delta T a\right) = -4\left(\frac{F_1 a}{EA} + \alpha\Delta T a\right) - 3\left(\frac{F_2 a}{EA}\right) \quad \Rightarrow$$

$$4F_1 + 3F_2 + 5F_3 = -9\alpha\Delta T E A$$

   Combining with (1) and (2):

$$4\left(\frac{4}{5}F_3\right) + 3\left(\frac{3}{5}F_3\right) + 5F_3 = -9\alpha\Delta T E A \quad \Rightarrow \quad F_3 = -\frac{9}{10}\alpha\Delta T E A \text{ (compression)}$$

$$(1) \Rightarrow F_1 = -\frac{18}{25}\alpha\Delta T E A \text{ (compression)}$$

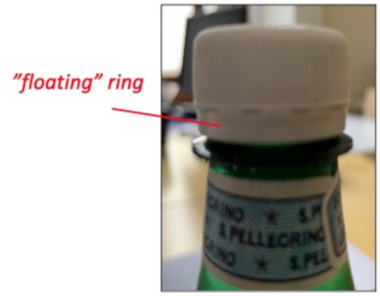
$$(2) \Rightarrow F_2 = -\frac{27}{50}\alpha\Delta T E A \text{ (compression)}$$

ME 323: Mechanics of Materials *Midterm exam - Summer 2024*

Problem No. 4

***PART 4.A** – 3 points*

Suppose you open a twist-off cap on a water bottle for which the cap has a floating ring. In opening the bottle, does the cap fail due to *normal stress* or *shear stress*? Provide a brief explanation.

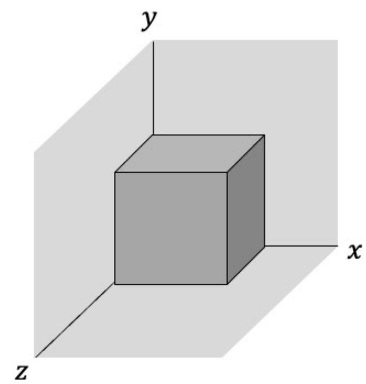


Before opening

As discussed in lecture, twisting on the cap produce a thrust force on the cap from the bottle threads. This produces normal stresses in the tabs connecting the cap to the floating ring.

***PART 4.B** – 3 points*

A uniform block made up of a material with a Young's modulus E and Poisson's ratio rests in a rigid, smooth corner. Uniform *compressive* stresses of σ_0 and $3\sigma_0$ act on the xz-faces of the block, respectively, with the y-face left stress-free. Determine the *three components of strain* experienced by the block. Consider the weight of the block to be negligible.



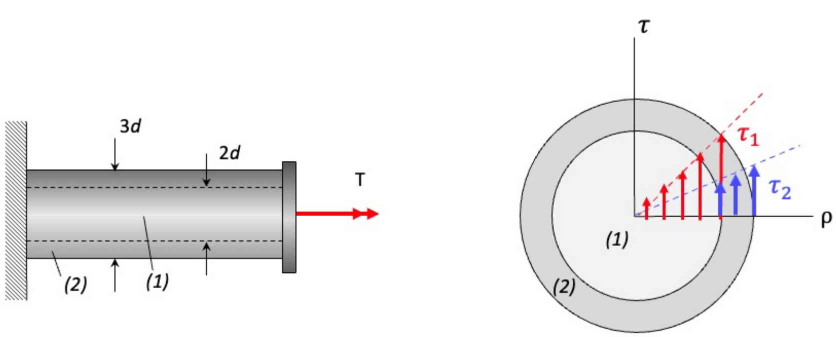
$$\epsilon_x = \frac{1}{E}[-\sigma_0 - \nu(0 - 3\sigma_0)] = \frac{\sigma_0}{E}(-1 + 3\nu)$$

$$\epsilon_y = \frac{1}{E}[0 - \nu(-\sigma_0 - 3\sigma_0)] = \frac{\sigma_0}{E}(4\nu)$$

$$\epsilon_z = \frac{1}{E}[-3\sigma_0 - \nu(-\sigma_0 + 0)] = \frac{\sigma_0}{E}(-3 + \nu)$$

***PART 4.C** – 4 points*

A composite shaft is made up of an outer shell (2) and an inner core (1). The shear moduli for (1) and (2) are known to be $2G$ and G, respectively. On the figure of the cross-section below, make a sketch of the distribution of shear stress τ vs. the radial position ρ on the cross-section of the composite shaft. Show the location of the maximum shear stress on the cross-section.



