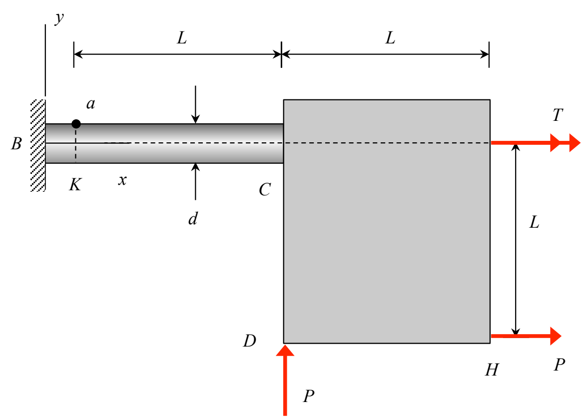
ME 323: Mechanics of Materials — *Homework Sets H31*

Summer 2024 — *Assigned/Due: July 24/July 26*

A circular shaft is mounted on a fixed wall at end B. A rigid plate is welded to the shaft at end C. Two loads, both of magnitude P, act on the plate at locations D and H, as shown. A torque T also acts on the plate, as shown in the figure. Ignore the weight of the shaft and plate. It is desired to know the maximum shear stress at point "a" on the cross-section K of the shaft. To this end:



a) Determine the resultants acting on the left side of the cut at the cross-section K of the shaft. Show these on the figure provided below.

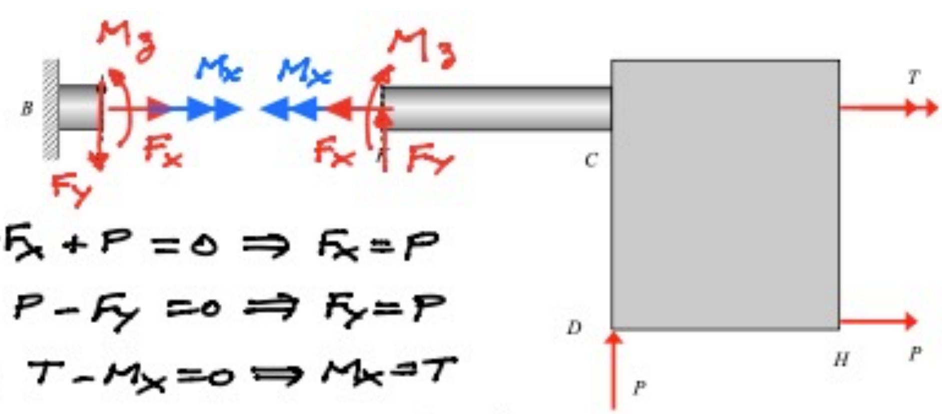
b) Show the different components of stress at point "a"on the cross-section K. Use the figures provided on the next page.

c) Make of list of the components of stress from b), and provide the equations for these components as related to the resultants found in (a). Use the table provided on the next page.

d) Show the stress components on the stress element provided on the next page.

e) Determine the maximum shear stress at point "a" corresponding to $T = 3PL$. Express your answer in terms of, at most, P, L and d.

f) If the material making up the shaft has a yield strength of σ_Y, what is the factor of safety against failure of the shaft at location "a" if the maximum shear stress theory of failure is used?



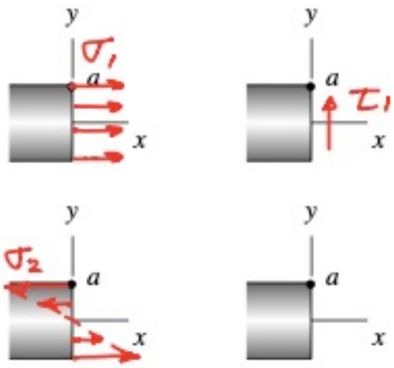
$$\Sigma F_x = -F_x + P = 0 \Rightarrow F_x = P$$

$$\Sigma F_y = P - F_y = 0 \Rightarrow F_y = P$$

$$\Sigma M_x = T - M_x = 0 \Rightarrow M_x = T$$

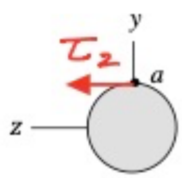
$$\Sigma M_z = -M_z + PL + PL = 0 \Rightarrow M_z = 2PL$$

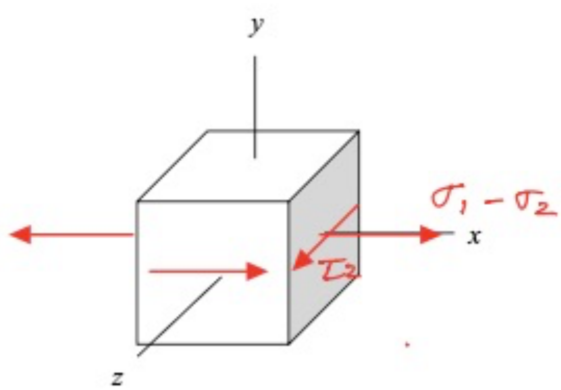
side view



resultant	stress at "a"
F_x	$\sigma_1 = F_x/A$
F_y	$\tau_1 = 0$
M_x	$\tau_2 = \frac{M_x\left(\frac{d}{2}\right)}{I_p}$
M_z	$\sigma_2 = \frac{M_z \frac{d}{2}}{I}$

end view





$$\sigma_x = \sigma_1 - \sigma_2$$
$$= \frac{P}{A} - \frac{PLd}{I}$$
$$\tau_{xz} = \tau_2 = \frac{Td}{2I_p} = \frac{3}{2}\frac{PLd}{I_p}$$
$$\therefore |\tau|_{max,abs} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2}$$
$$= P\sqrt{\frac{1}{4}\left(\frac{1}{A} - \frac{Ld}{I}\right)^2 + \left(\frac{3}{2}\frac{Ld}{I_p}\right)^2}$$

w/ $A = \pi(d/2)^2$

$I = \frac{\pi}{4}(d/2)^4$

$I_p = \frac{\pi}{2}(d/2)^4$