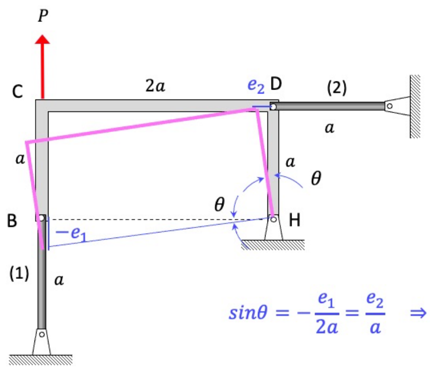
If the shear strain is written as: $\gamma = m\rho$ then: $\tau_1 = 2mG\rho$ and $\tau_2 = mG\rho$.
Therefore, $\tau_{1,max} = 2mGd$ and $\tau_{2,max} = 3mG/2$.

ME 323: Mechanics of Materials   ***Midterm exam - Summer 2024***

Problem No. 4 *(continued)*

PART 4.D – *4 points*

Rigid frame member BCDH is supported by elastic members (1) and (2). A force P acts at location C on frame member BCDH. Let e_1 and e_2 represent the elongation of members (1) and (2), respectively. Determine the compatibility equation relating e_1 and e_2.



$$\sin\theta = -\frac{e_1}{2a} = \frac{e_2}{a} \Rightarrow \boxed{e_1 = -2e_2}$$

PART 4.E – *6 points*

Draw the shear force/bending moment diagrams for the simply-supported beam shown below.

$\Sigma M_D = -B_y(2a) + p_0 a(a/2) - p_0 a(3a/2) = 0 \;\;\Rightarrow\;\; B_y = -p_0 a\,/\,2$

$\Sigma F_y = B_y + D_y + p_0 a - p_0 a = 0 \;\;\Rightarrow\;\; D_y = -B_y = p_0 a\,/\,2$

$V(a) = V(0) + p_0 a = -p_0 a/2 + p_0 a = p_0 a/2$

$V(2a) = V(a) - p_0 a = p_0 a/2 - p_0 a = -p_0 a\,/\,2$

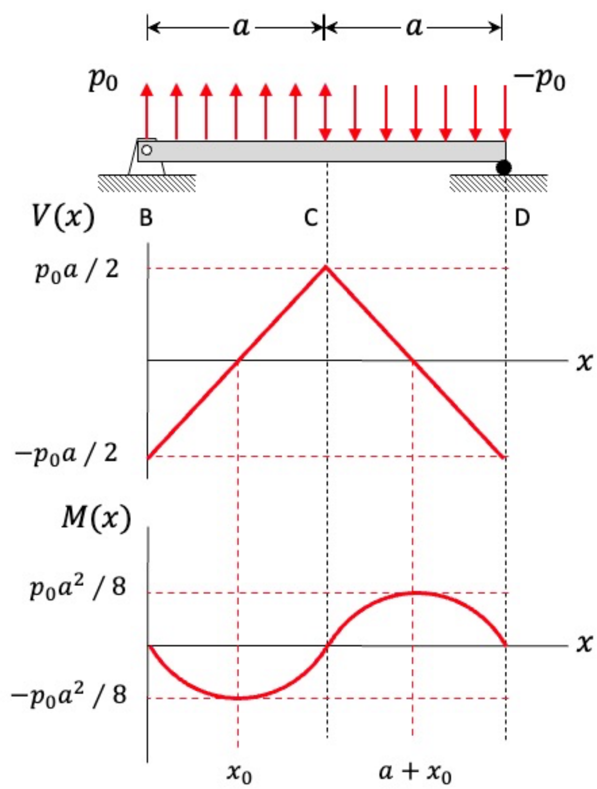
$x_0 = a\,/\,2$

$M(x_0) = M(0) - (p_0 a\,/\,2)x_0\,/\,2 = -p_0 a^2\,/\,8$

$M(a) = M(x_0) + p_0 a^2\,/\,8 = 0$

$M(a + x_0) = M(a) + p_0 a^2\,/\,8 = p_0 a^2\,/\,8$

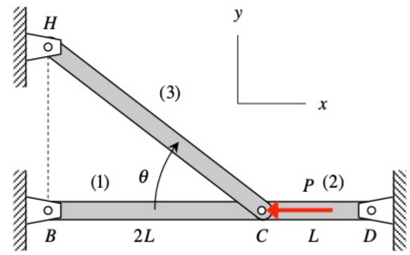
$M(2a) = M(a) - p_0 a^2\,/\,8 = 0$



Problem No. 5

***PART 5.A** – 4 points*

The truss shown is made up of truss elements (1), (2) and (3). A horizontal force P is applied to Joint C. Let σ_3 represent the stress developed in element (3). Circle the correct response. Provide a short explanation.

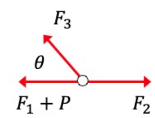


a) $\sigma_3 > 0$ (tension)

b) $\sigma_3 = 0$

c) $\sigma_3 < 0$ (compression)

d) More information is needed in order to answer this question.

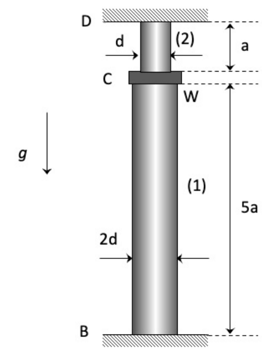


$\Sigma F_y = F_3 sin\theta = 0 \;\Rightarrow\; F_3 = 0$

$\Rightarrow\; \sigma_3 = 0$

***PART 5.B** – 4 points*

Rod elements (1) and (2) have circular solid cross-sections, and are made of the same material having a Young's modulus of E. Rigid connector C has a weight of W, whereas the weights of the rod elements are negligible. Let F_1 and F_2 be the axial loads carried by (1) and (2), respectively. Choose the correct response below. Provide a short explanation.



$$|e_1| = \frac{|F_1|(5a)}{E\pi(2d/2)^2} = \frac{5}{\pi}\frac{|F_1|a}{Ed^2}$$

$$|e_2| = \frac{|F_2|(a)}{E\pi(d/2)^2} = \frac{4}{\pi}\frac{|F_2|a}{Ed^2}$$

$$|e_1| = |e_2| \;\Rightarrow\; |F_1| = \frac{4}{5}|F_2|$$

a) $|F_1| > |F_2|$

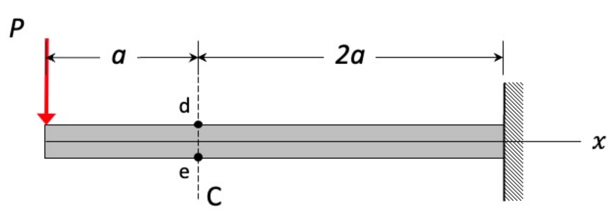
b) $|F_1| = |F_2|$

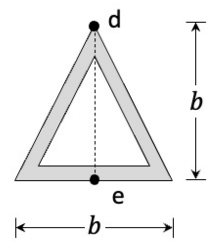
c) $|F_1| < |F_2|$

d) More information is needed in order to answer this question.

***PART 5.C** – 4 points*

Consider the end-loaded cantilevered beam shown that has a triangular-shaped tubular cross-section of uniform wall thickness. Let σ_d and σ_e be the *normal stresses* at points "d" and "e" on the cross-section at C, respectively. Choose the correct response below. Provide a short justification.





beam cross-section

a) $|\sigma_d| > |\sigma_e|$

b) $|\sigma_d| = |\sigma_e|$

c) $|\sigma_d| < |\sigma_e|$

d) More information is needed in order to answer this question.

"d" is further away from the neutral axis (centroid) than is "e". Therefore, the magnitude of the normal stress at "d" is greater.

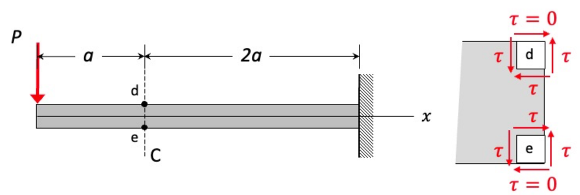
Problem No. 5 *(continued)*

***PART 5.D** – 4 points*

Same cantilevered beam as Part 5.C, except here the shape of the beam cross-section is *unknown*. Choose the correct *TRUE/FALSE* response below. Provide a short justification.

TRUE *FALSE*: *Shear stress* at "d" and "e" is zero, for any cross-section shape.

HINT: Consider the stress elements at "d" and "e".



Since top and bottom surfaces are free from shear stress, no shears stress exists at points "d" and "e".

***PART 5.E** – 4 points*

Again, the same cantilevered beam as Part 5.C, except here the beam has the rectangular cross-section shown. Let τ_p and τ_q represent the shear stress on the beam cross-section at points "p" and "q". Choose the correct response below. Provide a short justification.

a) $|\tau_p| > |\tau_q|$

b) $|\tau_p| = |\tau_q|$

c) $|\tau_p| < |\tau_q|$

d) More information is needed in order to answer this question.

Here the maximum shear stress occurs at the neutral axis. Point "p" is closer to the neutral axis than is "q"; therefore, its shear stress has a larger magnitude.